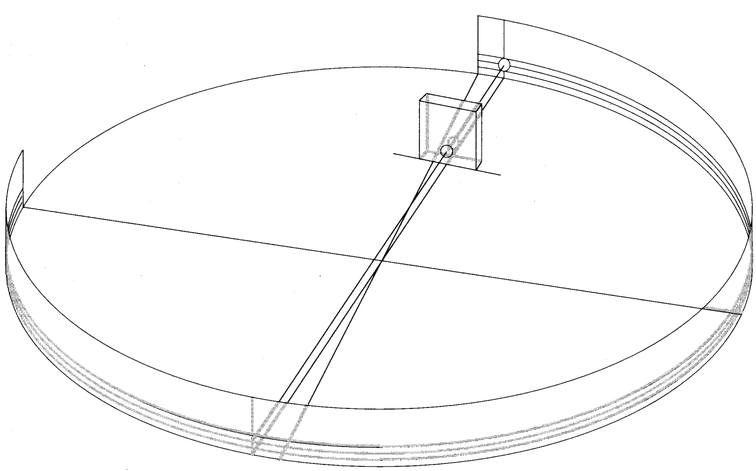
ALHACEN ON REFRACTION

A Critical Edition, with English Translation and Commentary, of Book 7 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*

Volume Two

English Translation



A. Mark Smith

American Philosophical Society

Transactions
of the
American Philosophical Society
Held at Philadelphia
For Promoting Useful Knowledge
Volume 100 Part 3, Sections 1 & 2

ALHACEN ON REFRACTION

A Critical Edition, with English Translation
and Commentary, of Book 7
of Alhacen's *De aspectibus*

VOLUME ONE
Introduction and Latin Text

VOLUME TWO
English Translation

A. Mark Smith

American Philosophical Society
Independence Square • Philadelphia
2010

ISBN: 978-1-60618-006-8
US ISSN: 0065-9746

Library of Congress Cataloging-in-Publication Data

Alhazen, 965-1039.
[Manazir Book 7. English & Latin]
Alhacen on refraction: a critical edition, with English translation and commentary, of book 7 of Alhacen's De aspectibus. / [edited by] A. Mark Smith.
p. cm. — (Transactions of the American Philosophical Society, v. 100, pt. 3)
Includes bibliographical references (p.) and indexes.
Contents: v. 1. Introduction and Latin text — v. 2. English translation and commentary.
ISBN v. 1 978-1-60618-003-7 (pbk.)   ISBN v. 2 978-1-60618-006-8 (pbk.)
1. Optics—Early works to 1800. I. Smith, A. Mark, II. Title. III. Transactions of the American Philosophical Society; v. 100 pt. 3.
QC353
[.A32313 2001]
535'.09'021—dc21 2001041227

TRANSACTIONS

of the

American Philosophical Society

Held at Philadelphia for Promoting Useful Knowledge

VOLUME 100, Part 3, Section 2

Alhacen on Refraction

A Critical Edition, with English Translation
and Commentary, of Book 7
of Alhacen's *De aspectibus*

VOLUME TWO

CONTENTS

VOLUME II

A. Mark Smith is Professor of History at the University of Missouri—Columbia. He teaches courses in Medieval History as well as the History of Science from antiquity to the late Enlightenment. Previous publications with the American Philosophical Society include all previous volumes of the Alhacen series *(Alhacen's Theory of Visual Perception*, 2001; *Alhacen on the Principles of Reflection*, 2006; *Alhacen on Image-Formation and Distortion in Mirrors, 2008)*, *Descartes's Theory of Light and Refraction* (1987), *Ptolemy's Theory of Visual Perception* (1996), and *Ptolemy and the Foundations of Ancient Mathematical Optics* (1999).

BOOK SEVEN OF ALHACEN'S *DE ASPECTIBUS*

Topical Synopsis

[1.1-2] The agenda for book 7: Explaining refracted vision and the misperceptions arising from it.

[2.1-15] Construction of experimental apparatus to test refraction of light from air to water, air to glass, glass to air, and glass to water. **[2.16-34]** Experimental test of refraction from air to water to determine: that light passes in a straight line through both media, that refraction occurs in a single plane perpendicular to the water's surface, that light is refracted toward the normal dropped from the point of refraction when it enters the water, and that light striking the water's surface orthogonally is not refracted. **[2.35-46]** Experiment with a succession of glass cubes forming a rod to show that, when it strikes the exposed face of the rod at one end orthogonally, light radiates straight through the entire rod as well as through any combination of cubes comprising it. **[2.47-49]** Formation of glass quarter-sphere for testing refraction into and out of glass. **[2.50-58]** Experiment showing that, when light strikes either the convex or flat face of the glass quarter-sphere orthogonally, it passes straight through the glass and continues unrefracted into air or water. **[2.59-78]** Experimental test of refraction from air to glass, glass to air, and glass to water to determine: that light refracts toward the normal when it passes from air to glass, that light refracts away from the normal when it passes from glass to air, that light refracts away from the normal when it passes from glass to water, and that all such refraction occurs within a single plane perpendicular to the refractive interface. **[2.79-80]** Gen-

intersecting lines, and if the two chords intersect outside the circle, the angle of intersection is equal to the angle at the circumference subtended by an arc equal to the difference between the two arcs cut off by those lines on the circle. **[5.50-61]** PROPOSITION 6: If an object point lies inside a sphere, if the center of sight lies outside the sphere and on the same normal as the object point, and if the sphere consists of a denser medium, the object point will be seen along the normal and only along the normal; moreover, if the object point and center of sight do not lie on the same normal, only one refracted image will be seen. **[5.62-64]** PROPOSITION 7: In the previous case, image location will depend upon where the object point lies in the sphere, so the image may be located in front of the center of sight, behind it, at it, or at the point of refraction. **[5.65-66]** PROPOSITION 8: If the sphere consists of a denser medium, then any object point in the rare medium outside it will have a single image for any center of sight inside it. **[5.67-72]** PROPOSITION 9: If a center of sight in a rarer medium faces a sphere consisting of a denser medium, if the area beyond the surface of the sphere on the other side of the center of curvature from the eye is filled with a medium of equal density to that within the sphere, and if there is an object line in that area coincident with the normal dropped from the center of sight through the sphere, that line will form a circular image on the convex surface of the sphere facing the center of sight. **[5.73-75]** PROPOSITION 10: If an object point lies in a denser medium and the center of sight in a rarer one, if the refracting surface facing the center of sight is concave spherical, and if the object point and center of sight lie on the same normal to that surface, the object point will be seen along the normal and only along the normal. **[5.76-88]** PROPOSITION 11: If an object point lies in a denser medium and the center of sight in a rarer one, if the refracting surface facing the center of sight is concave spherical, and if the object point and center of sight do not lie on the same normal to that surface, the object point will have only one refracted image. **[5.89-90]** Concluding points.

[6.1-6] Everything seen through a refractive interface is seen by means of a cone of refraction with its base on the refractive interface and its vertex at the center of sight. **[6.7]** All the normals dropped from points on an object to a refractive interface form a body of sorts,

such that the intersection of this body and the cone of refraction constitutes the image of that object for the center of sight at the cone of refraction's vertex. **[6.8-9]** Implications of this analysis for image formation in refraction from plane, spherical convex, and spherical concave refracting interfaces. **[6.10-16]** Explanation for why an object seen by means of refraction is seen as single in binocular vision. **[6.17-24]** Every point in the field of vision is seen by means not only of the orthogonal rays, or radial lines, within the cone of radiation with its vertex at the center of sight, but also of rays refracted through the transparent tunics and humors of the eye; forms refracted in the eye along those rays are seen by means of virtual radial lines. **[6.25-26]** Empirical demonstration on the basis of peripheral vision that objects outside the cone of radiation are seen by means of refracted rays. **[6.27-32]** Empirical demonstration on the basis of a needle held right in front of the eye that objects within the cone of radiation are also seen by means of refracted rays. **[6.33]** Visual clarity within the cone of radiation is due to the reinforcement of orthogonal impressions on the anterior surface of the lens by oblique ones.

[7.1-4] Images seen by means of refraction are subject to many of the misperceptions that occur in reflected vision as well as in direct vision; one of the key misperceptions arising from refraction is that objects can look magnified or diminished in size. **[7.5-7.12]** PROPOSITION 12: Object lines in a denser medium seen through a plane refractive interface by a center of sight in a rarer medium appear magnified when they lie in the plane of the normal dropped from the center of sight to the refractive interface. **[7.13-18]** PROPOSITION 13: Object lines in a denser medium seen through a plane refractive interface by a center of sight in a rarer medium appear magnified when they lie outside the plane of the normal dropped from the center of sight to the refractive interface. **[7.19-23]** PROPOSITION 14: Every segment of an object line seen in a denser medium appears magnified when the refractive interface is plane. **[7.24-26]** PROPOSITION 15: Object lines in a denser medium seen through a convex spherical refractive interface by a center of sight in a rarer medium appear magnified when they lie in the plane of the normal dropped from the center of sight to the refracting interface and when they are located between the center of curvature and the refracting interface. **[7.27-30]** PROPOSITION 16: Object lines in a denser medium seen

through a convex spherical refractive interface by a center of sight in a rarer medium appear magnified when they lie outside the plane of the normal dropped from the center of sight to the refractive interface and when they are located between the center of curvature and the refractive interface; moreover, the apparent magnification applies if the object lines are slanted; segments of those lines are also magnified. **[7.31-32]** Experiment to show that things actually do look magnified when seen in water. **[7.33-34]** Preliminary points leading to the next proposition. **[7.35-38]** PROPOSITION 17: If the center of sight and an object line lie within the same medium on either side of a sphere consisting of a denser medium, and if both the center of sight and the object line lie on a normal to the sphere's surface, the object line will form an annular image on the surface of the sphere facing the center of sight. **[7.39-40]** Empirical verification of proposition 17. **[7.41]** PROPOSITION 18: If the center of sight and an object line lie within the same medium on either side of a cylinder consisting of a denser medium, and if both the center of sight and the object line lie on a normal to the cylinder's surface, the object line will appear as two separate lines on the surface of the cylinder facing the center of sight. **[7.42-46]** Preliminary observations to introduce the analysis of how atmospheric refraction and location with respect to the horizon affect the apparent size of celestial bodies. **[7.47-51]** PROPOSITION 19: When viewed at zenith, a celestial body or an interval between celestial bodies appears diminished because of atmospheric refraction. **[7.52-56]** PROPOSITION 20: When viewed between zenith and horizon, a celestial body or an interval between celestial bodies appears diminished along its horizontal cross-section because of atmospheric refraction. **[7.57-61]** PROPOSITION 21: When viewed between zenith and horizon, a celestial body or an interval between celestial bodies appears diminished along its vertical cross-section because of atmospheric refraction. **[7.62-71]** Explanation of the Moon Illusion. **[7.72-75]** Thick vapor at the horizon will also magnify celestial bodies, but this magnification is secondary in causing the Moon Illusion.

BOOK SEVEN

Of the treatise *De aspectibus* by Alhacen,
son of Alhaycen

There are seven chapters [in this book]. The first chapter [consists] of an introduction; the second [establishes] that light passes through transparent bodies along straight lines and is refracted[1] when it encounters a body whose transparency is different from the transparency of the body in which it [originally] lies;[2] the third [deals with] how light is refracted in transparent bodies; the fourth chapter [shows] that whatever is perceived by the visual faculty through transparent bodies whose transparency differs from the transparency of the body in which the visual faculty lies is perceived by means of refraction when [the visual faculty] lies to the side of the normals dropped [from the object's surface] to the interface between the two transparent bodies; the fifth [deals] with the images [yielded by refraction];[3] the sixth [explains] how the visual faculty perceives visible objects[4] by means of refraction; [and] the seventh [is concerned] with the visual misperceptions that arise from refraction.

CHAPTER ONE

[1.1] It was pointed out in the introduction to the fourth book of this treatise [in Smith, *Alhacen on the Principles*, p. 295] that the visual faculty perceives visible objects in three ways—i.e., directly, or by reflection from polished bodies, or by refraction through transparent bodies that differ in transparency from the transparency of the air [in which the eye lies]—and [it was pointed out] that the visual faculty perceives nothing about visible objects except in one of these three ways; and [it was also pointed out] that in any one or all of these [three] ways the visual faculty perceives visible objects [as a whole] as well as all the properties of visible objects that were examined in detail in the last chapter of the second book.[5]

[1.2] In the previous books, moreover, it was shown how the visual faculty perceives visible objects directly and by means of reflection, and

we explained the various ways in which the visual faculty perceives visible objects according to each of those modes. It therefore remains to show how the visual faculty perceives visible objects by means of refraction through transparent bodies. And [so] in this book we will deal only with refraction; and we will clarify the formal nature of refraction, distinguish its modes, parse its specific characteristics, and explain how misperception arises in the visual faculty in this sort of vision. But first we will lay some foundations that confirm what will be entailed in this study.

CHAPTER TWO

That light passes through transparent bodies, extends through them along straight lines, and is refracted when it strikes a transparent body whose transparency differs from the transparency of the body in which [the light source] lies

[2.1] That light in fact passes through air and extends [through it] along straight lines was demonstrated in the first book of this treatise.[6] Air, however, is [only] one among [several] transparent bodies; light also passes through water, glass, and transparent stones, and it extends [through them] in straight lines. Indeed, this is observable by means of experiment.

[2.2] Accordingly, if one wants to conduct [such] an experiment, he will take a round plate of bronze [henceforth referred to as the "register plate"], whose diameter is no less than a cubit, and it should be fairly thick [so as to remain rigid]. It should also have a round rim [attached] upright to its surface [around its perimeter], and its rim should be no less than two digits high [as illustrated in the bottom diagram of figure 7.2.1, p. 147].[7] A small, round, cylindrical body no less than three digits long should be [attached] to the back side of the register plate at its center, and it should stand upright on the register plate's surface [as illustrated in figure 7.2.2, p. 148]. We should insert this [entire] apparatus in a lathe, whose turning mechanism [is designed to] round off copper implements, and we should place one of the teeth of the turning mechanism at the center of the register plate [on its inner surface] and the other at the center of the outer end of the body attached to the back of the register plate. We should then even the [entire] apparatus out by turning it with a precise grinding until its rim is properly rounded both inside and out and the inner and outer surfaces [of the register plate] are smoothed out to form two parallel surfaces. We will also grind the body [attached] at the back [of the register plate] until it becomes [appropriately] round.

[2.3] When this apparatus is completely finished by grinding and polishing, we should mark off two diameters intersecting one another orthogonally on the inside surface [of the register plate] and therefore passing through its center [as represented by FG and BE in the top diagram of figure 7.2.1, p. 147]. We should subsequently mark a point at the base of the [inside wall of the] apparatus's rim one digit to the side of the endpoint of either of the two intersecting diameters. Next we should draw a third diameter from this point passing through the center of the register plate and extending along its entire surface [as represented by AD in the top diagram of figure 7.2.1]. From the two endpoints of this [third] diameter we will draw two lines on the [inner] surface of the apparatus's rim perpendicular to the surface of the register plate [i.e., FK and GL in figure 7.2.1]. On either of these two [perpendicular] lines, finally, we will mark off three short lines of the same length [end to end], the first of which will abut the surface of the register plate, and each of them should be half a grain of barley long.[8] Three points will therefore be set on the perpendicular line, [and] these [points] constitute the endpoints of those [three short] lines.

[2.4] We should then reinsert this apparatus into the lathe, and we should mark off three parallel circles in it [on the inner wall of the rim] passing through the three [previously set] points that lie on the line perpendicular to the endpoint of the diameter [on the register plate's face, as illustrated in the bottom diagram of figure 7.2.1]. The opposite line [on the inner wall of the ring] that is perpendicular to the other endpoint of this diameter should therefore be cut by those three circles, and they will form three points on that same [perpendicular]. In each of the three circles, moreover, two opposed points will be marked out at the endpoints of a diameter among [all] the diameters of that circle.

[2.5] We should then divide the middle of the three circles into 360 parts and into minutes, if that is possible. Next we should bore a round hole in the rim of the apparatus with its center being the middle point of the three points that lie on either of the two lines that are perpendicular to the endpoint of the plate's diameter, and its radius should be the distance between the circles [i.e., half a grain of barley]. Hence, the perimeter of the hole will extend between the two outer parallel circles [that flank the middle one passing through the hole's center].[9]

[2.6] Afterwards we will take a small, moderately thick, square panel, whose length is equal to the height of the apparatus's rim and whose width is around the same as that. Its surface should be ground as flat as it can be, and [the bottom surface along] its depth at either of its ends should be ground flat until the common section [that forms the edge] between the surface of its face and the [bottom] surface of its depth will form a straight

line; we should bisect that line, and from the midpoint [so determined] we should draw a straight line on its face perpendicular to that straight line forming the common section [of the face and the bottom surface].

[2.7] From the endpoint of that perpendicular line on the side of the common section [at the bottom] we should mark off three lines equal to one another and equal to each of the short lines that were marked off on the perpendicular line on the [inner wall of the] register plate's rim [at the endpoint of the third diameter]. Three points will thus be marked on the perpendicular line [drawn] on the face of the panel. Then we will drill a round hole through the short line [bounded by the three points just marked on the face of the panel] with its center at the middle point of the three points marked off by the [three short] lines [drawn] on it, and its radius should be equal to [the length] of any of the [three] short lines [i.e., half a grain of barley]. This hole will therefore be the same size as the hole in the rim of the apparatus.[10]

[2.8] On the diameter of the register plate at whose endpoints the two perpendicular lines [on the rim] lie we will then mark a point at the center of the line [forming the radius] between the register plate's center and the endpoint of the diameter on the side of the hole [in the rim], and we should pass a line through this point perpendicular to the diameter. We should next place the base of the small panel on this line such that the common section [of the face and the bottom] of the small panel is flush with this line [drawn] perpendicular to the diameter and [so that] the point that bisects the common section of the small panel will coincide with the [mid]point marked on the register plate's diameter.

[2.9] When all this is done, the small panel should be firmly and immovably attached [to the surface of the register plate]. Accordingly, the hole in the small panel will face the hole in the rim of the apparatus directly, the imaginary straight line that connects the centers of the two holes will lie in the plane of the middle of the three circles that are [inscribed] on the inner wall of the apparatus's rim and will be parallel to the diameter on the register plate, and the small panel that will be attached at [that] point will be like the alidade of an astrolabe.[11]

[2.10] When this [step] is complete, the quadrant on the apparatus's rim next to the quadrant in which the hole [in the rim] lies should be excised from among the four quadrants defined by the two initial diameters that intersect orthogonally, that quadrant being next to the turning mechanism outside of which the rim lies, and the [bottom] edge along which the cut is made should be smoothed until it it is flush with the surface of the register plate.[12]

[2.11] We should then take a bronze strip that is not less than but more than a cubit long [i.e., longer than the diameter of the register plate], it should be rectangular in shape so that the four flat surfaces [along its length] form a square two digits [on each] side [at the two ends], and its surfaces should be planed down as much as possible so as to be flat and form right angles. A round hole should then be drilled through the midpoint of one of its [lengthwise] surfaces, it should be just large enough to accommodate the [cylindrical] body that is [attached] at the back of the apparatus such that [the apparatus] may be turned in it not loosely but tightly, and the hole should be perpendicular to the surface of the strip and should pass through the strip to the other side.

[2.12] We should then attach the apparatus to the strip and insert the [cylindrical] body at the back of the apparatus into the hole in the middle of the strip until the [back] surface of the apparatus nests against the surface of the strip. Once this is done, the excess at the ends of the strip along the diameter of the register plate should be cut off, given that the strip is longer than the diameter of the plate because we constructed it that way. Thus, when we cut the two excess pieces from the two ends of the strip, we will remove these two excess pieces, and we will put them upon the two ends of the strip so that we will place the two ends of the excess pieces on the two ends of what remains of the strip. We will then attach the surface of the ends in the plane of the back of the apparatus, and the portion of the two excess pieces that will be attached to the remainder of the strip will amount to one digit [of overlap]. When this arrangement is in place, the two excess pieces will protrude beyond the two ends of [the long, central portion of] the strip [inserted into the axle], and it will be preferable if the overlapping portion of the excess piece is perforated and a copper pin inserted into the hole so as to keep it from sliding off. When this is done, the apparatus will be finished, and this is how the back of the apparatus takes form.[13]

[2.13] The experimenter should then take a thin copper ruler whose width should be twice the diameter of the hole in the rim of the apparatus [i.e., two grains of barley], whose thickness should be equal to the diameter of the hole [i.e., one grain of barley], and whose length should be no less than half a cubit. That ruler will be evened out [on all four of its faces] until it is perfectly straight and true and its surfaces are rendered plane and parallel. Then we will cut it slantwise at one of its ends along the width until the edge along [one of] its lengths forms an acute angle with its [newly cut] edge along the width so that a person can incline it and move it however he wishes [by pivoting it about its sharpened edge], and [the experimeter] will set its width at the other end perpendicular to its longitudinal edge. We will then bisect this widthwise edge, and from the point of bisection we

will draw a line on the face of the ruler that extends its [entire] length, and [this line] will be perpendicular to the widthwise edge.[14]

[2.14] Hence, when this ruler is applied [with its back face] on the surface of the register plate, its top face will lie in the plane of the middle circle of the three circles inscribed on the inner wall of the apparatus's rim, for the thickness of this ruler is equal to the diameter of the hole [i.e., one grain of barley], and the diameter of the hole is equal to the perpendicular dropped from the center of the hole in [the inner wall of] the apparatus's rim to the register plate's surface, since the diameter of the hole is equal to two of the three short lines that were marked off on the perpendicular line [drawn] on the inner wall of the apparatus's rim. Thus, when this ruler is stood on its edge with the surface of its edge applied to the surface of the register plate, the line drawn lengthwise through its middle will lie in the plane of the aforementioned middle circle because the perpendicular dropped from any point on this [mid]line to the edge along the length of the ruler [applied to the register plate's surface] is equal to the perpendicular dropped from the center of the hole to the register plate's surface, for both of those perpendiculars are equal to the diameter of the hole.

[2.15] Therefore, when the experimenter wants to test the passage of light into water empirically with this apparatus, he will take a vessel, such as a copper pot, a turner's pottery jar, or the like, whose rim stands perpendicular [to its bottom]. The height of its rim should be no less than half a cubit and the diameter of its circumference no less than the diameter of the apparatus. [The lip of] its rim should be smoothed out until the plane passing through [the lip] of its rim is even, and we should place an object [consisting] of different segments or colors, such as a ring or a painted silver object, on its bottom, or a clear picture should be depicted at the bottom of the water [with which the vessel is to be filled].

[2.16] Clear water should then be poured into the vessel until it is full, and [the experimenter] should wait until the water is perfectly calm. Accordingly, when its motion subsides, the viewer should stand straight up or sit upright and look into the vessel, directing his line of sight on the object lying at the bottom of the water or the picture lying at the bottom of the water until the line between [his] center of sight and the midpoint of that object or that picture is perpendicular to the water's surface as far as can be empirically determined, and he should look at the object or picture at the bottom of the water. He will therefore find [that] it [appears] as it actually is, and he will find the arrangement of its parts among each other as they would be arranged if he looked at it when the vessel was empty. Having reached this determination, he will ascertain that, when he looks at anything according to the same vantage from which he looked at the object or picture at the bottom of the water, [whatever] is perceived at the bottom of

the water is perceived according to the [true] arrangement of its parts [i.e., without any apparent distortion].[15]

[2.17] With this point established, if one wants to test the passage of light empirically, he should choose a place upon which sunlight shines and put the vessel there, and he should make sure that the plane of the vessel's [lip around its upper] circumference is parallel to the horizon. This can be done as follows to ensure that the plane formed by the water line is parallel to the [plane] of the vessel's [lip]: if a circle is drawn inside the vessel or near its circumference parallel to the vessel's lip, it will be best for that purpose that the circumference of the water's surface be matched to the circumference of the circle.[16]

[2.18] Then the experimenter should insert the round apparatus into this vessel so that the two small [excess] pieces applied to the two endpoints of the longer strip [to which the apparatus is attached by its axle] are hung from the rim of the vessel on each side. The midpoint of the apparatus, along with the strip that extends [behind and] across the apparatus [to which the back of the register plate is attached], will thus lie inside the vessel [below its lip]. Water should then be added or removed from it until the surface of the water reaches the center of the apparatus, and the water should be clear. Then the apparatus will be moved around [the lip of] the vessel until the portion of the [apparatus's] rim that lies in the water is shaded by the portion of the rim that lies above the water. The [thin copper] ruler should therefore be held with one hand while the apparatus is rotated around [the axle at] its center with the remaining hand until the hole in the apparatus's rim faces the sun [so that] the sunlight passes through the hole, reaches the other hole [in the small panel between it and the water], and passes through [that] other hole. Thus, when the [sunlight's] form passes through the two holes, it will reach all the way down to the water. The experimenter will then make sure that the light [cast] on the ruler from the second hole is even [throughout].[17]

[2.19] Now that the apparatus is set up this way with the light reaching the water's surface, the experimenter should remove his hands from the apparatus and stand or sit upright, and he should look to the bottom of the water through the quadrant excised from the [apparatus's] rim while maintaining the position he assumed when he looked at the object that lay at the bottom of the water in order to be certain that what he sees is according to its actual situation [in terms of location and the arrangement of parts]. Thus, when he will examine the portion of the apparatus's rim that lies under water, he will find the light that passes through the two holes [shining] on the bottom rim of the apparatus that lies under water.

[2.20] He will also find that the light [falls] between the two parallel outer circles of the three circles inscribed on the inner wall of the apparatus's rim, or it will exceed the distance between [those] circles somewhat, but its excess will be equivalent on both sides of the circles. It therefore follows from [this] situation that the midpoint of the light appearing in the water on the inner wall of the apparatus's rim lies on the middle circle of the three parallel circles [incised] on the inner wall of the apparatus's rim. Moreover, the light that lies under water will be quite apparent because the upper portion of the apparatus's rim that contains the upper hole shades the lower portion of the apparatus's rim that contains the light on the inner wall of the apparatus's rim, and so in that place there will be no sunlight on the inner wall of the apparatus's rim other than the light that shines from the two holes.[18]

[2.21] Then the experimenter will take a fine wooden [stylus shaped] like a needle, and he should place it outside the upper hole in the apparatus's rim, and he should ensure that the stylus[19] is in line with the center of the hole. He should then look into the vessel from above, and he should maintain the position that he previously determined [by looking into the water along the perpendicular]. He will therefore see the shadow of the stylus in the middle of the [circle of] light [cast on the apparatus's rim under water]. He should then move the stylus, drawing it inward [across the hole] until its [sharpened] endpoint lies at the center of the hole, and he should look at the light that is inside the water as well as the light on the water's surface. He will thus find the shadow of the stylus's [sharpened] endpoint at the middle of the light under water and at the middle of the light on the water's surface.

[2.22] Next he should shift the position of the stylus [so as to bring it to the hole in the panel], and he should again place its [sharpened] endpoint at the middle of the hole and look at its shadow.[20] Accordingly, he will find the shadow of the stylus's [sharpened] endpoint at the middle of the light [on the rim and on the water's surface]. He should then remove the stylus, and he will find that the light returns to its [original] situation in the water and on the water's surface [i.e., replacing the shadow cast by the needle]. Then he should pose the stylus to the side of the hole, and he should place it along a chord, not a diameter of the hole, and he should look at the light inside the water and on the water's surface. He will therefore find that the shadow at both of those [locations] forms a chord [in the circle of light]. Finally, he should remove the stylus. As a result, he will find that the light returns to its [original] place, and if he moves the stylus to the sides of the hole, he will always find the shadow at the side of the [circles of] light [cast on the water's surface and on the rim under water].

[2.23] From this experiment it will therefore be manifest that the [light at the] point in the center of the light inside the water, which lies on the circumference of the middle circle [inscribed on the rim's inner wall], has reached that point only from the point in the middle of the light on the water's surface, [and it is also clear] that the point at the middle of the light on the water's surface will reach it only from the point at the center of the upper hole, and it passes through the point at the center of the lower hole, i.e., the hole that is in the panel, for if it did not pass through the center of the lower hole, the center of the light on the water's surface would not be visible when the [point of the] stylus was in the middle of the lower hole, but instead some light other than that at the center [of the hole] would be visible at [the midpoint of the light on] the water's surface.

[2.24] Consequently, the light that reaches the point at the center of the light on the water's surface and the light that extends through the air extend only along straight lines. The light that passes through the centers of the two holes therefore extends along the straight line passing through the centers of the two holes. This, moreover, is the light that reaches the center of the light on the water's surface. Consequently, the point at the center of the light on the water's surface lies on the straight line passing through the centers of the two holes, and this line lies in the plane of the middle circle among the three circles incised on the inner wall of the apparatus's rim, and this is a diameter of it because this line is parallel to the diameter of the circle on the surface of the register plate. Since, therefore, the point at the center of the light on the water's surface lies on this line, that point lies in the plane of the aforesaid middle circle. Moreover, the point at the center of the light [cast on the apparatus's rim] inside the water lies on the circumference of the middle circle, so these two points lie in the plane of the middle circle.[21]

[2.25] If the light on the water's surface is faint and not clearly visible, then the experimenter will place the ruler in the water, and he should apply its edge to the surface of the register plate and pose the face on which the [mid]line is inscribed up to the waterline and adjust it until its surface is flush with the water's surface. Hence, when the face of the ruler is flush with the surface of the water, and when the ruler is stood upon its edge, the [mid]line on its surface will lie in the plane of the middle circle that passes through the centers of the two holes. When this arrangement is in place, the light [shining] on the water's surface will appear on the ruler's surface, and [the experimenter] will find the center of the [resulting] light on the line through the middle of the ruler. And if the stylus is placed across the middle of the upper hole, the line that lies at the middle of the ruler will be darkened by its shadow, and if the [sharpened] endpoint of the stylus is placed at the center of the hole, the shadow of the stylus's [sharpened]

endpoint will appear in the center of the light cast on the ruler. Also, if the stylus is removed, the light will return as it was.

[2.26] Therefore, since the light [shining] on the water's surface will appear clearly on the ruler, it will also be evident that it lies on the line passing through the centers of the two holes. But earlier we brought the water up to the center of the register plate. Consequently, since the surface of the ruler coincides with the water's surface, the surface of the ruler will pass through the center of the register plate, and so the distance of the center of the light [shining on the ruler's surface] from the center of the register plate will be equal to half the width of the ruler, and that is equal to the perpendicular dropped from the center of the hole to the surface of the register plate [i.e., one grain of barley]. The center of the light on the ruler's face will therefore lie at the center of the middle circle.

[2.27] The experimenter must then remove the thin ruler, place it again in the water, apply it to the surface of the register plate along its wider face, and put its acute angle, i.e., the angle at its top end, at the center of the light that lies in the water. He should then adjust the ruler [in place] until its lower edge, which lies on the upper part of the register plate [above the water], passes through the center of the register plate, and so its top edge will pass through the center of the middle circle. Therefore, the point on the upper edge of the ruler at the surface of the water [where it meets that surface] is [coincident with] the center of the middle circle, so it is [coincident with] the center of the light [shining] on the water's surface, and [the line along its edge extending] its [full] length will form one of the diameters of the middle circle.

[2.28] Now when [the apparatus] is set up in this way, the experimenter should take a long needle,[22] place it in the water, put its point at the top point [on the edge] of the ruler [where that edge meets the water's surface], and look at the light inside the water. Accordingly, he will find that the shadow of the needle cuts the light, and he will find that the shadow of the needle's point [which he holds] at the vertex [on the edge] of the ruler lies at the center of the light [cast on the rim inside the water]. He should then change the position of the needle, but its point should [still] lie on the edge of the ruler. Accordingly, the location of the shadow in the light at the bottom of the water will change, but the shadow of the point will be inseparable from the middle of the light. Then he should remove the needle, and the light will return to its [original] location. He should then put the needle back into the water and place its point at another point on the ruler's edge, and he should look at the shadow. He will therefore find that it cuts the light inside the water, and he will find the shadow of the needle's point at the center of that light. Then he should shift the location of the needle to

several points on the ruler's edge, and he will invariably find the shadow of its point at the center of the light [in the water].

[2.29] It will thus be patently obvious from this experiment that the light at the centerpoint of the light in the water, which lies on the circumference of the middle circle, reaches that point from the point at the center of the light on the water's surface. And it will be manifest on that account that this light extends along the straight line formed by the edge of the ruler, for testing it with the [sharpened] endpoint of the needle at various spots on the ruler's edge shows that it passes through every point on the edge of the ruler. Accordingly, in this way the passage of light through the body of water will be empirically tested, from which it will be demonstrated that the light extends through the body of the water along straight lines.[23]

[2.30] The experimenter will then need to put a permanent mark with an incising tool at the center of the light [at the bottom of the rim inside the water]. Accordingly, when the experimenter looks at the point at the center of the light inside the water, he will find that it is not in line with the two endpoints of the diameter on the register plate, i.e., that it lies outside [the plane containing] the two lines [incised on the inner wall of the apparatus's rim] perpendicular to the endpoint of the diameter on the register plate in the water. And he will find that it inclines away from that line [i.e., the aforementioned perpendicular on the inner wall of the rim] in the direction of the sun, and I maintain that he will find a noticeable discrepancy between the point that lies at the center of the light [on the bottom of the rim] and the point that forms the common section of the line [on the inner wall of the rim] perpendicular to the endpoint of the diameter on the register plate and the midpoint at the end of the diameter of the middle circle, which passes through the centers of the two holes.

[2.31] Having established this point, [the experimenter] should place the thin ruler in the water, apply it to the surface of the register plate, pose the endpoint of the ruler at the center of the register plate, and adjust the ruler until its acute angle lies [in line with the] perpendicular to the water's surface as far as can be empirically determined. He will therefore find that the center of the light [on the rim] inside the water lies between the [point of the] ruler's acute angle and the line [drawn on the apparatus's rim] perpendicular to the diameter on the register plate [at its endpoint below the water]. It will therefore be manifest from this that this refraction occurs toward the normal dropped perpendicular to the water's surface from the point of refraction. Accordingly, when the experimenter has ascertained this point, he will need to put a permanent mark at the endpoint of the ruler, which lies on the perimeter of the middle circle at the endpoint of the normal

dropped from the center of the middle circle at the water's surface, just as he earlier marked the point at the center of the light [cast on the bottom of the rim].

[2.32] Now it has already been demonstrated that the light reaching the point at the center of the light inside the water is the [same] light that extends along the straight line connecting the two centers of the holes, and this line reaches the center of the middle circle that is parallel to the surface of the register plate and forms a diameter on it. If this line is imagined to continue in a straight line below the water until it reaches the rim [on the circumference] of the register plate, then it will be parallel to the register plate's diameter, and it will reach the perpendicular line on the inner wall of the register plate's rim. But since the center of the light in the water does not lie upon the line [drawn] perpendicular on the [inner wall of the] register plate's rim, the light that extends from the center of the light on the water's surface to the center of the light inside the water does not extend along the straight line passing through the centers of the two holes; instead, it is diverted.[24]

[2.33] It has also been demonstrated that this light extends [in a] straight [line] from the center of the light on the water's surface to the center of the light in the water. Consequently, the refraction of this light occurs at the water's surface. It has just been shown, as well, that this light passes through the centers of the two holes and through the midpoint of the light on the water's surface, which constitutes the center of the middle circle parallel to the surface of the register plate, and through the midpoint of the light in the water that lies on the circumference of the middle circle; from this it is evident that, when it extends through the air and is subsequently refracted in the water, the light reaching the center of the light in the water lies in the same plane, i.e., in the plane of the middle circle of the three circles that are [inscribed] on the inner wall of the apparatus's rim.

[2.34] But this refraction is observed when the line passing through the center of [each of] the holes is inclined, not perpendicular to the water's surface, and this line will never be perpendicular to the water's surface at the time of the sun's transit [to its highest point in the sky] unless the sun reaches the [viewer's] zenith. This, however, will be [the case] in some places, not in all places, and at certain times, not at all times, and [so] the sun does not pass through the zenith of those who live in many locations [on Earth], but in those places [where it does] this experiment will be feasible at any time [the sun transits the zenith]. If those through whose zenith the sun passes do want to test [the perpendicular passage of light through water], they will be careful [to note] the time at which the sun passes through their zenith.[25]

[2.35] Now the experimenter should take some pieces of clear glass that are cubical in shape, and each [side] of them should be twice as long as the diameter of the hole in the apparatus's rim [i.e., two grains of barley]. Their faces should be smoothed down by vigorous rubbing until their surfaces are flat and parallel and their edges are straight. They should then be polished. When this is done, a straight line should be inscribed on the face of the register plate through its center; it should be perpendicular to the diameter on it at whose endpoints the two perpendicular lines [drawn] on the inner wall of the apparatus fall, and it should extend on both sides [of that diameter]. This line should be incised with a steel [instrument] so that it cuts into the body of the register plate and remains there [permanently].

[2.36] [The experimenter] should then place one of the glass cubes upon the register plate's surface and apply one of its edges to this perpendicular [just drawn], and he should position the midpoint of the glass cube's [bottom] edge right at the center of the register plate while posing the body of the glass on the side of the [two] holes [on the upper side of the apparatus above the waterline]. Hence, the diameter on the register plate at whose endpoints the two perpendicular lines [drawn on the inner wall of the apparatus's rim] fall passes through the middle of the face of the glass [cube] applied to the plate. When it is set up in this way, the glass [cube] should be attached solidly [to the plate] with adhesive in such a way that it can [nonetheless] be removed.[26]

[2.37] [The experimenter] should then take a second glass [cube] and place it in front of the first one, i.e., on the side of the [two upper] holes, and he should nest one of its faces against the face of the first glass [cube]. When this is properly set up, he should attach the second glass [cube] solidly to the face of the plate [so that it can nonetheless be removed]. He should then take a third glass [cube], nest it against the second glass [cube], align its surfaces [on each side] with the two surfaces on the sides of the second glass [cube], and affix it to the plate. He should do the same with more of the glass [cubes] until they reach [successively] to the panel [with their upright faces] perpendicular to the surface of the [register plate of the] apparatus, or [until they] nearly [reach that panel].

[2.38] Therefore, when they are attached [in a line] to the surface of the register plate according to the aforesaid disposition, the diameter of the register plate upon whose endpoints the two perpendicular lines [drawn on the inner wall of the rim] fall will pass through the middle of the [bottom] surface of the glass [cubes] applied to the register plate. Moreover, the height of those glass cubes [above the register plate] is twice the diameter of the hole [in the panel], and the diameter of the hole [in the panel] is equal to the perpendicular dropped from the center of the hole to the surface of the

register plate and [intersecting that surface] on the register plate's diameter. Thus, each of the perpendiculars dropped from the centers of the faces of the glass [cubes], i.e., the faces upright on the register plate's surface and cutting the diameter [on that surface] aligned with the two holes, is equal to the perpendicular dropped from the center of the hole to the surface of the register plate and to the register plate's diameter. In addition, the perpendiculars dropped from the centers of the faces of the glass [cubes] to the surface of the register plate will fall on the diameter of the register plate at whose endpoints the perpendicular dropped from the center of [each] hole [in the apparatus's rim and the panel] falls. Hence, if it is imagined to extend rectilinearly, the line passing through the centers of the two holes will pass through the centers of the faces of the glass [cubes], i.e., the faces that are perpendicular to the surface of the register plate and directed toward the two holes.

[2.39] The experimenter should then take the thin ruler described before, stand it edgewise on the surface of the register plate, and pose the [wider] face on which the line was drawn toward the first glass [cube] at the center of the register plate. He should place the ruler near the glass [cube] and situate the lengthwise [bottom] edge of the ruler so that it intersects the diameter on the register plate orthogonally. Given this disposition, then, he should fasten the ruler firmly to the register plate so that it can [nonetheless] be removed from it. When the ruler is set up in this way, the line on the ruler's face will lie in the plane of the middle circle among the three circles inscribed on the inner wall of the apparatus's rim, and the straight line passing through the centers of the two holes, as well as through the centers of the faces of the glass [cubes], will intersect the line on the ruler.

[2.40] When all this is finally arranged, [the experimenter] should place the apparatus in the vessel described earlier. The vessel should be empty of water, though, and he should place the vessel in sunlight and adjust the apparatus until the sunlight passes through the two holes, and [so] the light at the second hole will be equivalent to that which lies at all the holes, and if it exceeds the [size of the] hole, the glass [cube can accommodate the excess because it] will embrace the hole [insofar as it extends beyond the hole on all sides]. The experimenter should therefore look at the surface of the ruler facing the [first] glass [cube], and he will find that the light passing through the two holes is [cast] on the ruler's surface; he will also find that the shadow circumscribing the light on the ruler's surface is the shadow [cast] by the [solid portion of the] apparatus's rim [surrounding the hole], and he will find that the center of the light [on the ruler] lies on the [mid]line [drawn] on the ruler's surface.[27]

[2.41] When he has made this determination, [the experimenter] should take the thin stylus [shaped] like a needle,[28] place it at the upper hole, pose

its [sharpened] endpoint perpendicular to the center of the hole, and look at the light on the ruler. He will then find the shadow of the stylus's [sharpened] endpoint at the center of the light [on the ruler], and he will find it on the line on the ruler's face. The experimenter will take a pen dipped in ink, and he should mark the point where the shadow of the [stylus's sharpened] endpoint [falls] on the midpoint of the light on the ruler. Accordingly, that point will lie on the [mid]line on the ruler's face.

[2.42] Then he should remove the stylus from the upper hole and place it at the lower hole, i.e., the one in the panel, and he should place the [sharpened] endpoint of the stylus at the center of the hole and look at the light on the ruler. He will thus find the shadow of the stylus's [sharpened] endpoint on the point [previously marked] on the ruler's face. Then he should remove the stylus, and the light will return to its [original] place. It will therefore be manifest from this experiment that the light at the point [marked] on the ruler's face is the light passing through the centers of the two holes.

[2.43] The experimenter should then take a pen dipped in ink and mark the point right at the center of the face of the glass [cube] on the side of the ruler. If, however, he cannot determine the midpoint of the [face on the] glass [cube] empirically, he should draw the two diagonals that intersect one another [on that face], and the point of intersection constitutes the center of the glass [cube's] face. Having done this, he should look at the light on the ruler, and he will find the shadow of the point at the center of the glass [cube's face cast] on the point [previously marked] on the ruler's face. It will thus be manifest from this that the light passing through the two centers of the two holes passes through the point at the center of the glass [cube's face].

[2.44] Once this has been determined, the experimenter should remove the [first] glass [cube] and set up the apparatus with the second one, and he should adjust it until the light passes through the two holes. He should then look at the [point marked on the] ruler's face that constitutes the center of the light and at the light [actually] reaching the center of the light on the ruler's face, and this is the light that passes through the centers of the two holes. It will therefore be manifest from this that the light passing through the centers of the two holes also passes through the point at the center of the face of the second glass [cube], and [this is] the [same] location as that of the light passing through the centers of the two holes to the faces of the glass [cubes] in the first experiment, and since this light passes through the point at the center of the [face of the] second glass [cube], it follows that the light passing through the centers of the two holes in the first experiment

also passes through the point at the center of [the face of] the second glass [cube].

[2.45] The experimenter should then remove the second glass [cube] and test the third, and so on to the last one. It will therefore be obvious from this experiment that the light passing through the centers of the two holes and reaching the ruler's surface also passes through the centers of the faces of all the glass [cubes] placed on the register plate's surface. It is therefore evident that [all those centerpoints] lie on the straight line passing through the centers of the two holes, and, when all the glass [cubes] are tested, the light passing through the centers of the two holes extends [through the glass] in a straight line connecting the centers of the two holes.

[2.46] So it is clear that the light extending along the straight line passing through the centers of the two holes also passes through the centers of the faces of [all] the glass [cubes], from which it is obvious that the light passes into the body of the glass through which it extends and then continues on along straight lines and that the light passing through the centers of the two holes also extends through the body of the glass along the [same] straight line according to which it extended in the air before it passed into the glass. Moreover, the line along which the light extends through the air is perpendicular to the surface of the glass [cube] facing the hole, for the line passing through the centers of the two holes is parallel to the diameter on the register plate that is perpendicular to the first face of the glass [cubes], since it is perpendicular to the common section of the glass [cube's] face and the surface of the register plate.[29]

[2.47] Next the experimenter should take a hemisphere of clear, white glass or crystal, whose radius is less than the distance between the panel [containing the second hole] and the register plate's center, and he should find the center of its base [circle] and draw a fine line through it with ink. Then to the side of the base's center, which constitutes the center of the sphere [out of which the hemisphere is formed], he should mark off a line segment [from that centerpoint] equal to the diameter of the hole in the apparatus's rim [i.e., one grain of barley]. This line will therefore be equal to the line dropped perpendicular to the register plate's surface from the center of the hole in the apparatus's rim [i.e., one grain of barley]. At the endpoint of the line [just] marked off on the diameter we should erect a perpendicular line, and we should extend it on both sides [to the circumference of the base circle of the sphere].

[2.48] We should then cut the glass [hemisphere] along this line in a grinding machine or a lathe until the surface of the section cut off is flat and perpendicular to the surface of the semicircular [remnant at the] base, and we should measure the angle between the two surfaces with a right angle formed from copper until that surface is properly aligned. Consequently,

the common section of this surface and the surface at the base of the [newly cut quarter-]sphere will be a straight line, and the line joining the center of the sphere [encompassing the newly cut quarter-sphere] and this line will be perpendicular to the surface [just] formed.[30] Afterwards a tiny spot should be made at the middle of this line that forms the common section, and it will mark its midpoint.

[2.49] When this is finished, the glass should be vigorously polished and placed on the surface of the register plate with its convex side facing the [two] holes, and the section of the glass [quarter-sphere forming its semicircular base] should lie on the register plate's surface. In addition, the straight line forming the common section of the two flat surfaces of the glass [quarter-sphere] should be placed on the line [previously] drawn on the register plate to cut its diameter orthogonally [through its center], and the middle of that line [formed by the two flat faces of the quarter-sphere] should be placed at the register plate's center. When things are so arranged, the glass [quarter-sphere] should be firmly attached to the plate [so that it can nonetheless be removed].[31]

[2.50] We should next apply the thin ruler to the surface of the [register plate in the] apparatus, just as we applied it during the experiment with the glass cubes, and [the experimenter] should pose the face of the ruler on which the line is [drawn] toward [the upright face of] the glass [quarter-sphere] and near it. He should then insert the apparatus in the aforementioned vessel, place the vessel empty of water in sun[light], and adjust the apparatus until the sunlight passes through the two holes, and the light from the second hole should be disposed as usual [i.e., with its axis in line with the two holes].[32] [The experimenter] should look at the ruler, and he will find the light passing through the two holes [cast] on the ruler's surface. He should then put the stylus up to the top hole and place the [sharpened] endpoint of the stylus at the hole's center, and he should look at the light on the ruler. Accordingly, he will find the shadow of the stylus's [sharpened] endpoint at the center of the light [on the ruler]. Then he should remove the stylus, and the light will return to its [original] place.

[2.51] Next he should put the stylus up to the second hole and place its [sharpened] endpoint at the center of [that] second [hole], and he should look at the light on the ruler. He will thus find the shadow of the stylus's [sharpened] endpoint at the center of the light [on the ruler]. He should then put the [sharpened] endpoint of the stylus at the center of the [upright] face of the glass [quarter-sphere], which constitutes the center of the sphere [encompassing the quarter-sphere], and he should look at the light on the ruler, and he will find the shadow of the stylus's [sharpened] endpoint at the center of the light [on the ruler].[33] Finally, he should place the [point of the]

stylus at the middle of the light that is [cast] on the convex surface of the glass [quarter-sphere] facing the second hole near it, and he should look at the light on the ruler, and he will find the shadow of the stylus's [sharpened] endpoint in the middle of the light [on the ruler], from which it is evident that the light passing through the centers of the two holes also passes through the centerpoint [of the sphere] on the face of the glass [quarter-sphere] and through the midpoint of the spot of light [cast] on the convex surface of the glass [quarter-sphere].

[2.52] It is therefore obvious that the light passing through the body of the glass extends along the straight line passing through the centers of the two holes. This line, moreover, forms a diameter of the sphere [encompassing the] glass [quarter-sphere], for the perpendicular dropped from the center [of that sphere] on the [upright] face of the glass [quarter-sphere] to the register plate is equal to the diameter of the hole. The diameter of the hole, in turn, is equal to the perpendicular dropped from the center of the hole to the surface of the register plate. Thus, the perpendicular dropped from the center [of the sphere encompassing the quarter-sphere] on the [upright] face of the glass [quarter-sphere] to the surface of the register plate is equal to the perpendicular dropped from the center of the hole to the surface of the register plate, and these two perpendiculars fall on the register plate's diameter.

[2.53] If it is extended in a straight line, then, the line passing through the centers of the two holes will reach the center of the sphere [encompassing the] glass [quarter-sphere]. It will thus constitute a diameter of this sphere, so it is perpendicular to the surface of this sphere. Furthermore, it was shown in the experiment with the glass cubes that the light extending through the body of the glass follows the straight line along which it extended in the air, and the line along which it extended through the air was the one that was perpendicular to the [upright] face of the glass [quarter-sphere].

[2.54] The experimenter should then remove the thin ruler that was attached to the register plate's surface, insert the apparatus back in the vessel, adjust it until the light passes through the two holes, and look at the rim of the apparatus that lies inside the vessel. He will find that the light [appears] on the apparatus's rim, and he will find the center of the light [cast on the apparatus's rim] at the point that forms the common section of the middle circle's perimeter and the perpendicular line on the apparatus's rim at the endpoint of the middle circle's diameter, which passes through the centers of the two holes. Also, the [axial] light that extends along this line will form a common section reaching to the center of the glass sphere [encompassing the quarter-sphere]. Thus, the center of the light on the apparatus's rim, the center of the sphere [encompassing the] glass [quarter-sphere], and the

centers of the two holes lie on the same straight line, from which it is evident that, when it extends through the air, the light passing into the body of the glass and reaching its centerpoint extends along the [same] straight line according to which it extended through the body of the glass.

[2.55] This line, moreover, is perpendicular to the plane of the [upright] face of the glass [quarter-sphere], and it is parallel to the diameter of the register plate, which is perpendicular to the plane of the [upright] face of the glass [quarter-sphere], since it is perpendicular to the straight line that forms the common section of the two plane surfaces of the glass [quarter-sphere], one of them being applied to the surface of the register plate and the other standing upright upon the surface of the register plate. Hence, the line passing through the centers of the two holes and through the center of the sphere [encompassing the] glass [quarter-sphere] is perpendicular to the [upright] face of the glass [quarter-sphere], so it is perpendicular to the surface of the air that is in contact with this face. And if the experimenter fills the vessel with water while leaving the glass in place, and if he brings the water up beyond the center of the glass [quarter-sphere] and looks at the light [cast] on the apparatus's rim, he will find the center of the light at the endpoint of the middle circle's diameter.[34]

[2.56] Furthermore, if [the experimenter] pulls the glass [quarter-sphere] off [the register plate's surface] and [re]applies it to the register plate with the opposite orientation, i.e., so that the flat surface faces the holes and the convexity of the glass faces the inside portion of the vessel [which has been emptied of water], and if he applies the straight line that forms the common section of the two plane surfaces of the glass [quarter-sphere] along the straight line on the face of the register plate that cuts the register plate's diameter orthogonally and puts the center of this line, i.e., the line [forming the common section of the two plane faces] on the glass [quarter-sphere], at the register plate's center, then when he looks at the light as he did in the first situation, he will find that the light falls on the apparatus's rim, and he will find the center of the light at the point that forms the common section of the middle circle's circumference and the line standing [perpendicular] on the [inner wall of the] apparatus's rim, from which it will be manifest that the light passing through the centers of the two holes also passes through the body of the glass along the [same] straight line along which it extended in the air, and [it will also be manifest that] after it exits the body of the glass, it extends along the [same] straight line through the air [below the glass] along which it extended through the glass.

[2.57] In addition, the line passing through the centers of the two holes in this situation is also perpendicular to the surface of the glass [quarter-sphere] on the other side of the holes, i.e., the face that forms the [upright

face that constitutes slightly more than half the] base of the hemisphere, and this line is also perpendicular to the convex surface [of the glass], for in this situation it also forms the diameter of the sphere [encompassing the glass quarter-sphere]. It is therefore perpendicular to the surface of the sphere, so it is perpendicular to the surface of the air enclosing the sphere's surface. And if the experimenter fills the vessel with water and leaves the glass in place, and if he brings the water up beneath the center of the sphere [on the upright face of the glass quarter-sphere] and looks at the light [cast] on the apparatus's rim, he will find the center of the light [cast on the rim] at the endpoint of the middle circle's diameter.[35]

[2.58] Hence, from these experiments conducted with the glass cubes and the glass [quarter-]sphere it is evident that, if light encounters a transparent body of different transparency from that of the body in which it lies, and if the line along which it extends is perpendicular to the surface of the second body, the light extends through the second body in the [same] straight line along which it extended in the first body, and it makes no difference whether the second body is denser or rarer than the first.[36]

[2.59] Now the experimenter should remove the glass [quarter-sphere] and put it back on the register plate, placing the middle of the straight line on it [that forms the common section of the two flat faces] at the center of the register plate and posing the [upright] flat face [of the quarter-sphere] toward the two holes with the line on the glass [quarter-sphere] forming the common section of its two [plane] faces inclined to some extent to the register plate's diameter, and he should slant it with respect to the register plate's diameter in the same direction it slanted in the experiment with water. The normal dropped from the center of the [sphere encompassing the] glass [quarter-sphere] and passing orthogonally through the [plane] surface of the glass into the body of the glass must therefore be oblique to the line passing through the centers of the two holes on the side of the two holes. The experimenter should attach the glass firmly [to the register plate] according to this disposition, insert the apparatus in the [evacuated] vessel and the vessel in sunlight, adjust the apparatus until the light passes through the two holes, and look at the light inside the vessel.

[2.60] Accordingly, he will find it [cast] on the inner wall of the apparatus's rim and will find the center of the light on the circumference of the middle circle but outside the point that forms the common section of the middle circle's circumference and the line standing [perpendicular] on the rim's [inner] wall, and its inclination will be toward the sun. It will therefore incline toward the normal dropped from the point of refraction, and this light extends through the air along the straight line passing through the centers of the two holes, and in this situation this line reaches the center of

the sphere [encompassing the] glass [quarter-sphere] but is oblique to the flat surface of the glass [facing the holes].

[2.61] Moreover, the endpoint of this light extending into the glass is at the center of the [sphere encompassing the] glass [quarter-sphere], so it extends through the body of the glass along a straight line passing from the center of the sphere, [and] so it forms a diameter of that [sphere]. Hence, this light extends in the body of the glass straight along one of its diameters; so when it reaches its [convex] spherical surface, it will be perpendicular to it, and when it will extend into the air, it will be perpendicular to the air enveloping the [convex] spherical surface.

[2.62] It is therefore not refracted in the air, nor does it continue along the straight line [it followed through the air to reach the upright flat face of the quarter-sphere]. So it is refracted, but not in the body of the glass, nor at its convex surface, nor in the first [part of the] air [from which it passes into the glass], nor in the second [part of the air into which it passes from the glass]. Consequently, it is refracted at the center of the [sphere encompassing the] glass [quarter-sphere], and this light is oblique to the flat face of the glass on which the centerpoint lies, from which it is evident that, when the light extends in air and passes into the glass, and when it is oblique to the glass's surface, it is refracted [there] rather than passing straight [through].[37] Also, its refraction will be in the direction of the normal dropped from the point of refraction, and the body of glass is denser than the body of air.

[2.63] From this experiment, as well as the first one dealing with the refraction of light from air to water when the light struck the water's surface obliquely, it is manifest that, if light extends through a rarer body and encounters a denser body, and if it is oblique to [the surface of] the denser body, it will be refracted at that [surface], and its refraction will be toward the line dropped from the point of refraction normal to the surface of the denser body.[38]

[2.64] The experimenter should now remove the glass [quarter-sphere] and replace it in the opposite way, i.e., so that its convex surface faces the holes, and he should place the midpoint of the common section of the [two plane faces of the] glass at the center of the register plate, pose the common section at a slant to the diameter of the register plate, and affix the glass firmly [to the plate's surface]. He should then draw the line on the register plate's surface that is normal to the common section of the [two plane faces of the] glass at the [register plate's] center. This line will thus be normal to the [upright flat] face of the glass, for the [upright] flat face of the glass is [erected] perpendicular to the surface of the register plate.

[2.65] Then the experimenter should insert the apparatus in the vessel, which remains without water, and adjust the apparatus until the light passes through the two holes, and he should look at the light inside the vessel.

Accordingly, he will find it on the inner wall of the apparatus's rim, and he will find the center of the light on the circumference of the middle circle but outside the point that forms the common section of the middle circle's circumference and the line [drawn] perpendicular on the [inner wall of the] apparatus's rim, which lies at the endpoint of the middle circle's diameter. He will also find that it inclines away from the normal.

[2.66] Moreover, this light extends through the glass along the straight line passing through the centers of the two holes because this line also forms the diameter of the [sphere encompassing the] glass [quarter-sphere] in this situtation, since it passes through the center of the [sphere encompassing the] glass [quarter-sphere]. In this situation, then, the refraction of the light also occurs at the center of the [sphere encompassing the] glass [quarter-sphere], and this light is oblique to the [upright] flat face of the glass as well as to the surface of the air in contact with the glass, from which it is clear that, when the light passes through the glass and exits into the air [below it], and when it is oblique to the air's surface, it will be refracted, and its refraction will be in the plane of the middle circle but in the direction away from the line dropped normal to the air's surface from the point of refraction.[39]

[2.67] Now if the experimenter pours water into the vessel while the glass remains in its position, if he brings the water up above the center of the [sphere encompassing the] glass [quarter-sphere], and if he looks at the light in the vessel, he will find the light [cast] on the inner wall of the apparatus's rim. He will also find the center of the light on the circumference of the middle circle, and he will find [that] it [lies] outside the endpoint of the middle circle's diameter, inclined in the direction away from the normal dropped [from the point of refraction]. He will also find that the distance between the center of the light and the endpoint of the middle circle's diameter is less than the distance between the center of the light and that point in the experiment where the light passes from glass into air because air is rarer than water, whereas water is rarer than glass.

[2.68] It is thus evident from this experiment and the aforementioned one that, when light extends through a denser body and encounters a rarer body, and when it is oblique to the surface of the rarer body, it will be refracted and will not pass straight through, and its refraction will be in the direction away from the normal dropped from the point of refraction to the surface of the rarer body. In addition, the more it will incline away from the perpendicular, the rarer [that] body will be.[40]

[2.69] Next the experimenter should remove the glass and place it again on the register plate's surface, he should apply the straight line on it [formed by the common section] upon the straight line on the register plate [that passes through the center perpendicular to the diameter], and he should

pose its convex surface toward the two holes with the straight line on the glass [quarter-sphere] eccentric to the register plate's center [i.e., so that the centerpoint of the quarter-sphere's common section at the bottom edge lies to either side of the register plate's center]. He should then attach the glass firmly [to the register plate's surface] and pose the thin ruler on its edge upon the register plate's surface [so that] the line marked on its surface faces the [flat face of the] glass [quarter-sphere] and [so that] its edge cuts the register plate's diameter orthogonally, and he should attach the ruler [to the register plate] accordingly. Thus, the line passing through the centers of the two holes does not pass through the center of the sphere [encompassing the glass quarter-sphere] but through some other point on the flat face of the glass [quarter-sphere], and [so] it will be oblique to the [convex] spherical surface.

[2.70] The experimenter should then insert the apparatus in the vessel and the vessel in sunlight, adjust the apparatus until the light passes through the two holes, and look at the ruler's surface. He will therefore find the light [cast] on the ruler's surface, and its midpoint [will lie] on the line marked on the ruler's surface, but the center of the light [will] lie outside the straight line passing through the centers of the two holes. He will also find that it inclines toward the center of [the sphere encompassing] the glass [quarter-sphere], and he will find that the line passing through the centers of the two holes is perpendicular to the flat face of the glass [quarter-sphere], for it is parallel to the diameter [of the register plate], and the diameter of the register plate is perpendicular to the flat face of the glass [quarter-sphere]. But if the light were to pass through the two centers of the holes and extend straight to the flat face, then it would extend [through that face] in [the same] straight line [along which it passed] through the air. However, since the center of the light on the ruler does not lie directly on this line, the light does not extend along the same straight line to the flat face. But the light does extend straight through the body of the glass. Hence, the light that extends through the body of the glass does not lie on the [same] straight line that passes through the centers of the two holes.

[2.71] It is refracted, then, but not in the air nor in the body of the glass, so it is refracted at the spherical surface of the glass [quarter-sphere], and it is oblique to the spherical surface because the line passing through the two centers of the holes does not pass through the center of the [sphere encompassing the] glass [quarter-sphere], and [so] when it exits the flat face of the glass, this light is refracted. Since the thin ruler is quite near the [flat] face of the glass [quarter-sphere], though, the inclination of the center of the light on the ruler away from the straight line [of light] that extends through the glass's body will not be noticeable insofar as it might [otherwise] interfere

with the refraction of the light in the body of the glass or [a significant] part of that refraction. And this refraction will incline toward the center of the [sphere encompassing the] glass [quarter-sphere], so it inclines toward the normal dropped from the point of refraction perpendicular to the spherical surface of the glass [quarter-sphere] because the line dropped from the center of the [sphere encompassing the] glass [quarter-sphere] to the point of refraction is the normal dropped from the point of refraction to the spherical surface [of the glass].[41]

[2.72] The experimenter should then remove the glass and replace it in the opposite way, i.e., he should pose its flat surface toward the two holes and place the common section of the two plane surfaces of the glass on the line intersecting the register plate's diameter orthogonally, and he should place the midpoint of the common section outside the register plate's center. With the glass attached in this way, the line passing through the centers of the two holes does not pass through the center of the [sphere containing the] glass [quarter-sphere]; rather, it will reach its flat face, upon which the centerpoint lies, at some point beyond its centerpoint, but it will be perpendicular to the flat face, as was pointed out earlier. When the line passing through the centers of the two holes is extended straight on in the imagination, moreover, it will reach the point that lies at the endpoint of the middle circle.

[2.73] When the experimenter has placed the glass accordingly, he will insert the apparatus in the vessel and the vessel in sunlight, and he should adjust the apparatus until the light passes through the two holes and look at the rim of the apparatus, and he will find the light [cast] on the inner wall of the apparatus's rim. He will also find the center of the light on the circumference of the middle circle but outside the point at the end of the middle circle's diameter and inclining toward the center of the [sphere encompassing the] glass [quarter-sphere]. Furthermore, the line imagined to exit from the center of this sphere to the point of refraction is normal to the surface of this sphere, so it is perpendicular to the surface of the air that envelops the surface of the [quarter-sphere encompassed by this] sphere. This refraction thus occurs away from the normal dropped from the point of refraction to the surface of the air enveloping the surface [of the quarter-sphere] and extending through the air's body.

[2.74] In addition, the light passing through the centers of the two holes passes straight through the body of the glass because it is perpendicular to the flat surface of the glass facing the two holes, and it will reach the convex surface of the glass [quarter-]sphere. When it reaches that surface, it will not be perpendicular to it, since it does not form a diameter in the sphere [encompassing the quarter-sphere], and every [line that is] perpendicular to

the surface of a sphere forms a diameter on it or lies in a straight line with a diameter on it [if the line lies outside the sphere]. But the light that extends through the body of the glass in this way is not perpendicular to the surface of the air enveloping the convex surface of the glass, and [so] this light is found to be refracted. It is therefore refracted at the convex surface of the [quarter-]sphere.[42]

[2.75] If the experimenter pours water into the vessel while the glass remains in place, and if he brings the water up below the center of the register plate and examines the light that is [cast] on the apparatus's rim, he will find that the light is still refracted toward the center of the [sphere encompassing the] glass [quarter-sphere]. It will therefore occur in the direction away from the normal dropped from the point of refraction, and it extends into the body of the glass from the body of the air [along a line] perpendicular to the concave surface of the air enveloping the convex surface of the glass.[43]

[2.76] From all these experiments, therefore, it is evident that sunlight passes through every transparent body along straight lines, and if it encounters a transparent body whose transparency differs from the transparency of the body in which it lies, and if the lines along which it extends through the first body are inclined to the surface of the second body, then the light will be refracted in the second body along straight lines other than the first ones along which it extended in the first body. But if the straight lines along which it extended in the first body are perpendicular to the surface of the second body, the light will extend directly into it and will not be refracted.

[2.77] When the light passes obliquely from a rarer to a denser body, moreover, it will be refracted toward the normal dropped orthogonally from the point of refraction to the surface of the second body. And when the light passes obliquely from a denser into a rarer [body], it will be refracted away from the normal dropped orthogonally from the point of refraction to the surface of the second body. Accordingly, since sunlight passes through all transparent bodies along straight lines, all [other kinds of] light will extend through all transparent bodies [along straight lines] because it was shown in the first book of this treatise that it is a property of light always to extend along straight lines, whether the light is essential or accidental, or whether it is intense or faint.[44]

[2.78] Furthermore, the experimenter can test accidental light in the aforementioned apparatus according to the procedures described earlier if he encloses the [apparatus with its] register plate in a room into which daylight enters through a window of some [moderate] size, poses the apparatus toward the window, looks at the light [cast] on the apparatus's rim inside the water or beyond the glass, and proceeds in the ways also shown

in the experimentation dealing with sunlight.[45] Accordingly, when the experimenter has tested the accidental light in these aforementioned ways, he will find that the accidental light passes through the body of water as well as through the body of glass, he will find that it extends through the glass along straight lines, and he will find that it is refracted if it is oblique to the surface of the second body or [continues] straight through if it is perpendicular to the surface of the second body. Furthermore, it was shown in the first book [of this treatise] that all light, whether essential or accidental, intense or faint, invariably extends in a straight line from any point on any [luminous or illuminated] body.

[2.79] On the basis of everything we have shown by experiment and reason, then, it is clear that all light in any given luminous body, whether it does so essentially or accidentally, intensely or faintly, extends from any point on that body through a transparent body contiguous with that body by every straight line along which it can extend, no matter if that contiguous body is air, water, or a transparent stone. In addition, if the light extending through the contiguous body in which the light originates encounters a body of transparency differing from the transparency of the body in which it lies, it passes straight into the second body when it strikes the second body's surface along perpendicular lines. If, on the other hand, it strikes the second body's surface along oblique lines, it will be refracted in the second body because it will extend through the second body along straight lines other than the original ones [along which it traveled through the first body].

[2.80] If, moreover, the light is refracted, the line along which the light extended through the first body and the line along which it was refracted in the second body will lie in the same plane, and [it follows] that, when it passes from a rarer into a denser body, its refraction will occur in the direction of the normal dropped from the surface of the denser body at the point of refraction. But when it passes from a denser into a rarer body, its refraction will occur in the direction away from the normal passing through the surface of the rarer body at the point of refraction.

[2.81] The reason light is refracted when it encounters [the surface of] a transparent body of different transparency is that the passage of light through transparent bodies occurs with an extraordinarily swift motion, as we showed earlier in the second book [of the treatise].[46] Consequently, light extending through transparent bodies extends with a swift motion that cannot be perceived because of its speed. In addition, its motion in rare bodies, i.e., in those that are quite transparent, is quicker than its motion in those that are denser, i.e., in those that are less transparent. For when light passes through any transparent body, [that body] resists the light to some extent according to how dense it is because in every physical body there

must be some density, since slight transparency has no limit in the imagination when it conceives of translucency, and what is [imaginable is] that all physical bodies [can] reach a limit [of rarity] they cannot transgress.[47] As a result, transparent physical bodies cannot avoid some density. When it passes through transparent bodies, then, light travels [through them] according to the transparency that is in them, and so they impede the light according to the density that is in them.

[2.82] Therefore, when light passes through a transparent body and encounters another transparent body that is denser than the first one, the denser body will resist the light more intensely than the first one resisted it, and when anything that moves in a given direction according to essential or accidental motion meets something to resist it, its motion must be altered.[48] Furthermore, if the resistance is intense, that moving body will be reflected back in the opposite direction.[49] If, on the other hand, [the resistance is] weak, the moving body will not be reflected back in the opposite direction, nor can it continue in the direction it originally followed, but its motion will be altered.

[2.83] Now when physical bodies move straight through some permeable body, their passage will be easier along the normal to the surface of the body through which they pass. This [fact] is observed in [various cases of moving] physical bodies, for if someone takes a thin plank [of wood] and attaches it against a wide opening, then faces the plank, takes an iron ball, and hurls it forcefully at the plank while making sure that the ball travels along a path perpendicular to the plank's surface, the plank will yield to the ball, or it will break, if the plank is thin [enough] and the force with which the ball moves is strong [enough]. But if he faces the plank obliquely at the same distance as before, and if he hurls the ball at that plank as forcefully as he did before, the ball will glance off the plank if the plank is not too thin, yet it will not move in the same direction it followed originally but will incline to a given side.

[2.84] Likewise, if he takes a sword, sets a log up in front of himself, and strikes it with the sword so that the sword [stroke] is perpendicular to the log's surface, the log will be cut through to a considerable extent. On the other hand, if the sword is [swung] at an angle and strikes the log at a slant, the log will not be cut through entirely but may be partially cut, or the sword may deflect off [the log], and the more oblique the sword [stroke], the slighter its effect on the log. There are many other examples like these, from which it is evident that motion along the normal is easier and stronger and that among oblique motions the one that occurs [along a path inclining] more toward the normal is easier than [the one that occurs along a path inclining] farther away [from the normal].

[2.85] Hence, if light encounters a transparent body that is denser than the body in which it [originally] lies, it will be impeded by that body so as not to continue in the direction it [originally] followed, but since the [denser body] does not resist it [too] forcefully, it will not return back in the direction along which it [originally] moved. Consequently, if the light's motion follows a perpendicular path, it will pass straight through on account of the force of [its] motion along the perpendicular, but if it moves along an oblique line, then it cannot pass [straight] through because of the weakness of its motion. It will happen, therefore, that it inclines toward the direction in which it will move more easily than the direction in which it [originally] moved [if it were to continue in that direction through the denser body]. The easiest of motions, however, is along the perpendicular, and motion nearer the perpendicular is easier than [motion] farther [from the perpendicular].

[2.86] Furthermore, if it is oblique to the surface of the body through which it passes, the motion is composed of motion along the normal passing through the [surface of the] body in which the motion occurs and motion in the direction of the line that is perpendicular to the normal that passes through [the surface of] that body. Therefore, when light moves into a dense transparent body along an oblique line, its passage in that transparent body will occur with a motion composed of the two aforementioned motions. Since the density of the body resists that [motion] in the direction along which [the light] was incident, and since its resistance is not particularly intense, it would follow that [the light] would incline toward the direction in which it would pass more easily, and because motion along the normal is the easiest of motions, light that extends along an oblique line must move toward the normal dropped from the point at which the light strikes the surface of the dense transparent body.

[2.87] In addition, since its motion is composed of the two motions, one of them being along a line normal to the surface of the dense body and the other along a line perpendicular to this normal, and since the composite motion that is in it is in no way diminished but only interfered with, the light must incline in the direction [of motion that is] easier than [the motion that would result were it to continue] in the direction along which it [originally] moved if its composite motion stayed the same. But, given that its [overall composite] motion does stay the same, the direction of easier [motion] in comparison to that in which it originally moved is a direction closer to the normal, so if it strikes a transparent body denser than the body in which it [originally] lies, the light extending into the [denser] transparent body will be refracted along a line closer to the normal dropped from the point

at which it strikes the denser body through which it extends than the line along which it moved [in the rarer body].[50]

[2.88] This, then, is why radiant light refracts in transparent bodies that are denser than the transparent bodies in which it [originally] lies, and so refraction is found to be characteristic of oblique light. Hence, when light passes through a transparent body and encounters a transparent body of different transparency that is denser than the body in which it [originally] lies, and when it is oblique to the surface of the transparent body it encounters, it will be refracted toward the normal dropped to the denser body [at the point of refraction] on the surface of the [denser] transparent body.

[2.89] The reason light refracts from a denser to a rarer body in the direction away from the normal is that, when light moves through a transparent body, [that body] pushes against it with some amount of repulsive force, and the denser body pushes against it with greater repulsive force, just as [happens] when a stone moving through air moves more easily and more swiftly than it moves in water because the water pushes against it with greater repulsive force than does air. Hence, when light passes from a denser into a rarer body, its motion will be faster [because it is less hindered], and when the light is oblique to the two [coincident] surfaces of the transparent body that form the interface of the two bodies, its motion will occur along a line that lies between the normal dropped from the initial point of its motion [into that interface] and the line perpendicular to the normal dropped from the initial point of motion. Consequently, the resistance of the denser body will be in the direction of the second perpendicular. Thus, when the light leaves the denser body and enters the rarer body, the resistance of the rarer body to the light posed in the direction of the second perpendicular will be less than the resistance in the first [body, which is denser], and so the motion of the light [will be] increased [after refraction] in the direction along which it was more forcefully resisted, and so it is that light [passing] into a rarer body [follows a path] away from the normal.[51]

CHAPTER THREE
How light refracts in transparent bodies[52]

[3.1] In the preceding chapter it was shown that all light [rays] refracted from one transparent body to another transparent body will always lie in a single plane [before and after refraction]. Consequently, [a] light [ray] refracted from air into water always lies in the same plane, so the straight line along which the light extends through the air and the straight line along which it is refracted into the water will always lie in the same plane. Fur-

thermore, according to what what was observed in the apparatus described earlier, this plane is [the one passing through] the middle circle among the three previously discussed [circles] inscribed on the inner wall of the apparatus's rim.

[3.2] But the inside surface of the register plate is parallel to its back surface, to which the surface of the rectangular strip [that holds the apparatus in the vessel] is attached [through its axle]. Therefore, the plane of the middle circle [on the rim's inner wall] is parallel to the surface of the rectangular strip [that nests against the back surface of the register plate]. In addition, the surface of the rectangular strip nesting against the back of the register plate is perpendicular to either of the [two] surfaces cutting the surface that nests against the back of the register plate, and [either] of these surfaces of the strip is conjoined with the surface of the two excess pieces attached at the two ends of the strip. But the surface [at the ends] of the two excess pieces is flush with the rim of the apparatus.

[3.3] Hence, the plane of the middle circle [on the rim's inner wall] is perpendicular to the surface of the apparatus's rim [i.e., to all its lines of longitude], and [every line of longitude on] the surface of the apparatus's rim is parallel to the horizon according to [the way the] experiment [is set up]. The plane of the middle circle is therefore perpendicular to the plane of the horizon. Accordingly, since it has been shown experimentally that light extending through air and refracting in water does so within the perimeter [i.e., in the plane] of the middle circle, it is evident that the light extending through air and refracting in water always lies in the same plane [perpendicular] to the plane of the horizon.

[3.4] We may also imagine a line [extending] from the center of the middle circle to the center of the world. This line will thus be perpendicular to the water's surface, since it is [a segment] of the diameter of the world.[53] However, this line lies in the plane of the middle circle, so it lies in the plane of refraction. Consequently, the plane of refraction is perpendicular to the surface of the water. And it has already been shown that, when light is refracted from air into water, it will do so between [the continuation of] the original line along which it extends through the air, that line being along a diameter of the middle circle, and the normal dropped from the center of the middle circle to the water's surface. It has also been shown that the light at the point constituting the center of the light under the water reaches it only from the light extending from the center of the middle circle. Consequently, the light that is refracted from the air into water is refracted in a plane perpendicular to the water's surface, and it will be refracted toward the normal dropped to the water's surface from the point of refraction but will not reach the normal [itself].[54]

[3.5] Refraction of light from air into glass occurs in this way too, for it was shown in the experiment with glass that, when the line [of light] passing through the centers of the two holes is oblique to the flat face of the glass and passes through the center of the [sphere encompassing the] glass [quarter-sphere], and when the flat face of the glass faces the holes, the line [of light] will be refracted at the center of the [sphere encompassing the] glass [quarter-sphere], and its refraction will occur in the plane of the middle circle [and will be] toward the normal dropped from the center of the [sphere encompassing the] glass [quarter-sphere] on the flat face of the glass.

[3.6] It has also been demonstrated that, when the line [of light] passing through the centers of the two holes is oblique to the spherical surface of the glass, and when the spherical surface faces the holes, the light will be refracted into the body of the glass at the spherical surface of the glass. Its refraction will also occur in the plane of the middle circle and toward the normal dropped from the point of refraction on the spherical surface of the glass. Furthermore, the flat face of the glass, upon which lies the center of the circle [forming the base] of the glass [hemisphere from which the quarter-sphere is formed], is perpendicular to the surface of the register plate, so it is perpendicular to the plane of the middle circle.

[3.7] Hence, the plane of the middle circle is perpendicular to the flat face of the glass, and the plane of the middle circle also passes through the center of the sphere [encompassing the] glass [quarter-sphere], so in all the experiments with glass [the middle circle] is also perpendicular to the spherical surface of the glass. When it extends through air and refracts into the body of the glass, then, the light invariably lies in a plane perpendicular to the surface of the glass while it extends through the air and after it is refracted into the glass, and its refraction will invariably be toward the normal dropped from the point of refraction on the surface of the glass, whether the glass's surface is flat or spherical.

[3.8] Likewise, it has been shown as well that, when the line [of light] passing through the two centers of the holes is perpendicular to the surface of the glass and extends straight through the body of the glass, then if the spherical surface faces the holes while this line [of light]—i.e., the one passing through the centers of the two holes—is oblique to the flat face of the glass but passes through the center of the [sphere encompassing the] glass [quarter-sphere] and is refracted in the body of the air in contact with the flat surface of the glass at the sphere's center, its refraction will occur in the plane of the middle circle and away from the normal dropped from the [sphere's] centerpoint on the flat face of the glass.

[3.9] It has also been shown that, when the line [of light] passing through the centers of the two holes is perpendicular to the flat face of the glass and passes through the body of the glass in a straight line, then if the flat face lies toward the holes while this line—i.e., the one passing through the centers of the two holes—is oblique to the spherical surface of the glass so as not to pass through its center, and if it is refracted at the spherical surface into the body of air enveloping the spherical surface, its refraction will occur in the plane of the middle circle and away from the normal dropped from the point of refraction on the surface at which refraction occurs. In these two cases, moreover, the plane of the middle circle is perpendicular to the flat surface as well as to the spherical surface of the glass. Therefore, when it extends through the glass and is refracted in the air, the light that extends through the body of the glass and is refracted in the air always lies in a plane perpendicular to the surface of the air [in contact with the glass], and its refraction will always be away from the normal dropped from the point of refraction on the surface of the air.

[3.10] From everything that has already been shown, it is clear that all light that is refracted from one transparent body to another [transparent] body is invariably refracted in a plane perpendicular to the surface of the second body, and if the second body is denser than the first, its refraction will be toward the normal dropped from the point of refraction on the surface of the second body, but it will not reach the normal [itself]. If, on the other hand, the second body is rarer than the first, the refraction will occur away from the normal dropped from the point of refraction on the surface of the second body according to the different shapes of the surfaces of transparent bodies [i.e., according to whether the surfaces are plane or curved].

[3.11] From these things it is also clear that, when light is refracted from one transparent body to a second transparent body and then from the second to a third, it will also be refracted at the surface of the third, if the transparency of the third [body] differs from the transparency of the second. But if the third [body] is denser than the second, the refraction of light will occur toward the normal dropped from the point of refraction on the surface of the third. If, on the other hand, the third [body] is rarer than the second, the refraction of the light will occur away from the normal, and the same [holds] if the light is refracted at a fourth body, or at a fifth, or at more.

[3.12] Now what we have shown [so far] in this chapter is how all light is refracted in transparent bodies of varying transparency. The reason refraction occurs in a plane perpendicular to the surface of a transparent body is that the line along which the light extends through the first transparent body is refracted toward the normal within this plane, i.e., the plane containing the normal and the initial line [of incidence], for the normal [forming] part

[of this couple] lies in this plane.[55] Therefore, the refraction occurs in a plane perpendicular to the surface of the transparent body.

[3.13] The sizes of the angles of refraction vary according to the sizes of the angles [of incidence] that the first line along which the light extends through the first body forms with the normal dropped from the point of refraction on the surface of the second body according to the transparency of the second body, for the larger the angle [of incidence] the first line forms with the normal becomes, the larger the angle of refraction becomes, whereas the smaller the angle [of incidence] becomes, the smaller the angle of refraction becomes. But the angles of refraction do not maintain the same ratio with the angles [of incidence] that the first line forms with the normal; rather these ratios vary in the same transparent body. Therefore, if the first line along which light extends through the first body forms two unequal angles with the normal at two different times or at two different locations, the ratio of the angle of refraction [resulting] from a smaller angle [of incidence] to [that] smaller angle [of incidence] will be less than the ratio of the angle of refraction [resulting] from a larger angle [of incidence] to the larger angle [of incidence].[56]

[3.14] So if the experimenter wants to investigate those angles empirically, he should measure off an arc on the middle circle [inscribed on the inner wall of the rim] at the outer edge of the apparatus consisting of ten of the 360 degrees into which the middle circle is subdivided [and he should measure that arc off] on the [right-hand] side of the center of the hole on the circumference of the apparatus. From the point [just] marked off we should then draw a straight line [on the inner wall of the apparatus's rim] perpendicular to the surface of the register plate, connect the endpoint of this line on the register plate to the center of the register plate with a straight line, and extend this line to [the base of the apparatus's rim on] the other side.

[3.15] Then we should measure off an arc of 90° on the periphery of the middle circle next to the first one, and we should mark the endpoint of this arc. Accordingly, the line extending from the center of the middle circle to this mark will be perpendicular to the line passing from the center of the middle circle to the first mark on the periphery of the middle circle. And the arc that remains between the second mark and the endpoint of the diameter of the middle circle that passes through the centers of the two holes will be 80°. We should therefore mark the endpoint of this diameter as well.[57]

[3.16] Next we should insert the apparatus in the vessel, and we should make sure that the lip of the vessel is parallel to the horizon, and we should begin the investigation at sunrise. We should fill the vessel with clear water until it reaches the center of the register plate, and we should adjust the ap-

paratus until the first line marked on the register plate's surface coincides with the water's surface. In this situation, then, the line passing through the center of the middle circle is parallel to the first line marked off on the register plate's surface, the endpoint of that line reaching the first mark on the periphery of the middle circle [which forms the plane of refraction]. It will also coincide with the water's surface, for the location of these two lines does not differ with respect to the water's surface as far as can be empirically determined. This line also forms a right angle with the line passing from the center of the middle circle to the second mark on the periphery of the middle circle, which is normal to the water's surface, and the diameter of the middle circle passing through the centers of the two holes forms an angle that will be 80° with this normal dropped from the center of the middle circle on the water's surface, for this angle subtends the arc on the middle circle between the second and third marks. The arc between the center of the hole and the first mark, which is 10°, subtends the angle of inclination.

[3.17] The experimenter should then take account of [the position of] the sun and adjust the apparatus until the light shines through the two holes, and he should thus observe the light on the rim of the apparatus under water and mark the center of that light. This mark will therefore lie on the periphery of the middle circle. Then he should remove the apparatus and look at the third mark, which lies between the endpoint [of the diameter coincident with the water's surface] on the middle circle and the second mark at the endpoint of the normal dropped from the center of the middle circle on the water's surface. On the basis of this experiment, then, it will be clear that the angle of refraction is the one subtended by the arc between the center of the light and the third mark at the endpoint of the line passing through the centers of the two holes along which the light extended [to the water]. According to the number of degrees in this arc, the size of the angle of refraction will be revealed, as will the size of the angle of refraction in proportion to the 80° that constitutes the angle that the line along which the light extended [to the water] forms with the normal dropped from the point of refraction on the water's surface [i.e., the angle of incidence].[58]

[3.18] Now the experimenter should erase the mark and the line drawn on the register plate and measure off an arc 20° in size on the periphery of the middle circle to the [right-hand] side of the center of the hole in the apparatus's rim. He should mark the endpoint of this [arc] and from this mark draw a line [on the inner wall of the rim] perpendicular to the register plate's surface, and from its endpoint he should draw a line to the center of the register plate. We should extend this line to the other side, cut the arc next to the arc that was [marked off at] 20° with an arc of 90°, and mark that point. The arc between this second mark and the end of the line passing

through the centers of the two holes should be 70°, and we should mark the endpoint of this line.

[3.19] Then we should insert the apparatus in the vessel and turn it until the line drawn on the register plate coincides with the water's surface. Accordingly, the line dropped from the center of the middle circle to the second mark will be normal to the water's surface, as was described earlier, and the line passing through the centers of the two holes forms an angle of 70° with this normal. The experimenter should then take account of the [position of the] sun and adjust the apparatus until the light shines through the two holes, and we should mark the center of the light [refracted to the apparatus's rim under water. The experimenter] should remove the apparatus and look at the marks on the periphery of the middle circle, [and] from this experiment he will have the size of the angle of refraction as well as its size in relation to the angle that the line along which the light extends [to the water] forms with the normal dropped from the point of refraction, which in this case is 70°.[59]

[3.20] The experimenter should then remove the apparatus, erase the marks and the line on the register plate, and measure off an arc of 30° to the [right-hand] side of the hole. He should proceed as he did in the first [cases] when removing [the marks and lines], and so he will have the size of the angle of refraction as well as its relation to the angle that the line along which the light extended [to the water] forms with the normal dropped from the point of refraction, which is 60° in this case. Next we should measure off an arc of 40° [to the right-hand side of the hole], then an arc 50° in size, then 60°, then 70°, then 80°; and [the experimenter] should take account of each of those arcs, and so he will have the sizes of the angles of refraction as well as the [respective] angles of inclination subtended by the first arc marked off to the side of the hole's center, and he will [also] have the ratio of the angles of refraction to the angles that the first lines along which the light extended [to the water] form with the normal to the water's surface, these angles increasing in increments of ten degrees. Moreover, if the experimenter wants the angles to increase by [increments of] five [degrees], he can certainly make [them do so], and if he wants [them to increase] by [increments of] less than five [degrees], he can certainly make [them do so] in the order just described.[60]

[3.21] Furthermore, if the experimenter wants to do the experiment with glass, he should mark off the [appropriate] arc [of 10°] and make the marks described before, and he should apply the glass [quarter-sphere] described earlier to the surface of the register plate and place its common section on the line drawn on the register plate. He will also pose the flat face of the glass toward the holes and affix the glass firmly [to the register

plate]. He should insert the apparatus in the vessel, adjust it until the light shines through the two holes, and mark the center of the light [cast on the rim opposite the holes]. Then he should remove the apparatus and look at the [resulting] arc. Subsequently he should erase the marks, measure off the other arcs [from 20° to 80°], mark the other points, and examine the arcs as he examined them for water, and so he will have the sizes of [the angles of] refraction in the passage of light from air to glass.[61]

[3.22] Now if he wishes to test the refraction of light from glass to air or to water, he should apply the glass in the opposite way from [that in] the first case, i.e., he should pose its convex surface toward the two holes, and he should place the midpoint of the common section on the glass at the center of the register plate. Accordingly, the light passing through the centers of the two holes reaches straight to the center of the [sphere encompassing the] glass [quarter-sphere] and is refracted at that point from the glass into the air. He should then mark off the arcs [from 10° to 80°] successively and change the position of the glass [accordingly], and thus he will have the angles of refraction specific [to those arcs] as well as their relation to the angles that the first line along which the light extends [to the glass] forms with the normal to the surface [of the air] in contact with the [flat] face of the glass.[62]

[3.23] And when the experimenter has put the two cases just discussed to the experimental test, he will see that the sizes of the angles of refraction from air to glass and from glass to air will always be equal if the angle that the line along which the light extends to the point of refraction forms with the normal, when [the light] is refracted from air into glass, is equal to the angle that the line along which the light extends from the point of refraction forms with the normal, when it is refracted from glass [into air].[63]

[3.24] If one wants to investigate the sizes of the angles of refraction [resulting from the incidence of light] at the convex surface of the glass, he should measure off an arc of ten degrees on the periphery of the middle circle from the [right-hand] side of the center of the hole in the apparatus's rim, and from its endpoint he should draw a line on the [inner] wall of the apparatus's rim perpendicular to the register plate's surface, just as he did earlier. Starting from the center of the register plate he should then measure off a distance [on the line drawn] from [the bottom endpoint of] that line [to the register plate's center] equal to the radius of the [sphere encompassing the] glass [quarter-sphere], and from the endpoint of this line [which lies between the register plate's center and the apparatus's rim] he should draw a line perpendicular to the diameter of the register plate at whose endpoints lie the two perpendicular lines [drawn] on [the inner wall of] the apparatus's rim, and he should extend this perpendicular to both sides. Then he should

place the glass on the register plate's surface and put its common section on the perpendicular just mentioned, and he should put the midpoint of that common section at the point from which the perpendicular [to the diameter] was drawn.

[3.25] Hence, the center of [the sphere encompassing] the glass [quarter-sphere] will lie in the plane of the middle circle, and the line passing through the centers of the two holes will be perpendicular to the flat face of the glass because it is parallel to the diameter of the register plate, which is perpendicular to the common section on the glass. Moreover, the center of the middle circle will lie on the convex surface of the glass, for the line extending from the center of the middle circle to the center of the register plate is equal to the line dropped from the center of the [sphere encompassing the] glass [quarter-sphere] to the midpoint of the common section [i.e., one grain of barley], and both of those lines are perpendicular to the register plate's surface. The two lines are thus equal and parallel, and the line joining the center of the [sphere encompassing the] glass [quarter-sphere] and the center of the middle circle is equal to the line joining the center of the register plate and the midpoint of the common section on the glass. This line, however, was constructed equal to the radius of the glass [quarter-sphere], so the line parallel to it is equal to the radius of the glass [quarter-sphere]. The center of the middle circle thus lies on the convex surface of the glass, so the line passing through the centers of the two holes, which [also] passes through the center of the middle circle, forms an angle with the line extending from the center of the [sphere encompassing the] glass [quarter-sphere] that is equal to the angle at the center of the register plate [formed by the respective parallels].

[3.26] Accordingly the two lines, i.e., the aforementioned diameter in the glass [quarter-sphere] and the line passing through the centers of the two holes, may be imagined to extend straight on both sides. [Each] will therefore reach the periphery of the middle circle, for they both lie in the plane of the middle circle. As a result, the two lines will measure off an arc on both sides of the periphery of the middle circle that is ten degrees in size, and the endpoints of the line passing through the centers of the two holes are already determined, one of them being the center of the hole [in the apparatus's rim], the other being the point opposite the center of the hole, and one of the two endpoints of the line passing through the center of the [sphere encompassing the] glass [quarter-sphere] is the endpoint of the arc that it had measured off on the periphery of the middle circle, and it lies ten degrees away from the center of the hole. The remaining endpoint of the line passing through the center of the [sphere encompassing the] glass [quarter-sphere] therefore lies ten degrees on the opposite side of the first

line marked off, which passes through the centers of the two holes. So we should mark the endpoint of this diameter as well as the endpoint of the line passing through the centers of the two holes, even though this point is [already] determined because it lies on the line [drawn] perpendicular [to the register plate's surface] on the [inner wall of the] apparatus's rim.

[3.27] [Having inserted the apparatus into the vessel while allowing the sunlight to shine through the two holes, and having subsequently marked the point at which the center of the refracted light strikes the inner wall of the rim],[64] the experimenter should examine the mark, and he will find that it lies farther away from the endpoint of the line passing through the center of the [sphere encompassing the] glass [quarter-sphere] than the endpoint of the line passing through the centers of the two holes does. This refraction is therefore in the direction away from the normal [dropped] from the point of refraction, since the normal dropped from the point of refraction is a line passing through the center of the [sphere encompassing the] glass [quarter-sphere]. Moreover, the arc on the periphery of the middle circle that lies between the center of the light and the endpoint of the line passing through the centers of the two holes is the size of the angle of refraction, for the angle of refraction lies [with its vertex] at the center of the middle circle because the light extends straight along the line passing through the centers of the two holes until it reaches the convex spherical surface of the glass. Consequently, the angle of refraction will lie [with its vertex] at the center of the middle circle, which lies on the convex surface of the glass, and the arc between the center of the light and the endpoint of the line passing through the centers of the two holes is the one subtending the angle of refraction that is [produced when the angle of inclination is] ten degrees.

[3.28] The experimenter should then remove the glass and measure off an arc 20° in size, starting from the center of the hole, and he should proceed as before. He will thus have an angle of refraction different in size from the size of the angle of [incidence of] 20°. He should measure off the other arcs accordingly in succession, and he should test their refractions, just as in the initial cases, and he will have the sizes of the angles of refraction at the convex surface of the glass, and these same angles will be the same size as the angles of refraction [for the passage] of light from air into glass, for this was shown in the two preceding experiments. But the refraction of air into glass is toward the normal, whereas the refraction from glass to air is away from the normal. And if one also wants to test [refraction between] glass and water at the convex surface of the glass [quarter-sphere] as well as at its flat face, he will have the angles of refraction for glass to water, for glass acts in place of the air.[65]

[3.29] If one wants to investigate the sizes of the angles of refraction [for light incident] on a concave glass surface, he should take a concave piece of glass whose hollow is semicylindrical in shape. The surfaces of the entire glass piece should be parallel, and it should be longer by one grain of barley than the radius of the glass [quarter-]sphere and wider by the same amount. It should be twice as thick as the diameter of the hole in the apparatus's rim [i.e., two grains of barley], and its concavity should be on one of its sides [only]. The [semi]cylindrical hollow should lie in the plane of the square surface of the glass, and the length [along the axis of the semi]cylinder should lie along the height of the glass. The radius of the [semi]cylinder's base should be the same as the radius of the glass [quarter-]sphere [used in earlier experiments], and the edges of the glass should form perfectly straight lines. This device can certainly be produced in a mold as long as that mold is formed according to the description just given and the glass is melted and poured into the mold just described.

[3.30] Accordingly, if the experimenter wants to test refraction with this device, he should measure off the arc he wants to test on the periphery of the middle circle and draw a line [on the inner wall of the apparatus's rim] from the endpoint of the arc that is perpendicular to the register plate's surface, as was described earlier. He should then join the endpoint of the perpendicular and the center of the register plate with a straight line that he extends to the other side [of the register plate], and on one segment of that line, i.e., the one on the [other] side of the two holes, he should measure off a line [from the center of the register plate] equal to the radius of the base of the [semi]cylinder [hollowed out of the glass]. From its endpoint he should drop a perpendicular to the diameter on the register plate and extend it on both sides. Then he should put the glass piece on the register plate and pose the back of the hollow toward the two holes. He should apply the [surfaces of the] two end pieces extending beyond the diameter of the cylinder [across the base of the semicylindrical hollow] flush with this perpendicular and make certain that the two endpoints of the diameter at the hollow's base are equidistant from the point at which the perpendicular was dropped. Consequently, the centerpoint [of the diameter] at the base of the [semi]cylindrical hollow will lie on the point from which the perpendicular was dropped and [thus] on the point whose distance from the centerpoint of the register plate is equal to the radius of the [semicylindrical] hollow's base. When everything is set up this way, [the experimenter] should affix the glass firmly [to the register plate].

[3.31] And [so] the plane of the middle circle [on the apparatus's rim] will cut the [semi]cylindrical hollow and will be parallel to its base because in this case its base lies on the surface of the register plate. Hence, the plane of

the middle circle [on the apparatus's rim] cuts a semicircle on the surface of the [semi]cylindrical hollow, and the diameter of this middle circle is parallel to the diameter [of the circle encompassing the semicircle] at the base of the hollow. The line dropped from the center of this semicircle to the center of the base of the hollow, which is perpendicular to the surface of the register plate, will thus be equal to the perpendicular dropped from the center of the middle circle [on the apparatus's rim] to the surface of the register plate [i.e., both will be one grain of barley in length]. Moreover, the perpendicular dropped from the center of the semicircle [at the hollow's base] to the center of the register plate is equal to the radius of the [semi]cylinder, so the line dropped from the center of the middle circle [on the apparatus's rim] to the center of the semicircle on the surface of the [semi]cylinder is equal to the radius of this semicircle. As a result, the center of the middle circle lies on the circumference of the semicircle formed [by the middle circle cutting the hollow], and so it lies on the concave [surface of the semi]cylinder [forming the hollow]. And since the edge of the glass piece is flush with the line [drawn] perpendicular through a point on the register plate, the diameter of the register plate will be perpendicular to the flat face of the glass, for the flat faces of the glass are perpendicular to one another. Thus, the line passing through the centers of the two holes will be perpendicular to the flat face of the glass on the side of the glass's convexity because it is parallel to the diameter on the register plate, and this flat surface of the glass faces the two holes.

[3.32] In this situation, then, the light extending along the line that passes through the centers of the two holes extends straight through the body of the glass until it reaches the concave surface of the glass. It is then refracted at the concave surface of the glass because it cannot pass through the center of the [middle] circle, which lies on the concave surface of the glass, and it is not perpendicular to the concave surface of the glass, so it is refracted at the concave surface of the glass. This line, moreover, will intersect the concave surface of the glass at one point, so the common section of this line and the concave surface of the glass is the center of the middle circle [on the apparatus's rim]. Accordingly, the light extending along the line that passes through the centers of the two holes is refracted at the center of the middle circle [on the apparatus's rim], so the arc between the center of the [refracted] light [cast on the inner wall of the rim] and the endpoint of the line passing through the centers of the two holes subtends the angle of refraction.[66]

[3.33] This, then, is how one could investigate the sizes of the angles of refraction produced in concave glass surfaces by increasing the [size of the] arcs [on the rim] bit by bit [according to the increments chosen]. Moreover,

this refraction occurs at the concave surface of the glass into air, and the angles obtained from this refraction will be the same as those produced [in refraction] from air to glass through the concave surface of the glass, for it was shown a bit earlier that the angle of refraction [when light passes] from glass to air and from air to glass is the same because the angle that the first line along which the light extends forms with the normal dropped from the point of refraction is the same angle [reciprocally in both cases]. Hence, in this way one could obtain the sizes of the angles of refraction from air to water, air to glass, glass to air, and glass to water at a plane, concave, and convex surface.[67]

[3.34] Thus, I maintain that, when these angles are tested and their relationships taken into account, the experimenter will discover [that the following rules invariably apply].

[1] Every pair of angles, one [being the angle of refraction and the other being the angle of incidence] that the first line along which the light extends forms with the normal dropped from the point of refraction on the surface of a transparent body, [is the same] in the same transparent bodies. Yet the two angles will vary [by pairs], for the angle of refraction [yielded] by the larger angle [of incidence] will be larger than the two angles of [incidence and] refraction [yielded] by the smaller angle [of incidence], and the difference between the [one] angle of refraction and the [other] angle of refraction will be smaller than the difference between the larger angle [of incidence] that the first line forms with the normal and the smaller angle [of incidence] that the first line forms with the normal.[68]

[2] Moreover, the ratio of the angle of refraction [yielded] by the larger angle [of incidence] to the larger angle [of incidence itself] will be greater than the ratio of the angle of refraction [yielded] by the smaller angle [of incidence] to the smaller angle [of incidence itself].[69]

[3] Also the difference between the angle of refraction [yielded] by the larger angle [of incidence and that larger angle] is greater than the difference between the angle of refraction [yielded] by the smaller angle [of incidence and that smaller angle].

[4] And when the light passes from a rarer to a denser body, the size of the angle of refraction will always be smaller than [the size of] the angle that the line along which the light extends to the point of refraction forms with the normal dropped from the point of refraction.[70]

[5] On the other hand, if the light passes from a denser to a rarer body, the angle of refraction will be [smaller than] half the two angles taken together.[71]

[6] Furthermore, if you compare the angles of refraction that are [produced in refraction] between one of those transparent bodies and another body that is denser than those to the angles of refraction that are [produced in refraction] between that same rare transparent body and another body that is denser than the first dense body, you will find that the ratios of the angles of refraction to the angles the first line forms with the normal in the case [of refraction] between the rarer body and the denser body [of the two dense bodies] are greater than the ratios of the [corresponding] angles of refraction to the angles that the first line forms with the normal in the case [of refraction] between the same rare body and the less dense body [of the two dense bodies], that is, [you will find] that, if the two angles that the first line along which the light extends forms with the perpendicular dropped from the point of refraction are both equal, and if one of them is [the angle of incidence for refraction] between the rarer body and the body denser than it, whereas the other [is the angle of incidence for refraction] between the same rare body and the body that is denser than the first dense body, then the angle of refraction that is [produced] in the denser body [of the two dense bodies] will be larger than the angle of refraction that is [produced] in the dense body that is less dense.[72]

[7] By the same token, if the refraction occurs from a denser body to the rarer body [of two rare bodies], the angle of refraction will be greater [in that rarer body] than it will be [in refraction] from that same denser body to the less rare body. This, then, is everything that pertains to the way light is refracted in transparent bodies.

CHAPTER FOUR

That whatever the visual faculty perceives through transparent bodies that differ in transparency from the body in which the eye lies is perceived by means of refraction when the eye lies to the side of the normals [dropped from the object] to the surface of the [transparent] body

[4.1] Now in the preceding chapter it was shown that light passes from glass to air, from air to glass, and from air to water, and since it passes from glass to air or to water, it is certainly the case that it passes from water to air, for, when it is clear, water is rarer than glass.[73] And since it passes from air to glass, it will pass from water to glass because water is denser than air. It

was further shown that all accidental or essential light, [whether] intense or faint, passes through these transparent bodies in these ways. Therefore, every body that is illuminated by any light whatever sends its light through every transparent body, and if it encounters another transparent body, it will pass into that other body either refracted or straight.

[4.2] In the first [book], moreover, it was demonstrated that light propagates from every point on any luminous body along any straight line that can be extended from that point, from which it obviously follows that light propagates from every point on any transparent body in contact with any body that is illuminated by any light whatever along every straight line that can be extended from that point, and it continues on through the transparent body in contact with that point. And if it encounters another transparent body, whose transparency is different from the transparency of the [original] body in contact with it, it will also pass through that body either refracted or straight, [no matter] whether the first body is rarer than the second, or whether the second is rarer than the first.

[4.3] It was also demonstrated in the first [book] that from every illuminated, colored body, color mingled with light is propagated along with the light in it, and [it was demonstrated] that, when it perceives the light, the visual faculty perceives the form of the color mingled with it. From these points it is evident that, if there is essential or accidental light, [whether] intense or faint, in colored bodies lying in water or beyond transparent bodies that differ in transparency from the transparency of air, light propagates from every point on them along with the form of the color at that point, and the light passes mingled with color through the body of the water as well as through every transparent body in contact with it, and the light coupled with the form of the color extends through the body of the water and through every [other] transparent body along straight lines until it reaches the surface of the water or [the surface] of that [other] transparent body.

[4.4] Furthermore, if air or another transparent body is in contact with the water, the light will pass through that transparent body with the form [of color] mingled with it in the air or in the other transparent body along straight lines, and for the most part these second [straight] lines will cut [i.e., form angles with] the first lines along which [the light and color forms] extended [through the first body], but some of them will be right in line with the first lines. Also, when any objects lie in water or beyond [other] transparent bodies that differ in transparency from the air, and when they lie in a lighted spot, i.e., when light is propagated into the water in which they lie, the light will reach them, for it is obvious that all light passes through every transparent body.

[4.5] Therefore, when light shines on water or on [another] transparent body, every body lying in the water or in the other transparent body will be illuminated, and from any point on it the form of light along with the form of the color in it will be propagated, and it extends throughout that water or that [other] transparent body along every straight line that can be projected from that point until the light at that point reaches the surface of the water or the surface of that [other] transparent body along with the form of the color.

[4.6] But only one perpendicular line can be dropped from a given point on any surface to that same surface. From any point on any luminous colored body lying in a transparent body, then, the form of light along with the form of color is propagated along straight lines throughout the transparent body in which it lies, and the form [of that light and color] reaches the entire facing surface of the transparent body. But [only] one of those lines [along which the form extends] will be perpendicular to the surface of the transparent body as well as to the surface [of another transparent body] in contact with the surface of the transparent body; the rest of the lines will be oblique to the surface of the transparent body.

[4.7] In the previous chapter, however, it was shown that, when light extends through a transparent body and encounters another transparent body whose transparency is different from the transparency of the first body, and when the line along which the light extends through the first body is perpendicular to the surface of the second body, the light will extend straight through the second body. On the other hand, if the line along which the light extends [through the first body] is oblique to the surface of the second body, the light is refracted. And [so] from any point on any colored and illuminated body lying in a transparent body the form of the light and color [at that point] extends throughout the transparent body, and it reaches the facing surface of the transparent body.

[4.8] If, moreover, another transparent body is in contact with that transparent body, and if it is of a different transparency, then the form that reaches the surface of that [first] transparent body passes into the body in contact with it, and all [such forms], except the form that passes along the perpendicular, will be refracted because [that particular form] extends straight into the [transparent] body in contact [with the original transparent body]. But if by chance the [form passing along the] perpendicular reaches a point on the surface in contact with the [original transparent] body's surface that is not on that transparent body, that form will be blocked, and so all the forms that pass into the [transparent] body in contact [with the original transparent body] will be refracted.[74]

[4.9] Consequently, the forms of all visible objects that lie in water, in the heavens, and in all [other] transparent bodies that are in contact with the air and that differ in transparency from the air extend throughout the facing air along straight lines, and a form extending along oblique lines among [all] the lines according to which the forms pass to the surface of the air in contact with the surface of the transparent body will be refracted, whereas a form extending to the surface of the air in contact with the surface of the transparent body along perpendicular lines among all these lines will be [propagated] straight through [that surface].

[4.10] And since it has just been shown that the form of light and color extends throughout [any] transparent body from any point on any colored and luminous body [in it] and [that it thereby] reaches its surface and is refracted at its surface, the form that extends from a single point to the surface of the transparent body will be continuous and coherent. And since the form is continuous, and since the surface of the transparent body is continuous and coherent, then, when the form is refracted into another transparent body, it will be refracted [as a] continuous [whole]. And since the refracted form is continuous and encounters a dense body [i.e., the second transparent medium], the form will reach that transparent body, and so the spot on the [denser] transparent body through which the form of the point in the first body extends is the spot at which the form is refracted from the first transparent body, when [that spot] is illuminated and colored, and it sends the form of [its] light and color to any point in that second body along every straight line that can be extended from that point.[75]

[4.11] It therefore follows from this that among the lines along which the form of that spot extends [through the second medium] there are refracted ones at that spot, and the form of every point on that spot has already extended along one of those refracted lines. The form of that spot on the dense body, which is luminous and colored, will therefore lie at that spot on the surface of the transparent body where the form of a single point extending to that spot on the surface of the transparent body is refracted to that same spot on the surface of the dense body, from which it follows that the form of the spot on the dense body that extends to that point on the transparent body is refracted along the same lines extending from the single point to that spot on the transparent body.[76]

[4.12] And when the form at the spot on the transparent body is refracted along those same lines, it will reach the same point, from which it is clear that, if you imagine a cone extending from any point in the air along straight lines, and if the cone is continuous and coherent, and if that cone reaches the surface of a transparent body of different transparency than the air, and if you imagine that every straight line that can be extended within that cone

is refracted at the surface of the transparent body at a point that requires it to be deflected, and if one of them is perpendicular, it will continue straight on. This [cone] thus forms a continuous body refracted in the transparent body that differs in transparency from the air. And when this refracted body reaches an opaque body, then, if that opaque body is colored and luminous, it sends the form of the light and color that are in it through this imaginary refracted body along every straight line that can be extended in this refracted body by means of a line extending in the body of the cone to a point in the air, for every colored and illuminated body by its nature sends its form from every point on it along every straight line that can be extended from that point.

[4.13] Accordingly, the form of the point at that spot on the opaque body will extend along any one of the refracted lines at that point on the opaque body. So the form of that point on the opaque, illuminated, colored body will reach a spot on the surface of the transparent body at which those lines are refracted, and when the form reaches that spot on the surface of the transparent body, it will necessarily be refracted along the same lines extending to that spot from one point in the air. And if a center of sight lies at that point in the air, the form of the spot on the opaque, colored body that lies in the transparent body that differs in transparency from the air and that extends [orthogonally] along one of those lines according to which the form extends to the center of sight, the form extending along that line reaches straight to the center of sight. On the other hand, the forms that extend along all the other lines forming the cone that extends from the center of sight will be refracted, not straight.[77]

[4.14] In addition, it was shown in the first book that air receives the form of visible objects and transmits it to every facing body and that, when the air transmitting that form is in contact with the eye, the form that is in it will pass into the body of the eye, and so the visual faculty perceives the [forms of the] visible objects that the air transmits to the eye. From all of these things, therefore, it is evident that the form of every colored and illuminated body lying in a body whose transparency is different from the transparency of the air extends through the transparent body in which it lies, is [then] refracted in the air, and [subsequently] continues through the air in straight lines, and [it is also evident] that some of the straight lines along which the form is refracted in the air converge at the same point in the air. And [so] when the center of sight is at that point, the visual faculty will perceive that visible object according to refraction, and if any of [the points on the form] is perceived by direct vision, it will only be a single point. This, then, is how the visual faculty perceives things that lie in water

and in the heavens, and [it is how it perceives] every visible object that lies behind transparent bodies that differ in transparency from the air.

[4.15] That this is in fact the case can be determined experimentally. Accordingly, the experimenter should take the apparatus discussed earlier, insert it in the vessel, and set the vessel up in a location illuminated by some light so that the light reaches inside the vessel. He should then pour water into the vessel until it reaches the center of the register plate. Next he should constrict the holes with wax so that only a narrow opening at the center of the holes is left, and he should push a [hollow] reed through the two holes so that the space between the two holes is determined [i.e., so that the axial line of sight through the hollow reed coincides with the line passing through the centers of the two holes]. He should then adjust the apparatus until the diameter on the register plate at whose endpoints the two perpendicular lines [drawn] on the [inner wall of the] apparatus's rim lie is perpendicular to the surface of the water. He should then take a thin white stylus, put it into the vessel, and place its endpoint at the point on the middle circle [inscribed on the inner wall of the rim] that constitutes the common section of the middle circle's circumference and the perpendicular line on [the inner wall of the] apparatus's rim at the endpoint of the diameter of the middle circle, which passes through the centers of the two holes. After that the experimenter should place one of his eyes at the top hole, close the other, and look at the rim of the apparatus under water, for he will then see the endpoint of the stylus.

[4.16] It will therefore be manifest from this experiment that he perceives the endpoint of the stylus along the normal dropped from the endpoint of the stylus to the water's surface, for the line passing through the centers of the two holes, upon which the center of sight and the endpoint of the stylus lie and along which the center of sight perceives the stylus's endpoint, is perpendicular to the water's surface. Moreover, in the first [book] it was shown that the visual faculty perceives nothing unless [it does so] along straight lines that extend through the center of sight. Therefore, the visual faculty perceives the endpoint of the stylus along the straight line that passes through the centers of the two holes, and this line extends straight to the endpoint of the stylus and is perpendicular to the water's surface.

[4.17] Now the experimenter should incline the apparatus [by turning it on its axis] until the line passing through the centers of the two holes is oblique to the water's surface. He should then put the stylus in the water, place its endpoint at the first point, i.e., at the endpoint of the middle circle's diameter that passes through the centers of the two holes, put his eye at the top hole, and look at the rim of the apparatus under water. In that case, he will in fact not see the endpoint of the stylus. Then he should move the

stylus in the direction away from that in which the eye lies, and he will therefore not see the endpoint of the stylus. Subsequently he should move the stylus to the same side as the eye and shift the stylus's endpoint on the circumference of the middle circle smoothly and gently, and he should [continue to] look at the rim of the apparatus [under water], in which case he will [eventually] see the endpoint of the stylus.[78]

[4.18] Accordingly, he should fix the endpoint of the stylus in place [and] then direct someone else to put a [wooden rod of some kind or a needle] that is neither [too] thick nor [too] thin upright[79] into the vessel and to place it at the water's surface facing the second hole so that it lies at the center of the middle circle. The experimenter should then look into the vessel, and he will not see the endpoint of the stylus. Next he should direct [the other person] to remove the rod, and in that case he will see the endpoint of the stylus.[80] Afterwards he should fix the endpoint of the stylus in place, raise his eye from the hole, remove his apparatus from the vessel, while leaving the endpoint of the stylus in its place, and look at where the endpoint of the stylus lies. He will then see that there is a noticeable discrepancy between that point and the [endpoint of the] diameter of the middle circle. And if at the time of the experiment he places the thin ruler on the rim under water and has its pointed edge pass through the center of the register plate, and if he marks the point on the middle circle at the endpoint of the ruler, removes the apparatus, and looks at where the endpoint of the stylus lies, he will see that the point at which the endpoint of the stylus lies is between the point marking the endpoint of the ruler and the diameter of the middle circle.[81]

[4.19] Next [the experimenter] should remove the apparatus and pour water into the vessel, and he should affix the glass [quarter-sphere] to the register, posing the flat face of the glass toward the holes and placing the common section [formed by the two flat faces] on it flush with the line cutting the diameter of the register plate orthogonally. Hence, the line passing through the centers of the two holes will be perpendicular to the flat face of the glass [quarter-sphere] as well as to its convex surface.[82] [The experimenter] should then place the apparatus in the water, put the stylus in the vessel, pose the endpoint of the stylus at the endpoint of the middle circle's diameter, put his eye up to the top hole, and look at the apparatus's rim [under water]. Accordingly, he will see the stylus's endpoint, and if he moves the stylus endpoint and shifts it from the point that marks the endpoint of the middle circle's diameter, he will not see the stylus's endpoint, from which it follows that he perceives the stylus's endpoint straight on, for the two centers of the holes and the endpoint of the middle circle's diameter lie on the same straight line, and in this situation, when the stylus's endpoint does not lie on the diameter's endpoint, the experimenter does not perceive

the stylus's endpoint. Furthermore, if he removes the glass [quarter-sphere] and places it in the opposite way, i.e., if he poses the convex surface of the glass toward the two holes with its common section [left] where it was in the first case, and if he does the test with the stylus's endpoint, he will see it when it lies at the endpoint of the middle circle's diameter. So in this case as well, the line passing through the centers of the two holes along which the visual faculty perceives the stylus's endpoint will be perpendicular to the flat face of the glass [quarter-sphere] as well as to its convex surface.[83]

[4.20] Now the experimenter should remove the glass and draw a straight line on the surface of the register plate from the center of the register plate so as to form an obtuse angle with the diameter on the register plate at whose endpoints the two perpendicular lines [drawn] on the [inner wall of the] apparatus's rim lie, and he should extend it until it reaches the apparatus's rim. From the center of the register plate he should then draw a line on the register plate's surface that forms a right angle with the first line, and he should extend it on both sides. This line will therefore form an acute angle with the diameter on the register plate, and the diameter on the register plate will be oblique to this line. Next he should apply the glass [quarter-sphere] to the register plate, put its common section flush with the line he drew last on the register plate, pose the flat face of the glass [quarter-sphere] toward the two holes, and place the midpoint of the common section at the register plate's center.

[4.21] Accordingly, the center of the [sphere encompassing the] glass [quarter-sphere] will lie at the center of the middle circle, as was shown before, and the line passing through the centers of the two holes will pass through the center of the [sphere encompassing the] glass [quarter-sphere]. This line will be oblique to the flat face of the glass, for the diameter on the register plate parallel to it is oblique to the common section on the glass. And yet this line will be perpendicular to the convex surface of the glass because it passes through its center.

[4.22] The experimenter should next draw a perpendicular line on the [inner wall of the] apparatus's rim from the endpoint of the line that he first drew on the register plate and extend it to the circumference of the middle circle, and these lines should be black. Accordingly, if a line is extended from the point [that this perpendicular on the rim] reaches [on the middle circle] to the center of the middle circle, which is the center of the [sphere encompassing the] glass [quarter-sphere], it will be normal to the flat face of the glass as well as to the [convex] spherical surface of the glass. On the one hand, it is normal to the flat face of the glass because it is parallel to the first line drawn on the register plate to the common section on the glass. On the other hand, [it is normal] to the [convex] spherical surface because it

passes through its center [of curvature]. Consequently, the point at which the [perpendicular] line drawn on the [inner wall of the] apparatus's rim intersects the circumference of the middle circle is where the normal dropped from the center of the [sphere encompassing the] glass [quarter-sphere] falls on the flat face of the glass.

[4.23] The experimenter should now insert the apparatus in the vessel and place the endpoint of the stylus at the endpoint of the middle circle's diameter, and he should put his eye up to the top hole and look at the apparatus's rim. In that case he will not see the stylus's endpoint. He should then move the stylus on the opposite side of where the normal is dropped, and in that case he will still not see the stylus's endpoint. Finally, he should move the stylus along the circumference of the middle circle on the side where the normal is dropped, for in that case, if the adjustment is smooth, he will [eventually] see the stylus's endpoint. Accordingly, he should fix the endpoint of the stylus in the place where it appeared. Then he should direct someone to block the center of the [sphere encompassing the] glass [quarter-sphere] with a fine, thin rod, and in that case he will not see the endpoint of the stylus, whereas if he removes the block, he will see it.[84]

[4.24] From this experiment it is therefore evident that, when the visual faculty perceives the endpoint of the stylus [as if it lay] along a straight line [of sight], it is [perceived] according to refraction, and [it is also evident] that the refraction occurs at the center of the [sphere encompassing the] glass [quarter-sphere] and that the form is refracted in the plane of the middle circle, which is perpendicular to the flat face of the glass, where the refraction occurs [away] from the normal, as was previously demonstrated. And if the experimenter looks at the location of the stylus's endpoint, he will find it between where the normal falls [on the apparatus's rim] and the endpoint of the middle circle's diameter, which passes through the centers of the two holes. Therefore, since it extends straight along that line through the air, and since the normal dropped from the center of the [sphere encompassing the] glass [quarter-sphere] on the flat face of the glass extends through the air along with it, the line dropped from the endpoint of the stylus to the center of the [sphere encompassing the] glass [quarter-sphere] will lie between the normal and the line passing through the centers of the two holes. And the form of the stylus's endpoint, which passes from the endpoint of the stylus to the center of the [sphere encompassing the] glass [quarter-sphere], extends along this line and continues straight along it to the center of the [sphere encompassing the] glass [quarter-sphere], for this line is perpendicular to the [convex] spherical surface of the glass, which faces the [stylus's] endpoint.

[4.25] Then, since this form is refracted along the line passing through the centers of the two holes, [it follows] that, in this case, [of all] the radial lines extending from the center of sight only the line passing through the centers of the two holes reaches the glass, for the reed extending between the two holes cuts every line that passes out from the center of sight to the glass except the line passing through the centers of the two holes. But the visual faculty perceives forms only along such straight lines, so [visible] forms only extend [along] straight [lines], [and] so the visual faculty perceives this form only along this line [which is] perpendicular [to the surface of the glass]. Hence, [the form] that extends straight through the air is perpendicular to the surface of the air in contact with the flat face of the glass. This refraction, then, will be away from the normal dropped from the point of refraction on the surface of the air, for the line passing through the centers of the two holes lies farther from the normal that extends through the air than the line dropped from the stylus's endpoint to the center of the [sphere encompassing the] glass [quarter-sphere] and continuing through the air. This form passes from the glass and is refracted in the air, and air is rarer than glass, and this is the way the form was refracted from water to air, for the visual faculty perceives the endpoint of the stylus at this location in water, i.e., it perceives the endpoint of the stylus when it lies between where the normal falls [on the apparatus's rim] and the endpoint of the diameter of the middle circle, which passes through the centers of the two holes. And that form too passed from the water and was refracted in the air, and air is rarer than water.

[4.26] Now the experimenter should remove the glass and place it on the register plate in the opposite way, i.e., he should pose its convex surface toward the two holes and put its common section on the straight line on the register plate where he put it in the previous case, and he should put the midpoint of [that] common section on the center of the register plate. Thus, the line passing through the centers of the two holes will be oblique to the flat face of the glass and normal to its convex surface. He should then affix the glass in this position, insert the apparatus in the vessel, put the endpoint of the stylus at the endpoint of the middle circle's diameter, just as he did before, and he should position his eye at the top hole and look at the apparatus's rim [under water], for in that case he will not see the stylus's endpoint. He should then move the stylus toward where the normal falls [on the rim], and in that case he will not see the stylus's endpoint. Next he should move it away from where the normal falls on the periphery of the middle circle, and he should do so smoothly, in which case he will [eventually] see the stylus's endpoint. It therefore follows that, when the straight line passing from the endpoint of the stylus to the center of the [sphere encompassing the] glass

[quarter-sphere] is extended straight into the glass, and when the normal is dropped from the center of the [sphere encompassing the] glass [quarter-sphere] to the [flat] face of the glass, the line passing through the centers of the two holes will lie between [these] two lines. And when it reached the center of the [sphere encompassing the] glass [quarter-sphere], the form of the stylus's endpoint extending along this line was refracted along the line that passes through the centers of the two holes. Hence, that refraction will be toward the normal dropped from the point of refraction on the surface of the glass, and this form passes from the air and is refracted in the glass, and glass is denser than air.[85]

[4.27] From all these experiments, then, it is obvious that the visual faculty perceives visible objects in water and beyond transparent bodies that differ in transparency from air according to refraction, except those that lie on lines that are perpendicular to the surface of the transparent body in which they lie, and [it is also obvious] that the refraction of their forms occurs in planes that are perpendicular to the surfaces of the transparent bodies, for everything tested with the aforementioned apparatus is found to be refracted in the plane of the middle circle, on the basis of which it was shown that [this plane] is perpendicular to the surfaces of the transparent bodies [in which the objects lie] as well as to the surfaces of the [transparent] bodies in contact with their surfaces. And from this series of tests it will also be manifest that, when they pass from a denser to a rarer [transparent] body, the forms that are perceived by the visual faculty according to refraction are refracted away from the normal dropped from the point of refraction on the surface of the transparent body, whereas those passing from a rarer to a denser [body] are refracted toward the aforesaid normal.

[4.28] Indeed, even the stars are perceived by means of refraction, for the body of the heavens is rarer than the body of air, i.e., of greater transparency.[86] This can actually be empirically determined by an experiment that will show that the stars are perceived by means of refraction, from which it will also be evident that the body of the heavens is more transparent than the body of the air. And if one wants to conduct this experiment, he should take an armillary sphere, set it up on a high location from which the eastern horizon can be seen, and arrange the armillary apparatus in the way appropriate to it, i.e., he should pose the ring representing the meridian circle in the plane of the meridian circle, and its pole should be [pointing] above the Earth according to the altitude of the celestial pole on the horizon at the place where the apparatus is posed.[87] Then at night he should track one of the large fixed stars that passes through the zenith of his location, or nearly so, and he should track it at its rising in the east. When the star rises, he should turn the ring that rotates about the equinoctial pole until it is in line

with the star, and he should determine the star's location on the ring, and he will thus have the [angular] distance of the star from the celestial pole. Then he should track the star until it reaches the meridian circle and adjust the ring, as he adjusted it before, until it is in line with the star, and so he will have the [angular] distance of the star from the celestial pole when the star lies directly overhead. When this is done, he will find that the distance of the star from the celestial pole at its rising is less than its distance from the celestial pole when it lies at the zenith, from which it is clear that the visual faculty perceives stars by means of refraction, not by means of direct vision.[88]

[4.29] [This is so] because the fixed star always moves on the same circle among circles parallel to the equator [which marks the division] of [equal night and] day, and to all appearances it never deviates from [that circle], unless [it does so] over an extremely long [period of] time.[89] If the star were perceived by direct vision, then the lines of sight would extend straight from the center of sight to the stars, and the forms of the stars would extend along straight radial lines until they reached the center of sight. And [so], if the form extended straight from the star to the center of sight, the visual faculty would perceive it in its [actual] location, and so [the experimenter] would find the distance of the fixed star from the celestial pole to be the same during the same night. But the distance of the star from the celestial pole changes during the same night, and so the visual faculty does not perceive the star by direct vision. Moreover, neither in the heavens nor in the air is there a dense, polished body from which forms can be reflected, and since the faculty of sight perceives the star neither by direct vision nor by reflection, it does so by refraction because things are perceived by sight according to these three modes alone [i.e., by direct, reflected, or refracted vision].[90] Consequently, on the basis of the variation in the same star's distance from the celestial pole during the same night it is evident without the slightest doubt that the visual faculty perceives heavenly bodies by refraction. Therefore, the body in which the fixed stars lies differs in transparency from the air.[91]

[4.30] The [relative] transparency of the heavens can be tested subsequently by an experiment with the moon, for [you can begin this experiment] if you compare the location of the moon at a certain time near its rising and if, later in the night [at a] determinate [time] and at a determinate location, you verify its position with regard to the [north] celestial pole [i.e., its angular distance from that pole. This can be done] if you set up an apparatus with sighting-holes before moonrise during that night, and you should [thereby] know the moon's altitude [from the pole]. You then track the moon until its rising, and the time should come to the very minute of the very hour that the moon has [risen], and you should take into account the moon's altitude

from the zenith at that time and make sure that the apparatus for [measuring that] altitude is divided into minutes and less than minutes, if it is possible. You will then find that the distance of the moon from the zenith at that time according to the apparatus is less than its distance from the zenith at that time according to calculation. Hence, the moon's light does not extend through the two [sighting] holes of the apparatus by means of which its elevation is supposed to be correctly found, for then its distance from the zenith would be the same as that which is arrived at by calculation, but the distance found by means of the apparatus differs from the distance [found] by calculation.[92] Thus, the moon's light does not extend from the heavens into the air along straight lines, so [it must do so] by refraction. From these experiments, therefore, it is evident that the visual faculty perceives all heavenly bodies by means of refraction. Therefore, the entire heavens differ in transparency from the air. It remains, then, to show [by geometry] that the body of the heavens differs in rarity from the air.

[4.31] **[PROPOSITION 1]** This will be demonstrated for the experiment just discussed. Accordingly, let ABG [in figure 7.4.45, p. 179] be [a segment of] the circle of the meridian at the location of the experiment, and let B be the zenith, D the [north] celestial pole, E the center of the universe [and circle AF the equator]. Let us connect B and E, let Z [on the Earth's surface] be where the center of sight lies, and let HT be the circle parallel to the equator whose distance from the celestial pole is that at which the star is found at the time the first distance is determined [i.e. at the horizon]. Let H be where the star is at that time, and let KB be the circle parallel to the equator, whose distance from the pole is that at which the star is found at the second time [i.e., at the zenith]. That circle will thus be the one on which the star will lie on a line [passing] straight [through the interface between the heavens and the air], for when the star lies at the zenith, or very nearly so, the visual faculty will perceive it by direct vision because the straight line [EZB] that passes through the center of sight and the zenith is perpendicular to the concave surface of the heavenly sphere as well as perpendicular to the convex surface of the air. And since [that line of sight] is perpendicular to both bodies, the visual faculty will perceive the star on that line by direct vision, whether these two bodies of heaven and air are of different or of the same transparency.

[4.32] Hence, when the star lies at the zenith, or very nearly so, the visual faculty perceives it on its actual circle, which is parallel to the equator and upon which it moved at the beginning of night until reaching the meridian circle. Circle KB is thus the one on which the star lay in the experimental determination [at that point]. Let BHK be the circle of the

zenith that passes through the star at the first time in the experiment, and let that circle intersect circle KB at point K.[93] Since the distance of the star from the celestial pole was less in the first than in the second [stage of the] experiment, circle HT [parallel to the equator] will lie closer to the pole than circle KB, so point H lies closer to the zenith than does point K.

[4.33] Let us join the two lines HZ and KZ. Accordingly, since the star is perceived by the visual faculty at point H at the first time of the experiment, and since it was thus in the plane of vertical circle BHK while at that time the star was [actually] on the circumference of circle KB, the star was at point K at that time, but it was perceived by the visual faculty at point H along straight line ZH, for the visual faculty perceives nothing unless [it does so] by means of straight radial lines along which forms reach the center of sight. Thus, the visual faculty perceives the star at point H because the form reaches it along straight line HZ. And [so], since the visual faculty perceives it along straight line HZ, and since the straight line between the star and the center of sight is [actually] KZ, it is obvious that the visual faculty does not perceive the star that is at point K by direct vision, so [it perceives] it by means of refraction.

[4.34] So let M [in figure 7.4.45a, p. 179] be the point of refraction [within the plane of refraction defined by circle BHK in figure 7.4.45], and let us join KM and extend it straight to Z.[94] Accordingly, the form of the star, which reaches Z, at which [point] the visual faculty perceives the star, extends from the star along line KM and is refracted along line MZ. Moreover, forms only refract when they encounter a body of different transparency from the transparency of the body within which they lie. Therefore, the body in which the star lies, i.e., the heavens, is of a transparency differing from the transparency of the air, and [it follows] that the point of refraction is at the interface passing between two bodies that differ in transparency. Point M is therefore a point on the concave surface of the heavens [contiguous with the convex surface of the atmosphere]. Let us draw a line between E and M, and let it be a [radial segment of a] diameter of the heavenly sphere. Line EM will thus be perpendicular to the concave surface of the heavens that is in contact with the air and [also] to the convex surface of the air. And since the form of the star at point K extends along line MK and will be refracted in the air along line MZ, it is clear that this refraction occurs toward normal EM, which passes through the point of refraction and is normal to the surface of the air. Since, therefore, the refraction in the air is toward the normal passing through the point of refraction, the body of the air is denser than the body of the heavens.[95]

[4.35] It is thus evident that what we discover about the appearance of the stars by experiment indicates conclusively that the visual faculty

perceives the stars solely by means of refraction, that the body of the air is denser than the body of the heavens, and that the body of the heavens is rarer than the body of the air. From all these [experimental demonstrations], then, it is clear that all objects that are perceived by the visual faculty through transparent bodies whose transparency is different from the transparency of air are perceived by means of refraction if the center of sight lies to the side of the normals dropped from those objects to the surface of the transparent bodies in which they lie.

CHAPTER FIVE
On Image[s]

[5.1] An image is the form of a visible object that the visual faculty perceives through a transparent body whose transparency differs from the transparency of air, when the center of sight lies to the side of the normals dropped from that visible object to the surface of that transparent body, for the form of the visible object that the visual faculty perceives through the transparent body is not the visible object itself because in this case the visual faculty does not perceive the visible object in its [actual] location nor in its [true] form but in another place and in another way, i.e., according to refraction, and yet it perceives that object in a directly facing position. Such a form is called an "image."[96] This [point], moreover, is understood on the basis of reason as well as empirical test.

[5.2] By reason, on the one hand, it is evident from the preceding chapter that a visible object [lying] in a transparent body that differs in transparency from the air is perceived by the visual faculty according to refraction if the center of sight lies to the side of the normals dropped from the visible object to the transparent body's surface. Since the visual faculty perceives the visible object in such a refracted way, [and since] the object does not face the eye directly, [and since] the visual faculty does not perceive it directly or sense that it is perceiving it according to refraction, it is obvious that it perceives the object outside of its [actual] location.

[5.3] By empirical test, on the other hand, this can be understood as follows. If one takes a container with a rim that is perpendicular [to its bottom] and places some conspicuous visible object, such as an *obolus* or *denarius*,[97] on the middle of its [bottom], and if he stands away from it so that he can see the visible object at the bottom of the container and then moves back from the visible object little by little until he cannot see the object [any longer], then if he stands at the point where the coin first disappears

from view and directs someone else to pour water in the container while he stays in place without moving his line of sight or his position, then when he looks into the water in the container, he will see the visible object after not having seen it.[98] He will also see it in a directly facing position, from which it is evident that the form he sees in the water is not where the visible object [actually] is, for the form he saw in the water in the container is not where the visible object [actually is] because, if the form were in the [actual] place [occupied by the] visible object, the visual faculty would perceive the visible object [there] when there was no water in the container. For in the second situation the visual faculty perceives the visible object in a directly facing position when it is not [actually there]. In this way, then, it will be demonstrated by both means, namely, reason and experiment, that the image of a visible object that the visual faculty perceives according to refraction does not lie where the visible object [itself] lies.

[5.4] I say, then, that the image of any point that the visual faculty perceives according to refraction lies at the point that forms the common section of the line along which the form reaches the center of sight and the normal dropped from that visible point to the surface of the transparent body.[99] This, moreover, will be proven by experiment in the following way. Let someone take a wooden disk whose diameter is no less than a cubit, and let him plane and smooth its surface as thoroughly as he can. He should then find its center, draw as many intersecting diameters through it as he wishes, score them with an iron [incising tool] so that they are [clearly] visible, and fill those [scored] lines with a white substance, such as white lead mixed with snow-white milk,[100] and the centerpoint should be black. When this is done, he should take a wide vessel, such as a wash-basin, that has a high brim, and he should place the vessel in a lighted spot. Then he should pour clear water into the vessel, and the depth of the water should be less than the diameter of the [wooden] disk but greater than its radius. He should determine this with the disk itself until the water rises above the center of the disk by some [number of] digits, i.e., above the two or more diameters scored [on the wooden disk] in that vessel, so that the water covers some portion of each diameter while the other portion remains above water.

[5.5] He should wait until the water in the vessel becomes still, then insert the wooden disk in the vessel and stand the disk upright on its rim, and he should pose the surface on which the lines are scored toward his eye and then turn the disk until one of its diameters is perpendicular to the water's surface. Afterwards he should lower his eye and set the vessel up so that his line of sight outside the rim of the vessel is nearly parallel to the

water's surface but high enough above the water's surface that the center of the disk can be seen, for the experiment will be clearer [if it is conducted] in this way.

[5.6] When this is all done, therefore, he should look at the center of the disk as well as the diameter on the disk that is perpendicular to the water's surface, for in that case he will find that the disk's center lies in a straight line with the perpendicular diameter. Then he should look at a slanted diameter that has a portion standing above the water, for in that case he will find that it is bent and that its bending will occur at the water's surface. Furthermore, the portion that is under water will form an obtuse angle with the portion that is above water. He will also find that the angle faces the perpendicular diameter, and he will find that the portion under water continues in a straight line, from which it is clear that the form of the point at the center of the circle, i.e., the form that the visual faculty perceives, is not [actually] at the center of the disk, for if it were at the disk's center, it would lie on the straight continuation of the slanted diameter because in actuality that is where it is.[101]

[5.7] Since, therefore, the visual faculty perceives this point outside the straight continuation of the slanted diameter, and since the angle that the segments of the slanted diameter form faces the perpendicular diameter, the point that constitutes the form of the center lies above the [actual] center. Moreover, because the visual faculty perceives this point on the straight continuation of the diameter that is perpendicular to the water's surface, this point, which constitutes the form of the point at the center, will lie outside the center and above the center, yet nonetheless it lies on the normal dropped straight from the [actual] center to the water's surface. It will also be clear from the bending of the slanted diameter at the water's surface, as well as from the straightness of the [portion] of that diameter and its continuation under water, that every point on the portion of the slanted diameter under water lies above its [actual] location.[102]

[5.8] Next the experimenter should revolve the wooden disk until the slanted diameter becomes perpendicular to the water's surface and the diameter that was perpendicular becomes slanted. He should then lower his eye and look at the disk's center, and in that case he will find the form of the center on the straight continuation of the diameter that is now perpendicular to the water's surface and outside of which the form of the center lay when the diameter was slanted, and he will find the form [of the center] outside the straight continuation of the diameter that is now slanted but was perpendicular to the water's surface before. He will also find the slanted diameter bent at the water's surface, and the

angle of [its] bending will be on the side [away] from the diameter [that was formerly] slanted. Furthermore, if there are several diameters on the disk, and if the experimenter revolves the disk so that each of them in turn is perpendicular to the water's surface, and if the diameter next to that diameter is slanted and part of it lies outside the water, then he will find that the form of the point at the center of the disk on the straight continuation of the perpendicular diameter always lies above the straight continuation of the slanted diameter. And he will always find that what lies under water is straight.

[5.9] From all these [observations] it is therefore evident that the form of any point perceived by the visual faculty in a transparent body denser than the body of air is perceived outside its [actual] location and above its [actual] location on a straight line with the normal dropped from that point to the surface of the transparent body, when the line connecting the center of sight with that point is not perpendicular to the transparent body's surface. Furthermore, every point is perceived by the visual faculty in a directly facing position on the straight line along which the form extends to the center of sight, so points that the visual faculty perceives by means of refraction are perceived in a directly facing position on the straight line along which the form reaches the center of sight.

[5.10] This will be demonstrated by means of an experimental test of the perception of visible objects according to refraction based on the apparatus discussed earlier [in chapters 3 and 4], for if the experimenter closes off the second hole in the apparatus [i.e., the one in the panel attached to the register plate], he will not perceive a visible object that he perceived [earlier through the holes] according to refraction, but when he closes the second hole, he has done nothing other than cut the imaginary straight line extending from the center of sight to the point of refraction, from which it is clear that the form extending from what is seen in the transparent body in which the visible object lies and refracted in the transparent body in which the center of sight lies extends along the straight line that passes from the center of sight to the point of refraction. And every point perceived by the visual faculty in a transparent body denser than the body of air is perceived at the point that forms the common section of the line along which the form reaches the eye and the normal dropped from the visible point to the surface of the transparent body on the side of the center of sight, provided that the center of sight lies outside the normal dropped from that point to the transparent body.

[5.11] Now if the experimenter wants to test the image of a visible object whose form is refracted from a rarer body to a denser body, he should take a block of glass that has flat, parallel surfaces and is eight digits long, four

digits high, and four digits wide. He should then take the wooden disk just described, mark off a chord ten digits long on its back side, bisect it, and connect the point of bisection with the center of the disk by a straight line extending [to the disk's edge] on both sides [of that point]. This line will thus be perpendicular to the first line. Then he should connect either endpoint of the chord to the center of the circle with a straight line that also extends on both sides, and these two diameters should be scored [into the wood] with the iron [incising too], and the perpendicular should be filled with the white substance while the other [is filled] with some other kind of [colored] substance.

[5.12] Next he should place the glass block lengthwise on the back of the circular wooden device [upon which the new lines have been incised] and apply one of its long edges on the middle of the [bisected] chord, and he should mark off three digits on the glass, two of which will lie beyond the slanted diameter outside the circle, and one digit of the glass's length will remain [to project] beyond the diameter that is perpendicular to the chord. The glass block should lie on the side of [i.e., above] the center, and [the experimenter] should affix the glass firmly to the wooden disk according to this position. Thus, the diameter that is perpendicular to the chord will be perpendicular to the parallel edges of the glass, and the other diameter will be inclined to these two surfaces.[103]

[5.13] The experimenter should then put the edge of the disk on which the projecting end of the glass lies toward his eye, placing one eye at the common section of the circumference and the [top] edge of the glass, which is the endpoint of the slanted diameter, and he should bring his eye as close as he can to the glass so that he cannot see any of its [top] surface except the endpoint of the slanted diameter. The remaining eye will lie on the side where the glass and the disk lie. Then he should block the part of the glass that faces the other eye with a piece of paper[104] that he places over some portion of the glass so that he may [only] see the slanted diameter, which is the last line [viewed] by the one eye that is up close to the glass, and [so that] he may not see beyond this line but may see the white perpendicular line with both eyes.

[5.14] When everything is in place this way, he should look at the disk's center, and he will find that it lies on the straight continuation of the white line that is perpendicular to the surface of the glass. He should also look at the slanted diameter at whose endpoint he holds his eye, and in that case he will see it bent at the surface of the glass on the side of the center, and he will find that the angle of bending faces the circumference [of the wooden disk]. Moreover, the visual faculty perceives the part of this slanted diameter below the glass in a straight line, for the eye is right up against

the surface of the glass, and one part of the perpendicular diameter lies below the glass, another beyond the [bottom edge of the] glass on the side of the center, and another beyond the glass on the side of the diameter's endpoint.

[5.15] Consequently, the part below the glass is perceived by the visual faculty outside the glass according to refraction, and the part on the side of the diameter's endpoint is perceived by the visual faculty outside the glass, and it is seen straight outside the glass without refraction. The part that lies on the side of the center is perceived by both eyes according to refraction because, when they reach the surface of the glass at the edge on the side of the center, all the lines extending from the center of sight that is right up against the glass and [then] continuing through the body of the glass will be oblique to the glass's surface. Therefore, the part of the perpendicular diameter on the side of the center is perceived according to refraction by the eye right up against the glass.

[5.16] On the other hand, the lines extending from the remaining eye to the top surface of the glass will be oblique to the top surface of the glass, and when they continue to the other surface of the glass that lies on the side of the center, they will also be slanted. Thus, the remaining eye also perceives the part of the perpendicular diameter on the side of the center according to two refractions, the part below the glass by one single refraction and the upper part without refraction, and despite all this both eyes perceive this diameter straight. And if the experimenter covers one or the other eye and looks with the eye that lies on the side of the glass, he will perceive the perpendicular [diameter as] straight. And if he raises his eye from the glass and looks at the perpendicular diameter behind the glass, he will perceive it as straight despite his perceiving it according to refraction.[105]

[5.17] The reason for this is that, when it is perceived by the visual faculty according to refraction, no point on the perpendicular diameter is perceived in its [actual] location, but [the visual faculty] still perceives it in a location on the normal that extends straight from it to the surface of the glass, and that [perpendicular] diameter constitutes a normal dropped from any point on it to the surface of the glass, and no point on it is perceived according to refraction except on that diameter. Therefore, since the visual faculty perceives this diameter as straight and perceives the form of the center on the straight continuation of this diameter, the form of the center that the visual faculty perceives through the glass lies in a straight line with the normal dropped from the center to the surface of the glass, when the eye is right up against the glass.

[5.18] Moreover, when [the visual faculty] perceives the slanted diameter as bent, it does not perceive the portion of it that passes out of the glass on the side of the center in its [actual] place. [Thus] the centerpoint is only perceived by the visual faculty [somewhere] other than in its [actual] place, and since the angle of bending faces the circumference [of the wooden disk], the point that constitutes the form of the center lies below the [actual] center, from which it is clear that the image of any point perceived by the visual faculty through a transparent body rarer than the transparent body on the side of the eye lies on the straight line dropped orthogonally from that point to the surface of the transparent body on the side of the eye, and it lies farther from the surface of the transparent body on the side of the eye than the [actual] point itself. And every point perceived by the visual faculty lies on the straight line along which the form reaches the center of sight, and the image of any point perceived by the visual faculty through a transparent body that is rarer than the transparent body on the side of the eye lies at the common section of the line along which the form reaches the eye and the normal dropped from the visible point to the surface of the transparent body on the side of the eye.

[5.19] Hence, from everything that has been shown in this chapter it is evident that, when the eye lies to the side of the normals dropped from the visible object to the surface of the transparent body on the side of the eye, the image of any point on any visible object perceived by the visual faculty through a transparent body differing in transparency from the transparency of the body on the side of the eye lies at the common section of the line along which the form of that point reaches the eye and the normal dropped from that point to the surface of the transparent body on the side of the eye, whether the transparent body on the side of the eye is rarer or denser than the transparent body on the side of the visible object.

[5.20] Why the visual faculty perceives a visible object at the image location and why the image lies at the intersection of the line along which the form reaches the eye and the normal dropped from the visible point to the surface of the transparent body will be explained as follows. Now it is manifest that the visual faculty perceives the form of a visible point that it perceives by means of refraction on the straight line along which the form reaches the eye, and the reason for this has been discussed in the preceding chapters, and it is because the visual faculty perceives nothing unless [it does so] along straight radial lines, for it is only affected along these lines.[106]

[5.21] But the reason it perceives the form on the normals dropped from the visible object to the surface of the transparent body is that, as we showed in the second [chapter], when light extends through a transparent

body, it does so with an extraordinarily swift motion. And in the fourth chapter of this book [actually chapter 2, paragraphs 2.81-87] we showed that the motion of light through a transparent body along a line slanted to the surface of that body is composed of motion along the normal dropped from the point to which the light extends on the surface of that transparent body and motion along a line orthogonal to this perpendicular line. Consisting of the form of the light in a visible point mingled with the form of color, the form that extends according to refraction from the visible point to the point of refraction always extends along a line that is oblique to the surface of the transparent body. Hence, this form extends to the point of refraction with a motion composed of the motion along the normal dropped from the visible point to the surface of the transparent body and the motion along a line orthogonal to this normal.

[5.22] The motion of a form moving along the normal to the surface of the transparent body and then diverted from that normal by another motion or along the perpendicular to the first perpendicular and diverted after its motion on the first perpendicular therefore occurs with a motion composed of the two aforementioned motions. But this point is perceived by the visual faculty on the straight line along which the form reaches the center of sight. Hence, the form at the point of refraction reaches it according to the motion of a form moving along a line normal to the surface of the transparent body and then diverted from that normal by the motion on the straight line along which the form reaches the center of sight.

[5.23] Moreover, a form that lies on the normal to the surface of the transparent body and then moves on the straight line along which the form extends to the center of sight is a form that extends from the visible point in a straight line with the normal dropped from it to the surface of the transparent body until it reaches the point of intersection between this normal and the line along which the form extends to the center of sight. Thus, the form of a point that the visual faculty perceives by means of refraction through a transparent body results from the motion of the form that reaches the center of sight from the image location. And the visual faculty perceives this form at the image location because it results from the motion of the form that the visual faculty perceives straight on without refraction, and this location lies as far from the center of sight as the image point whose location with respect to the center of sight is the location of the form at the image location, so the visual faculty perceives that form at the image location according to refraction.

[5.24] This, then, is why the visual faculty perceives a visible object through a transparent body at the image location and why the image of any point on a visible object that is perceived by means of refraction lies

where the line along which the form reaches the eye intersects the normal dropped from that point to the surface of the transparent body.[107]

[5.25] Now that this has been shown, we should point out that no visible object perceived by the visual faculty through any transparent body that differs in transparency from the body on the side of the eye has more than one single image if the [transparent] body is among the common [transparent] bodies.[108] The heavens, air, water, glass, and transparent stones are customary transparent bodies, and the surface of the heavens that faces the eye is concave spherical, so every plane that cuts it forms a circular line on it with its concavity facing the eye. The surface of the air in contact with it is convex spherical, so, if it is cut by a plane, [that cut] will form a circular line on it with its convexity facing the heavens. The surface of water facing the eye, on the other hand, is spherical convex, and if it is cut by a plane surface, [that cut] will form a circular line on it with its convexity facing the eye.[109]

[5.26] The customary shapes of glass [objects] and transparent stones are round or flat, so if they are cut by planes, [those cuts] will form either circles or straight lines on them. And [so] we say that, generally speaking, every point perceived by the visual faculty through any transparent body whose surface facing the eye is a single [simple] surface, has only one image and is perceived as only one point, if, when [that surface] is cut by a plane surface, [that cut] will form a straight or circular line on its surface.

[5.27] **[PROPOSITION 2]** So let A [in figure 7.5.49, p. 184] be the center of sight and B the visible point. Let the transparent body in which B lies have [point] G on its surface, and let the transparency of this body be denser than the transparency of the body on the side of the eye [at A]. Let the surface of the body on the side of the eye be plane, and let us erect perpendicular AGC from point A. Hence, point B will either lie on line AGC, or it will lie outside it.

[5.28] Accordingly, if point B lies on line GC, the center of sight at A will perceive B straight on without refraction, for when the form of B extends along BG, it passes out to the [transparent] body on the side of A straight along BG because BG is perpendicular to the surface of the [transparent] body on the side of the eye. Therefore, the center of sight at A perceives B in its [actual] location along straight line AGB.

[5.29] We say, then, that [the form of] point B is never refracted to A outside this line, but if it is possible [for it to do so, let us suppose that] the form of B is refracted to A from T. Let us produce the plane containing perpendicular AGB and point T. It will therefore cut a straight line on the surface of the transparent body. Let it be GTD, then, and let us extend a

perpendicular to line GD from point T, and let it be KTL. KTL will thus be perpendicular to the transparent body's surface. Let us then connect BT and extend it to H.

[5.30] Angle KTH will thus be [equal to] the angle [BTL] that the line along which the form extending [to the surface of the transparent body] makes with the normal dropped from the point of refraction to the surface of the transparent body. Consequently, since the [transparent] body on the side of A is rarer than the one on the side of B, then when [the form of] B reaches T, it will be refracted away from normal TK. Hence, the refracted form does not reach line AB, yet it is [supposedly] refracted to A, which is impossible. The form of B will therefore not be refracted to A from T, nor from any other point [outside line AGB]. Hence, [the center of sight at] A will only perceive B along straight line AGB, so it perceives it at only one point, and this [is what] we wanted to prove.

[5.31] **[PROPOSITION 3]** If B lies outside AGC [as in figures 7.5.50 and 7.5.50a, p. 185], let us form the plane containing line AGC and point B. It will thus be perpendicular to the surface of the transparent body, and let it form line GD on the surface of this body. GD will thus be straight. Hence, the form of B will be refracted to A only in the plane containing GD, for only one plane perpendicular to the surface of the transparent body passes through the two points A and B, or passes through perpendicular AC, and [also] passes through perpendicular AC and point B. Thus, the form of B is refracted to A only from line GD.

[5.32] So let the form of B be refracted to A from point E, and let us connect the two lines BE and EA and erect a perpendicular from E on line GED. Let it be HEZ, then. HEZ will thus be perpendicular to the two surfaces of the two [contiguous] transparent bodies [on the sides of A and B]. Let us extend BE straight to T. ET will therefore lie between the two lines EH and EA, for the transparent body on the side of A is rarer than that on the side of B. Consequently, when the form of B, which extends along line BE, reaches E, it will be refracted away from normal ZEH, so line ET will lie between the two lines EH and EA.

[5.33] From B let us drop a perpendicular, i.e., BK, to line GD. BK will thus be perpendicular to the surface of the transparent body on the side of B. Let us also extend AE straight so that it cuts angle BEK and intersects line BK at M. M will thus be the image of point B, and angle TEA will be the angle of refraction. I say, then, that B will have no image other than M, and its form will not be refracted to A from any point other than E.

[5.34] The proof of this claim is based on the proof that [the form of] B is perceived by the center of sight only on normal BK [i.e., the cathetus of

incidence]. Thus, if B has another image, it will lie on line BK and between the two points B and K, for the [transparent] body on the side of B is denser than that on the side of A. So let that other image be point N, if such is possible. N will therefore lie either between the two points M and K or between the two points M and B.

[5.35] [Let it lie between M and K, as in figure 7.5.50, p. 185] and let us connect AN. Hence, it will intersect line GD at point O. Let us connect BO, and let it continue to L. O will thus be the point of refraction because line AON is the one along which the form [of the point] at N extends to A, and [so] angle LOA will be the angle of refraction. From O let us drop perpendicular FOQ to line GD. Line FOQ will thus be perpendicular to the surface of the transparent body, and angle LOF will be [equal to] the angle [of incidence BOQ] that the normal makes with the line along which the form extends to the point of refraction.

[5.36] Now if N lies between the two points M and K, O will lie between the two points E and K, so angle EBK > angle OBK. Consequently, angle TEH [which = corresponding angle EBK] > angle LOF [which = corresponding angle OBK]. But angle TEA is the angle of refraction for angle [of incidence BEZ, which = angle] TEH, whereas angle LOA is the angle of refraction for angle [of incidence BOQ, which = angle] LOF. Therefore, angle TEA > angle LOA, as was demonstrated in the third chapter of this book [where it was shown that a larger angle of incidence yields a larger angle of refraction], so angle AEH > angle AOF, which is impossible.[110]

[5.37] If, however, N lies between the two points M and B [in figure 7.5.50a, p. 185], then point E will lie between the two points O and K, and angle EBK < angle OBK. Therefore, angle TEH < angle LOF, so angle TEA, which is the angle of refraction [for angle of incidence BEZ], is less than angle LOA, which is the angle of refraction [for angle of incidence BOQ]. Hence, angle AEH < angle AOF, which is impossible. It is therefore impossible for point N or any point other than M to be the image of point B, so with respect to center of sight A, point B has no image other than point M, and this [is what] we had to demonstrate.

[5.38] **[PROPOSITION 4]** To continue, let the denser [transparent] body lie on the side of the eye [at A in figures 7.5.51 and 7.5.51a, p. 186] and the rarer one on the side of the visible object [B], and let the common section of this plane [containing A and B] and the surface of the transparent body be line GD. Let us drop a perpendicular from B to line GD, and let it be BK. BK will thus be perpendicular to the surface of the transparent body. And let the form of B be refracted to A from E, and let us connect [lines] BE and EA. Let us then erect perpendicular HE and extend BE straight to T.

[5.39] Line AE will therefore lie between the two lines ET and EH, for the first line along which the form extends to the point of refraction is line BET. However, the refraction is toward normal EH because the [transparent] body on the side of A is denser than the one on the side of B, so line AE lies between the two lines ET and EH. Let us extend AE straight on the side of E until it meets line BK, for it will cut HEZ [parallel to BK]. So let it meet it at point M. M will therefore be the image of point B because the [transparent] body on the side of B is rarer than the one on the side of A. I say, then, that B has no image except for M.

[5.40] So if it is possible, let it have N [as an image]. N will therefore lie on normal BK and below point B, since the [transparent] body on the side of B is rarer than that on the side of A. Hence, it lies between the two points M and B or below M. Let us connect AN. It will therefore intersect line GD at O, so O is the point of refraction. Let us connect BO; let it continue to L, and let us erect perpendicular FOQ at O. Hence, line BO is the one along which the form extends to the point of refraction; so line AO will lie between the two lines OL and OF because the refraction occurs toward the normal [FO].

[5.41] If, therefore, N lies between the two points M and B [in figure 7.5.51], point O will lie between the two points E and K. Consequently angle OBK < angle EBK, so angle LOF < angle TEH. Angle LOA, which is the angle of refraction [for angle of incidence BOQ, which = angle LOF], is therefore less than angle TEA, which is the angle of refraction [for angle of incidence BEZ, which = angle TEH]. As we demonstrated in the third chapter of this book, moreover, angle AOF, which remains after angle of refraction [LOA is subtracted from LOF], is less than angle AEH, which remains after angle of refraction [TEA is subtracted from TEH]. But angle AOF = [corresponding] angle ANK, and angle AEH = [corresponding] angle AMK, so angle ANK < angle AMK, which is impossible.

[5.42] On the other hand, if N lies below M [in figure 7.5.51a], E will lie between the two points O and K, and angle OBK > angle EBK, so angle LOF > angle TEH. Therefore, angle LOA > angle TEA. But [that requires that] angle AOF > angle AEH, from which it follows that angle ANK > angle AMK, which is impossible. N is therefore not an image for B, nor is any point other than M, so B has no image other than M, and this is what we wanted [to demonstrate].

[5.43] **[PROPOSITION 5, LEMMA 1]** We will [now] make the following preliminary point about two convex or concave circular lines: if two chords intersect each other within a circle, the angle of intersection will be equal to the angle at the circumference that the two arcs those two chords mark

off subtend, and if the two lines intersect the circle but intersect each other outside the circle, the angle of intersection will be equal to the angle at the circumference subtended by the difference between the larger and smaller of those two arcs that those two lines mark off [on the circumference].

[5.44] For instance, [in figure 7.5.52, p. 187] let the two chords AG and BD in circle ABG intersect one another at E. I say, then, that angle AEB is equal to the angle on the circumference that the two arcs AB and GD subtend, and that angle BEG is equal to the angle on the circumference that the two arcs AD and GB subtend.

[5.45] The proof of this [claim is as follows]. We will draw line HBZ from [point] B parallel to line AG. Thus, arc GZ = arc AB, and arc GD is common, so arc DZ is equal to the two arcs AB + GD. But arc DZ subtends angle DBZ, so DZ spans an arc equal to the two arcs AB + GD. But angle DBZ = [alternate] angle AEB, so angle AEB = the angle on the circumference that the two arcs AB + GD subtend, and this is what we wanted [to demonstrate].

[5.46] To continue, let us connect DZ. Therefore, [exterior] angle HBE [of triangle BDZ] is equal to the two [interior] angles BDZ + BZD, and the two angles BZD and BDZ are subtended by the two arcs DB and BZ, so angle HBE is equal to the angle that arcs DB + BZ subtend. Arc AB = arc ZG [by construction], and arc DABZ is equal to the two arcs DA + BG, so angle HBE is equal to the angle that the two arcs DA + BG subtend. But angle HBE = [alternate] angle BEG, so angle BEG is equal to the angle on the circumference that the two arcs DA + BG subtend, and this is what we wanted to demonstrate.

[5.47] If, moreover, line HBZ is tangent to the circle [in figure 7.5.52a, p. 187], then angle EBZ will be equal to an angle [with its vertex] falling on segment BAD [of the circle, by Euclid, III.32], and so arc BGD subtends an angle at the circumference equal to angle EBZ. But angle EBZ = [alternate] angle BEA. Therefore, angle BEA is equal to an angle at the circumference that arc BGD subtends, and arc BG = arc BA because the diameter that passes from B is perpendicular to line AG, since it bisects it.[111] Therefore, arc AB = arc BG, so arc BGD will be equal to the two arcs BA + GD. Angle BEA is thus equal to the angle at the circumference that the two arcs AB and GD subtend. And it will be demonstrated likewise that angle BEG is equal to the angle at the circumference that the two arcs BG and AD subtend, and this is what we wanted [to demonstrate].[112]

[5.48] Now let E lie outside circle ABGD [in figure 7.5.52b, p. 188], let us extend two lines from E to intersect circle ABGD, and let them be EAD and EBG. I say, then, that angle GED is equal to the angle at the circumference that the difference between arc DG and arc AB subtends.

[5.49] The demonstration of this [claim is as follows]. Let us draw line [AZ] parallel to line BG. Arc ZG will therefore be equal to arc AB, so arc DZ is the difference between arc DG and arc AB. But arc DZ subtends angle DAZ, and angle DAZ = [corresponding] angle GED. Therefore, angle GED = angle DAZ on the circumference, and this is what we wanted [to demonstrate].

[5.50] **[PROPOSITION 6]** Now that these [points] have been established, let point A [in figure 7.5.53, p. 188] be the center of sight, let point B lie on some visible object, and let it lie in a transparent body denser than the body that lies on the side of the center of sight. Let the surface of the transparent body on the side of B be a circular surface with its convexity facing the center of sight. Therefore, [there is] a plane [that] passes through both points A and B and is perpendicular to the surface of the transparent body, but only one plane passes through the two points A and B orthogonal to the surface of the transparent body [and] in that plane the form of B is refracted to A. Let circle GED, whose center lies at Z, represent this plane on the transparent body, and let us draw AGD. Line GZD will thus be perpendicular to the surface of the transparent body, and point B will lie either outside line GD or on it.

[CASE 1]

[5.51] Accordingly, if B lies on line GD, then center of sight A will perceive B straight on without refraction, for the form extending along line GD extends straight through the transparent body on the side of center of sight A, since line GD is perpendicular to the surface of the transparent body on the side of the center of sight. Consequently, center of sight A perceives B in its [actual] location and [it does so] directly. I say, then, that the form of B on line GD is never refracted to A.

[5.52] The proof of this [point depends on the fact] that point B will lie either at the center or outside the center [of the circle]. If, then, it lies at the center, every line along which the form of B extends straight to circumference GED extends through the transparent body on its side, for every line extending from the center of circle GED is perpendicular to the surface of the transparent body, and no straight line other than line ZA extends from the center of circle GED to center of sight A. Thus, the form of B at the center is not refracted to A at circumference GED, so the form of B is never refracted to A if B lies at the center.

[CASE 2]

[5.53] If, on the other hand, it lies outside the center, it will lie either on line ZG or on [line] ZD. For a start, then, let it lie on line ZG. I say that the form of B is not refracted to A, but if it is possible, let it be refracted at E.

Let us connect BE and continue [that] line to H, and let us connect ZE and continue [that] line to T. Line ZET will therefore be normal to the surface of the transparent body on the side of the center of sight [at A]. Hence, when it extends along line BE and is refracted at point E, the form of B passes through normal TE[Z] toward H away from the normal. Thus, the form of B will not reach A by refraction if B lies on line ZG.

[5.54] Now let B lie on line DZ [at point B′]. I say, then, that the form of B′ is not refracted to A, but if it is possible, let it be refracted at E. Let us connect [line] B′E and extend the line to R, and let us connect ZE and extend the line to T. Let the form of B′ be refracted to A along line EA. Thus, angle REA will be the angle of refraction, and angle RET will be [equal to] the angle [of incidence B′EZ] that line [B′E] along which the form extends makes with normal [ZET] dropped from the point of refraction. Angle REA is therefore smaller than angle RET [by rule 5, p. 260], and line B′Z is either less than or equal to line ZE, for B′ lies either between points D and Z or at point D. Consequently, angle EB′Z is either greater than or equal to angle B′EZ. But angle AER > angle EB′Z, so angle AER > [angle] B′EZ. Therefore, angle AER > angle RET, although it was previously [assumed to be] smaller, which is impossible.

[5.55] [No matter where B is on line GD], then, the form of B will not be refracted to A from E, nor from any other point on circumference GED, nor from any other circumference on the [great] circles that lie on the surface of the transparent body in which B lies. Hence, if B lies on line GD, it is not perceived by the visual faculty according to refraction, so it is perceived as only one point.

[CASE 3]

[5.56] To continue, let B lie outside line GD [in figure 7.5.53a, p. 189], and we will produce the plane containing normal AD and point B. This plane will therefore be perpendicular to the surface of the transparent body, and [the form of] point B is refracted to A only in this plane, for no plane perpendicular to the transparent body's surface passes through the two points A and B except the one passing through line AD, and only one plane through line AD passes through B. So let this plane cut circle GED on the surface of the transparent body. The form of B will therefore be refracted to A only from the circumference of GED, so let it be refracted at E. I say, then, that it is refracted from no point other than E.

[5.57] So, if it is possible, let it be refracted from another point, which, as was [just] said, will lie on circumference GED. Let [that point] be M. Let us connect lines BE, EA, BM, MA, ZE, and ZM, and let lines BM and ZE intersect one another at C. Let us extend BE to H, BM to N, EZ to T, and ZM to L. Therefore, angle HET will be [equal to] the angle [of incidence BEZ]

that the line along which the form extends makes with the normal dropped from the point of refraction, and angle HEA will be the angle of refraction; NML will also be [equal to] the angle [of incidence BMZ] that the line along which the form extends makes with the normal dropped from the point of refraction, and angle NMA will be the angle of refraction.

[5.58] Angle HET will be equal to, less than, or greater than angle NML. If it is equal, then angle HEA, which is the angle of refraction [tied to HET], will be equal to angle NMA, which is the angle of refraction [tied to NML]. Therefore, angle AMB [supplementary to NMA] = angle AEB [supplementary to HEA], which is impossible. If it [i.e., HET] is less [than NML], then angle HEA < angle NMA, so angle AMB < angle AEB, which is impossible.

[5.59] If it [i.e., HET] is greater [than NML, which is the case in figure 7.5.53a], we will extend line EB to F on the side of B, and we will extend MB to O. Thus, angle EBM will be equal to the angle at the circumference that the two arcs EM + FO subtend [by proposition 5, lemma 1], and since angle HET > angle NML [by supposition], angle ZEB > angle NML. Since, moreover, angle ZEB > angle NML, angle MZT > angle MBE,[113] and angle MZE – angle MBE = angle ZEB – angle ZMB, for the two angles at C are equal.[114] Therefore, the arc subtending angle MZE will be twice arc ME, when [the angle subtended by it] lies at the circumference.

[5.60] If, therefore, angle MZE > angle MBE, twice arc ME [subtending an angle equal to MZE at the circumference] will be greater than the two arcs ME + FO [subtending an angle at the circumference equal to angle MBE]. And 2 arc ME – (arc ME + arc FO) = arc ME – arc FO. Therefore, angle MZE – angle MBE is equal to the angle at the circumference subtended by arc ME – arc FO. But arc ME – arc FO < arc ME + arc FO, so angle MZE – angle MBE < angle MBE. Consequently, angle ZEB – angle ZMB [which = MZE – MBE by previous conclusions] < angle MBE. Therefore, angle HET – angle NML < angle MBE. As a result, the difference between angle HEA, which is the angle of refraction [tied to HET], and angle NMA, which is the angle of refraction [tied to NML] is *a fortiori* less than angle MBE.

[5.61] But angle HEA – angle NMA = angle AMB – angle AEB, so angle AMB – angle AEB < angle MBE. However, angle AMB – angle AEB = the two angles MAE + MBE. Therefore, the two angles MAE + MBE < angle MBE, which is impossible. The form of B will thus not be refracted to A from any point other than E, and this is what we wanted [to demonstrate].[115]

[5.62] **[PROPOSITION 7]** Consequently, since the form of B is refracted to A from only one point, it will have only one image. The location of the image, however, varies according to the variation in where B lies. So let us

draw [lines] BZ [B′Z and B″Z in figure 7.5.53b, p. 189, according to three locations for B]. Line BZ [or B′Z, or B″Z] will intersect line EA, or it will be parallel to it, and the intersection will be either on the side of EB, as in K, or on the side of A, as in R. And when B′Z is parallel to line EA, it will be like line B′Z[X], which lies between the two lines KBZ and B″ZR.

[5.63] Hence, if the intersection of these two lines is at K, the image will lie in front of the center of sight, and the form will be clear and [will be] perceived by the visual faculty at K. If, however, the intersection is at R, point R will be the image, and in that case the form will be perceived by the visual faculty straight ahead, but not clearly because it is still perceived outside of its [true] location by the visual faculty. This was shown in the place where we discussed reflection. If line BZ is parallel to line EA the image will be indefinite, and the form will be perceived at the point of refraction. The reason for this is similar to the one we discussed in regard to reflection when reflection occurs along a line parallel to the normal.[116]

[5.64] From the foregoing it is therefore evident that an object perceived by the visual faculty through a transparent body that is denser than the body on the side of the eye has only one image, and it will only be perceived as single. This refraction, however, occurs at the concave surface of the transparent body on the side of the eye [and containing it], which is in contact with the convex [surface] of the transparent body on the side of the visible object [and containing it], and this is what we wanted [to demonstrate].[117]

[5.65] **[PROPOSITION 8]** Furthermore, if the denser transparent body lies on the side of the eye and the rarer one on the side of the visible object, then the visual faculty will grasp only one image, for in that case the center of sight will be at B and the visible object at A [in figure 7.5.53a, p. 189], and since the form of A will be refracted to B, the refraction will occur in a plane perpendicular to the surface of the transparent body, and the common section of that plane and the surface of the transparent body will be a circle, such as circle GED. Also the point of refraction will be E, and the line of refraction will be [B]EH.[118]

[5.66] It follows, then, that the form that extends along AE and that will be refracted along BE [according to the analysis in proposition 4] is refracted along line AE when it extends from B along line BE. Therefore, if the form of A is refracted to B from some point other than E, it follows that the form of B should be refracted to A from that [same] point. But it was just demonstrated that, when the form extends along line BE and is refracted along AE, another form from B will never be refracted to A, since [the form] of A will be refracted to B from only one point and will only

have one image. Moreover, if A lies on the normal dropped from B to the center of the sphere, B will perceive A straight along the normal, and it is clear that the form of A will not be refracted to B, from which it was evident that, when it lies on the normal, the form of B will not be refracted to A. Hence, when the denser body lies on the side of the eye and the rarer one on the side of the visible object, the visible object will have only one image and only one form, and this [is what] we wanted [to demonstrate].[119]

[5.67] **[PROPOSITION 9]** To continue, let us copy the diagram [in figure 7.5.54, p. 190], locating the point [of refraction] on circumference GED of the circle on the side of G. Let it be E, and from it we will draw line ET parallel to line AD. Let us connect ZE and extend it to H. And let the ratio of angle ZEK to twice angle KET be the size of the largest possible ratio between the angle that the line along which the form extends makes with the normal and the angle of refraction mandated by that angle, as far as can be empirically determined.[120] For the angles of refraction that are [produced] between two transparent bodies differing in transparency through which light passes vary in that regard, and, as far as the sense [of sight] is concerned, that difference has a limit, and if it is exceeded, the sense [of sight] will not perceive the amount of refraction, for it will perceive the center of the light passing through the two bodies on the straight line along which the light extends, i.e., as this was observed in the [experimental] apparatus [constructed in chapter 3].[121]

[5.68] Let us suppose that angle DZT = angle KET. Angle ZKE will thus be twice angle KET, and so angle ZEK to angle ZKE will be the largest possible ratio between the angle that the first line makes with the normal and the angle of refraction.[122] But line EK will intersect line AD, so let them intersect at B. Let us draw a line from E parallel to TZ. It will thus intersect ZG outside the circle on the side of G. Let the intersection be at A. Let us extend BE to L. Consequently, angle LEA = angle ZKE, and angle LEH = angle ZEK.[123] Angle LEA will therefore be the angle of refraction mandated by angle LEH [which = angle of incidence BEZ]. So if B lies on some visible object, and if the transparent body with its convex surface facing A is continuous from E to B and has no [refractive] interface at circumference GED on the side of B, the form of B will extend [straight] along line BE and will be refracted along line EA, and it is perceived by the center of sight at A along line AE [so its image will be located at center of sight A, where reflected ray AE and normal BZA intersect].

[5.69] But angle AEH can be subdivided into several ratios of those obtaining between the angles of refraction and the angles that the normals make with the first lines [of incidence] between the two transparent bodies.

There will thus be several points on line DB whose forms will extend to arc GE and will be refracted to A, and [so] the form of the entire line containing [each] such point will be refracted to A from arc GE.[124]

[5.70] Therefore, when the eye lies in a transparent body and the visible object lies in another, denser transparent body, and when the surface of the denser transparent body that faces the eye is convex spherical, and when the visible object lies outside the circle whose convexity faces the eye, and when it is farther from the center of sight than the farther of the two points of intersection of the normal and the circumference [i.e., beyond point D in figure 7.5.54], and when the dense, transparent body facing the eye is continuous up to where the visible object lies and there is no refractive interface at the segment on the circle facing the visible object, then the visual faculty will be able to perceive that visible object both directly and according to refraction, and the image of this visible object will lie at the center of sight.

[5.71] Furthermore, if we hold line AGB fixed and revolve the figure [containing triangle] AEB around AB [as an axis], and if the portion of the surface of the transparent body facing the visible object is spherical, then point E will describe a circumference in a convex circular plane facing the center of sight, and [the form of] B will be refracted to A from that circumference. But the image on the entire circumference of refraction will be single, i.e., [coincident with] the center of sight. Hence, the image of this kind of visible object is also single. But in this situation it happens that the visual faculty perceives the form of the visible object at the point of refraction for the reason that we discussed in [the case of] reflection from mirrors, when the reflection occurs from a circumference in some sphere and when the image is [at] the center of sight.[125]

[5.72] Consequently, the form of this visible object is perceived by the visual faculty [as] circular on the circle of refraction and [is perceived] at the same time in a straight line along the normal passing through the center of sight and the visible object, and this is what we wanted [to demonstrate].[126]

[5.73] **[PROPOSITION 10]** Now let A [in figure 7.5.55, p. 192] be the center of sight, and let B lie on some visible object in a transparent body denser than the one in which the center of sight lies. Let the [great circle on the] surface of the body facing the eye be concave circular, with its concavity facing the eye. I say, then, that B will have only one image and only one form from [the perspective of point] A.

[5.74] Let G be the center of concave curvature, and let us connect AG; and we will extend it straight to Z. AZ will thus be perpendicular to the concave surface, and B will lie either on AZ or outside it. First, let it

lie on line AZ. A will therefore perceive B straight along AB, since AB is perpendicular to the concave surface, and it is never refracted. But if that is possible, let the form of B be refracted to A from E, and let us draw BE and GE [and] extend BE to T.

[5.75] Angle TEG is thus [equal to] the [angle of incidence] that the line along which the form extends makes with the normal dropped from the point of refraction, and since the [transparent] body on the side of A is rarer than the one on the side of B, the refraction will occur away from [normal] EG. Consequently, when it will be refracted, line [B]ET is diverted away from line EG, and line ET does not intersect line BA in any way. The form of B will therefore not be refracted to A, so it will not be perceived according to refraction but will be perceived directly. Hence, from [the perspective of] the center of sight [at A] it will have only one form, and this is what we wanted [to demonstrate].[127]

[5.76] **[PROPOSITION 11]** To continue, let us redraw the diagram [figure 7.5.56, p. 193]; let B lie outside line AZ, and we will produce the plane containing AZ and B. This plane will thus be perpendicular to the concave surface, and the form of B will be refracted to A only in this plane, for no [plane] is erected perpendicular to the concave surface to pass through A unless it passes through AZ. But only one plane passes through AZ and through B. Thus, the form of B will be refracted to A only in the plane passing through [both] line AZ and [point] B. Let the common section of this plane and the concave surface be arc HDE, and let the form of B be refracted to A from H.

[5.77] I say, then, that it will not be refracted from any other point, but if it is possible, let it be refracted from M. Let us then connect lines AH, BH, GH, AM, BM, and GM, and let us extend HB straight to T, BM straight to N, GH straight to L, and GM straight to O. Let us complete circumference HDE, and let it cut line AG at K. A will therefore lie on line KD or beyond it on the side of K. So if A lies on line KD, it will either lie at G, or [it will lie] on one of the two line segments GD or GK.

[CASE 1]

[5.78] Accordingly, if A lies at G, the form of B will not be refracted to A, for the lines that connect the circular [section on the surface of the transparent] body and G are perpendicular to the surface of the body on the side of A. Refraction, however, will not occur along the normal itself but away from it, so the form of B is not refracted to A, if A lies at G.

[CASE 2]

[5.79] Furthermore, if A lies on GD, then line HT will lie between the two lines HA and HG, and so line MN will lie between the two lines MA

and MG, for the refraction occurs away from the normal because the transparent body on the side of the eye is rarer than the one on the side of the visible object. And if line HT lies between the two lines HA and HG, and if A lies on line GD, angle BHA will lie on the side of D, and likewise angle BMA will lie on the side of D, and B will lie beyond line GHL, i.e., on the side of K away from line GHL. Angle THG will be [equal to angle of incidence BHL] that the line along which the form extends makes with the normal, and so will angle NMG [be equal to angle of incidence BMO], and angle THA will be the angle of refraction [for angle of incidence BHL], and so will angle NMA [be the angle of refraction for angle of incidence BMO].

[5.80] However, angle NMG will be equal to, greater than, or less than angle THG. If it [i.e., NMG] is equal [to THG], then [angle] AMN = angle AHT, so angle BHA = angle BMA, which is impossible. If it [i.e., NMG] is greater [than THG], then angle AMN > angle AHT, and so angle BMA < angle BHA, which is impossible.

[5.81] If it [i.e., NMG] is less [than THG], then angle AMN < angle AHT, and so the entire angle AMG < the entire angle AHG. And [so] angle AHT – angle AMN < angle AHG – angle AMG. But angle AHG – angle AMG = [angle] HAM – angle HGM, for the two angles at the intersection of lines AH and MG are equal.[128] Therefore, angle AHT – angle AMN < angle HAM – angle HGM.

[5.82] Let us extend the two lines HA and MA to the two points E and C. Angle HAM will therefore be [equal to] the angle on the circumference that the two arcs HM + EC subtend [by proposition 5, lemma 1], and angle HGM [will be equal to the angle] on the circumference that twice arc HM subtends [by Euclid, III.20, since the vertex of angle HGM is at the center of the circle], and since angle HGM < angle HAM, 2 arc HM < arcs MH + EC. But arcs HM + EC – 2 arc HM = arc EC – arc HM. Therefore angle AHT – angle AMN is less than the angle at the circumference that arc EC – arc HM subtends, so it is less than angle HAM. Angle BMA [supplementary to AMN] – angle BHA [supplementary to AHT] is therefore less than angle HAM. But angle BMA – angle BHA = angle HAM + angle HBM.[129] Therefore those two angles [HAM + HBM] together are less than angle HAM, which is impossible.

[CASE 3]

[5.83] If A lies on line GK [in figure 7.5.56a, p. 193], then line HT will lie between the two lines HG and HA, and likewise line MN will lie between the two lines MG and MA. Consequently, angle BHA will face toward K, and likewise angle BMA will face K, and B will lie below line GMO, i.e., on the side of D from line GMO. Both angles THG and NMG are [equal to]

the ones that the line along which the form extends makes with the normal [i.e., angles of incidence BHL and BMO], and both angles THA and NMA will be the [respective] angles of refraction.

[5.84] Hence, if angle THG = angle NMG, then angle THA = angle NMA, and so angle BHA = angle BMA, which is impossible. If, however, it [i.e., THG] is greater [than NMG], then angle THA > angle NMA, and so angle BHA < angle BMA, which is impossible.

[5.85] If it [i.e., THG] is less [than NMG], then angle THA < angle NMA, and so the entire angle GHA < [the entire] angle GMA. Therefore, angle HGM < angle HAM, and angle HAM – angle HGM < angle GMA, as we demonstrated earlier [in paragraph 5.81], and angle NMA – angle THA < angle GMA – angle GHA, so it is less than angle HAM – angle HGM. Hence, angle NMA – angle THA < angle GMA. But angle NMA – angle THA = angle BHA – angle BMA. Angle BHA – angle BMA, however, is equal to the two angles HAM + HBM, so those two angles together are smaller than angle HAM, which is impossible.

[CASE 4]

[5.86] But if A lies beyond line KZ on the side of K [in figure 7.5.56b, p. 193], and if the body in which A lies is continuous up to A, we will draw the two lines AH and AM, and they will intersect the circumference at R and Q. If angle THG = angle NMG, then angle BHA = angle BMA, which is impossible. If it [i.e., THG] is greater [than NMG], then angle THA > angle NMA, and so angle BHA < angle BMA, which is impossible.

[5.87] If it [i.e., THG] is smaller [than NMG], though, then angle THA < angle NMA, and the entire angle GHA < the entire angle GMA, so angle HGM < angle HAM. But angle MGH is [equal to the angle] at the circumference that twice arc HM subtends, and angle HAM is [equal to the angle] at the circumference that arc HM – arc RQ subtends [by proposition 5, lemma 1]. Thus, 2 arc HM < arc HM – arc RQ, which is impossible.

[5.88] Therefore, if point B lies outside line AKG, its form will be refracted to A from only one point, so it will have only one image, and that image will be in front of the center of sight, behind it, or at the point of reflection, as we showed earlier, and this is what we wanted [to demonstrate].[130]

[5.89] If, however, the denser transparent body lies on the side of the eye and the rarer one on the side of the visible object, all things remaining the same in the figures, then the visible object will still have only one image, and this will be proven as [it was] in the converse of the seventh proposition [of this chapter, i.e., proposition 8, pp. 290-291].[131] And everything we have demonstrated about refraction from convex and concave circular sections applies to spherical and cylindrical surfaces,

except the circular refraction from the circumference of a circle, which only occurs on spherical surfaces. Moreover, what we have discussed involves images of visible objects that are perceived by the visual faculty through simple transparent bodies, which consist of one substance [with a surface] whose shape facing the eye is unitary [i.e., that yields circles or straight lines when cut by a plane].

[5.90] But if the transparent body consists of different [surface shapes] or is not uniform in its transparency, the images of a visible object [seen through them] vary, and if the surface of the transparent body facing the eye consists of different [surface shapes], the image locations of the visible object vary, since the modes of refraction from the body's surface also vary. And if one looks through a small sphere, a small round body, or a cylindrical piece of glass or crystal, behind which a visible object lies, he will find that the image of that object is different from the visible object itself, and he may find another image of the visible object and may thus be confused about it. But this sort of refraction is not singular but double, for the form of the visible object extends from the object to the sphere or other round, cylindrical body and is refracted into the body at the convexity of the sphere or cylinder, and it extends through the body until it reaches its [inner] surface and is then refracted from the sphere or cylinder at the concavity of the air enclosing the sphere or cylinder. Consequently, the perception of this sort of object will be due to two different refractions, so its image will be different from the image that is perceived according to one refraction.[132] But we will discuss this a bit, when we deal with the visual misperceptions that arise from refraction.

CHAPTER SIX

How the visual faculty perceives visible objects according to refraction

[6.1] We just showed in the previous analysis that, when a form is refracted from one transparent body to another body of different transparency, it extends along a straight line until it reaches the surface of the transparent body in which it lies; then it is refracted in that other transparent body along another straight line that forms an angle with the first line. And as the form extends along this other line according to which the form is refracted in the second body, some other form in the second body [which] is [extending] to the point of intersection between the two straight lines will be refracted [reciprocally] along the first straight line.

[6.2] And it is obvious from experience that, if someone looks through some transparent body that differs in transparency from the transparency of the air, he will perceive everything beyond [the two transparent bodies] that face the eye, and if he closes either eye and looks with the other, he will still perceive what lies beyond, whether the body [through which he looks] is air, water, or glass. Likewise, if a person places his eye in or on another body denser than air, be it glass or crystal, he will see everything in the air behind it. And if the viewer moves his eye to the right, or left, or in any direction whatever, and if he does not move it too far from its original location, he will still perceive everything he perceived before, whether the eye's motion occurs in air or in glass.

[6.3] But it has already been shown by experiment and [rational] demonstration that the visual faculty only perceives things through transparent bodies that differ in transparency from the air according to refraction, except for the one point that lies on the normal dropped from the center of sight to the surface of the transparent body. Thus, every point that is perceived by the visual faculty through a transparent body, except for that aforementioned point, is perceived by means of a form that extends from that point to the surface of the transparent body behind which it lies, and it will be refracted to the center of sight from the surface of that body. And since one eye perceives everything that lies behind the transparent body, the form of every point lying behind that transparent body extends along a straight line to the surface of that transparent body, and it will be refracted to that one eye, except for that aforementioned point.

[6.4] And since the forms of all the points on all the visible objects lying behind the transparent body are refracted to the center of the one eye at the same time, the form of the point that lies at the center of that eye, if it lies on some visible object, will be refracted at the same time and in the same way to all the points on all the visible objects that lie behind the transparent body and face the eye. The same holds for every point near the point at the center of the eye, for if the eye is moved in every direction but does not lie far from its [original] place, it will perceive [all those] visible objects. When it lies behind some transparent body, then, the form of any point on any visible object extends to the surface of the transparent body behind which it lies, and it is refracted throughout the body of the air that faces it. And there is no time more appropriate than another for this [to happen], but it is a natural characteristic of the light and color in visible objects that they always extend [continually] from any point on any luminous or illuminated body along a straight line extending from that point, and they are refracted in every body of differing transparency, except for the point on the normal.

[6.5] Moreover, every form of a point on any visible object lying in any [transparent] body different [in transparency] from the air extends through that body in which it lies and is refracted throughout the body of the air facing it. And that form leaves at any given point in the air, so the form of the entire visible object converges at any given point in the air, and the form of any entire visible object lying in any [transparent] body different from the air lies at any given point in the air facing that visible object. And that form extends from any given point on the visible object in the [transparent] body within which it lies, and it is refracted at the surface of that body to reach that point in the air. And so, if the visual faculty looks through any given transparent body different from the air behind which some visible object lies, the visual faculty perceives that object, for its form lies at the point at which the center of sight lies, and so also, if the visual faculty perceives some visible object through some transparent body different from the air, and if it then moves to the right or left of its [original] place while it remains facing the object and the visible object behind it during its motion, it will always perceive that object.[133] So, too, several viewers will perceive a single object in the heavens or in water, and [they will do so] at the same time, and this still occurs in the same transparent medium, i.e., [it remains the case] that the form of the visible object converges at any given point in the [transparent] body in which [the viewer] lies, for the form of any point on it extends along a straight line, and there is a straight line between any given point in the [transparent] body in which the center of sight lies and any given point on the visible object.

[6.6] Hence, the form of any point on a visible object extends to any point in the transparent body in which the visible object lies, and the form of any luminous or illuminated visible object converges at any given point in the [transparent] body in which it lies, and it converges at any given point in any transparent body different from the body in which it lies, as long as there is no obstacle between the visible object and that different transparent body. And the form of the visible object that lies at any given point in the transparent body through which it extends straight to that point and the form of that object at any given point in the different transparent body extends to that point by means of refraction, according to which a cone of refraction is formed between any given point in the air and some visible object lying in another body of different transparency from the air, the vertex of this cone lying at the point in the air, and its base lying on that visible object, and its refraction will occur at the surface of the transparent body different from the air. Consequently, when it is perceived by the visual faculty, every visible object in a transparent body different from the air is perceived according to a form extending through a cone of refraction

and united at the point lying at the center of sight. This, then, is how the visual faculty perceives what it perceives according to refraction.

[6.7] Now in chapter [five] on image[s] we showed that every visible object [seen according to refraction] is perceived by the visual faculty through an image, and the image location is the point at which the radial line along which the form extends to the center of sight and the normal dropped from the visible point intersect. Hence, if we imagine a normal dropped from every given point on a visible object to the surface of the transparent body in which the visible object lies, we will have a body of sorts extending from the visible object to the surface of the transparent body, so it follows that this body should cut the refracted cone, and the plane in which they intersect forms the image of that visible object.

[6.8] Accordingly, if the surface of the transparent body in which the visible object lies is plane, the body imagined to comprise all the normals will have a plane surface, so the image will be somewhat larger than the visible object. If the [transparent] body is [convex] spherical, and if its convexity faces the center of sight, while its center lies on that visible object, then the imaginary body will be conical, with its vertex at the center of the sphere, and the farther [that cone] extends toward the surface of the [transparent] spherical body, the more it will spread out. On the one hand, if the intersection [of this cone and the cone of refraction] lies between the visible object and the spherical surface, the image will be larger than the visible object itself. On the other hand, if the intersection lies beyond the visible object, the image will be smaller than the visible object. But if the visible object lies beyond the spherical surface, the imagined body will form two opposite cones [each] with its vertex at the center of the sphere. Because the intersection points will not fall between the imagined body and the cone, the area of intersection at which the image lies may be larger than, smaller than, or the same size as the visible object.[134]

[6.9] If the transparent body is [concave] spherical and its concavity faces the eye, however, the imagined body [comprising the normals] will be a cone with its vertex at the center of the sphere, so the farther this body extends toward the surface of the sphere, the narrower and more constricted it gets, and the farther it extends in the opposite direction, the more it spreads out, for the surface between the sphere's center and the sphere itself is [relatively] small. On the one hand, if the area of intersection between this [imagined] body and the cone of refraction lies nearer the center of concave curvature than the visible object, the image will be smaller than the visible object itself. On the other hand, if it lies farther from the center of concave curvature than the visible object, the image is larger than the visible object itself.[135]

[6.10] In addition, when a single visible object is perceived by several eyes at once, all the images that those eyes perceive at that time will [each] lie in one imagined [cone] that is perpendicular to the surface of the transparent body. And a single visible object is perceived by a single person at one time with both eyes through a transparent body differing in transparency from the body in which his eye lies, and yet he perceives it as single. For if a person perceives a given body among those in the heavens, or in water, or through glass, and if he closes either eye, he will nonetheless perceive it with the remaining eye, from which it is clear that a single object lying behind a transparent body different from air will be perceived by both eyes and by either eye.

[6.11] As we explained in the third book, moreover, the reason for this is that, when any point on any visible object is perceived directly by both eyes, and when two rays from both eyes that have a corresponding situation with respect to the two visual axes intersect on that point, it will be perceived as single, but if the rays converging on it have a divergent [non-corresponding] situation with respect to the two axes of the eyes, it will be perceived double. For the most part, however, the situation [of two such rays] corresponds when things are perceived. And the cases in which they have a divergent situation with respect to both eyes are quite rare, as we said in the third [book].[136]

[6.12] What is perceived by means of refraction is perceived at an image location. Moreover, the form at the image location is perceived by the visual faculty [as if it were perceived] directly, so it is as if it lay in air and were perceived directly by the visual faculty. In addition, the situation of this form, which is the image with respect to the center of sight, is like the situation of any of those visible objects that are perceived directly, so for the most part the situation of these images with respect to the center of sight is corresponding. And from both eyes two rays having a corresponding situation converge at every point on an image, so a single object will be seen as single by both eyes.

[6.13] In order for this to be shown more clearly, we should point out that we have just claimed that every point on an object that is perceived according to refraction is perceived at the image location, which is the point of intersection between the normal dropped from that point to the surface of the transparent body in which the visible object lies and the radial line along which the form extends to the center of sight. Hence, when the viewer perceives a point on any object with both eyes, the image of that point with respect to both eyes lies on the normal dropped from that point, which is the same line. And when the form of that point reaches two points on the surfaces of the eyes that have a corresponding situation

with respect to the visual axes, the two lines along which the forms extend to both eyes reach the two centers of both eyes. [These lines] are therefore either the [visual] axes [themselves] or [lines] that have a corresponding situation with respect to the [visual] axes.

[6.14] But the two visual axes always lie in the same plane, and all of the lines extending from the center of the two eyes that have a corresponding situation with respect to the common axis will lie in the same plane, for the common axis always lies in the same plane because, if something is perceived correctly by both eyes at the same time, the axes intersect at one point on that thing, so they lie in the same plane. Furthermore, the eyes are naturally disposed to correspond in orientation, and they do not deviate from [their] natural [disposition] unless by accident or force, so their axes lie in the same plane, for the origin of the axes is a single point at the center of the hollow in the common nerve, from which the common axis extends.

[6.15] Therefore, when the two eyes are in their natural situation, the axes will always lie in the same plane, whether they are moved or at rest. If, however, the situation of either of the eyes is altered with respect to the other on account of some obstacle, a single object will appear double, as we showed in the first [book].[137] Hence, for the most part the two axes lie in the same plane, so every pair of rays having a corresponding situation with respect to the two axes will lie in the same plane. The two lines along which the forms of one point extend to two places [on the eye] that have a corresponding situation thus lie in the same plane. But the images of that point with respect to [each of] the two eyes lie on those two lines, so they lie in the same plane, and the images of that point lie on the normal dropped from that point, so they lie at the intersection of the plane in which the radial lines lie, which is a single plane, and the normal, which is a single line.

[6.16] The intersection of a plane with a line, however, is a point, so, when they reach two places with a corresponding situation, the images of one point with respect to the two eyes form a single point, from which it is evident that the image of the entire visible object will be single with respect to both eyes if the situation of the image is corresponding, so the object is perceived as single by both eyes. If the situation is slightly divergent, though, the object will still be seen as single, but in a confused [or blurred] rather than a correct [or distinct] way. But if the divergence in situation is considerable, the form of the object will be seen double, although this happens extremely rarely. This, then, is how the visual faculty perceives visible objects according to refraction.

[6.17] Now that this has been confirmed, we may make the general claim that everything perceived by sight is perceived according to refraction,

whether the eye and the visible object lie in the same or different transparent [bodies], or whether the visible object faces the eye directly, or whether it is perceived by the eye according to refraction. In fact, nothing is perceived without refraction occurring at the surface of the eye, for the tunics of the eye, which are the cornea, the albugineous [humor], and the glacial [humor], are also transparent and denser than the air. And it has just been shown that the forms of objects in the air and in other transparent bodies extend through those bodies, and if they encounter a body whose transparency differs from that of the one in which they lie, [those forms] are refracted in that transparent body. Accordingly, the form of what is in the air always extends through the air, [and] so, if the air is in contact with the surface of any eye, the form in the air is refracted at the surface of the eye, and it will then be refracted throughout in the body of the cornea and the albugineous humor, for refraction is characteristic of all forms. Moreover, the reception and refraction of forms is characteristic of transparent bodies, so the forms of objects that face the eye are always refracted in the tunics of the eye.[138]

[6.18] It was just shown as well that, when forms extend along perpendicular lines to a second [transparent] body, they pass straight through the second body, so the forms of objects that face the surface of the eye are all refracted in the tunics of the eye [when they reach its surface at a slant], and the forms of those [points on the object] that lie at the endpoints of the radial lines that are perpendicular to the surface of the eye will pass straight through while at the same time their forms are [also] refracted in the tunics of the eyes. For many visible objects face the portion of the eye's surface in line with the aperture in the uvea [i.e., the pupil], and some of them lie at the endpoints of radial lines, and others lie outside [such lines].[139]

[6.19] Indeed, all the radial lines that are perpendicular to the surfaces of the eye's tunics are contained within a cone, whose vertex lies at the center of sight and whose base is [formed by] the circumference of the aperture in the uvea, and the farther this cone extends outward from the center of sight, the more it spreads out. All the forms of those objects lying within the cone extend straight along radial lines and pass straight through the tunics of the eye, and this cone is called the "cone of radiation." Furthermore, the lines that extend within this cone and have their endpoints at the center of sight [i.e., converge at that point] are called "radial lines."

[6.20] But the forms of objects that are outside this cone never extend along any of the radial lines, yet they will extend along straight lines [that pass] between [the object and] the surface of the eye facing the aperture in the uvea. The forms that extend along these lines are refracted by the

transparency of the eye's tunics, and the form of any point on objects that lie within the cone of radiation extends to the surface of the eye facing the aperture of the uvea within a cone whose vertex lies at that point and whose base is the surface facing the aperture of the uvea.[140] Of [all] the lines imagined [to exist] in this cone, [only] one is a radial line; all the rest that are within this cone are not radial [lines], and none of them is perpendicular to the surfaces of the eye's tunics.

[6.21] Moreover, the form of any point among those within the cone of radiation extends along every line that can fall within that cone, whose vertex lies at that point and whose base is the surface of the visible object that faces the aperture in the uvea.[141] And the form that extends straight through the tunics of the eye follows one of those lines, whereas all the other forms extending within the rest of the cone are refracted in the eye's tunics and do not pass straight through. Consequently, [the form of] everything in the air, the heavens, water, or the like, as well as what is reflected from polished bodies, is refracted in the tunics of the eye [when that form extends from an object] in front of the portion of the eye's surface facing the aperture in the uvea and [when it] reaches that portion of the eye's surface. And the forms of those objects that lie within the cone pass straight through the tunics of the eyes while the remaining forms of those objects, which extend within the cone [along oblique lines] are refracted throughout this portion of the eye's surface. So it remains to demonstrate that forms refracted in the eye's tunics are perceived by the visual faculty and sensed by the sensitive power.[142]

[6.22] Now in the first [book] we showed that, if the sensitive organ were to sense every form reaching it at every point on its surface, it would sense the forms of things all mingled together, which is why the sensitive organ senses forms exclusively along straight lines perpendicular to its surface so [that] the forms of visible objects pass [straight] through, and the forms of visible objects do not intermingle at it. In this book, in fact, we have demonstrated that refracted forms are perceived only on the normals extending from visible objects to the surfaces of transparent bodies. Consequently, the forms that are refracted in the eye's tunics are perceived by the visual faculty only along the normals extending from visible objects to the surfaces of the eye's tunics, and these normals are lines extending from the center of sight.

[6.23] Hence, all forms that are refracted in the eye's tunics are perceived by the visual faculty along straight lines extending from the center of sight, so the forms of all visible objects that are in front of the portion of the eye's surface facing the aperture in the uvea lie on this portion of the eye's surface, and they are refracted according to the transparency of the eye's

tunics, then reach the sensitive organ, which is the glacial humor, and are perceived by the sensitive power along straight lines connecting the center of sight with those visible objects. But the form of any point on the object in front of the surface of the eye facing the aperture in the uvea lies on the entire surface of this portion, and it is refracted at this entire portion and reaches the glacial humor, at which time the [glacial] humor senses the form reaching it. And the sensitive power perceives all the forms of the visible point that reach the glacial humor along a single line connecting the center of sight with that point. So this is how the visual faculty perceives all visible objects.

[6.24] In this chapter we claimed that some objects that face the surface of the eye lie within the cone of radiation and some lie outside it, and when I say "the surface of the eye" you must henceforth understand [that I mean] the portion facing the surface of the uvea. Thus, visible objects that lie within the cone of radiation are perceived by the visual faculty along straight radial lines according to the forms [of those objects] that extend to the center of sight straight along these same lines, for these lines are the normals within the cone that extend to the surface of the eye's tunics from visible points. On the other hand, those [points] outside the cone of radiation are perceived by the visual faculty according to refracted forms but along straight lines extending from the center of sight [to points] lying outside the cone of radiation, and these lines that lie outside the cone can also be called radial lines by analogy, for they are like [actual] radial lines in that they extend from the center of sight. So it remains [for us] to show by experiment that the visual faculty perceives objects outside the cone of radiation.

[6.25] Accordingly, we maintain that it is obvious that the tear ducts and what surrounds the eye lie outside the cone whose vertex lies at the center of sight and whose base is [formed by] the circumference of the aperture in the uvea, which is a small opening in the middle of the black portion of the eye.[143] And if one takes a small, thin needle and places its endpoint at the lower extremity of the eye between the eyelids, and if he holds his eye steady, he will see the needle's endpoint. By the same token, if he places the needle's endpoint at the tear ducts and brings it up to the eye, or if he puts the endpoint at or near the side of the eye's black portion, he will see the needle's endpoint. Furthermore, everything parallel to the surface of the visible object in the areas surrounding the eye lies outside the cone of radiation, and when I say "areas surrounding the eye" I mean those [areas] from which lines extending to the middle of the eye's surface intersect the axis of the cone of radiation.[144] Also, if a person holds his forefinger up to the side of his face and near the eyelid, he will see his forefinger, and

likewise, if he puts his finger at the lower eyelid so that the top surface of his forefinger is parallel to the eye's surface as far as can be empirically determined, he will see the forefinger's surface.

[6.26] But all these areas lie outside the cone of radiation, and this is obvious because the cone of radiation that the aperture in the uvea circumscribes is quite small and continues straight out [through the aperture], and its amplitude is slight, so none of it reaches the areas that surround the eye or approach the eye's body while remaining parallel to the eye's surface. But there are straight lines between the eye's surface and all the areas surrounding the eye and parallel to the eye's surface according to their refraction in the dense bodies [comprising the eye's tunics, and] since the air between those areas and the eye's surface is continuous, the form of these visible objects reaches the eye's surface along these lines that lie outside the cone. And since this form does not reach the eye along radial lines yet will be perceived by the visual faculty, it is clear that the visual faculty perceives it according to refraction. From this experiment, therefore, it is evident that the visual faculty perceives many objects that lie outside the cone of radiation by means of refraction.[145]

[6.27] We can also show empirically that the visual faculty perceives objects within the cone by means of refraction, notwithstanding that it perceives them directly [and we can do so] as follows. Take a thin needle, sit down in a place that faces a white wall, close one eye, put the needle in front of the other eye, and bring the needle near [the eye] so that it touches the eyelid. Then put the needle directly in front of the midpoint of [the front surface of the] eye, and look at the facing wall, for in that case you will see the needle as a transparent object with a bit of opacity in it, and you will see that portion of the wall beyond the needle and in line with the needle as a wide object that is many times wider than the needle.

[6.28] The reason for this has been shown in the second book, i.e., that, if a visible object lies quite close to the eye, it will appear larger than it is, and the closer it gets, the larger it will appear.[146] Its [apparent] transparency, moreover, is due to the visual faculty's perceiving what lies beyond it. But the needle is an opaque body blocking what lies beyond, and since the needle is quite close to the eye, it blocks a portion of the wall that is many times its width, for [when it is extended to the wall] the base of the cone whose vertex lies at the center of sight and whose base is the width of the needle will be many times as wide as the width of the needle. And yet the visual faculty perceives what lies behind the needle, and it blocks none of the wall [behind] from the center of sight, but [the visual faculty] perceives what lies behind as if [it lay] behind a transparent body.

[6.29] But since the needle faces the middle of the eye directly, it will not block the entire surface of the eye but only a part commensurate with

its width because of its narrowness, and [so] some of the surface of the eye remains [uncovered] to the sides of the needle, and the form of the object extends to that area of the eye's surface to the sides of the needle. But the form extending to the needle [itself] will never reach the center of sight or be perceived by it, whereas the form that reaches the sides of the eye's surface is refracted at the eye, since it cannot reach the center of sight directly. Consequently, if the eye were only to perceive the part of the wall that lies directly in line with the needle by means of direct vision [i.e., only along the orthogonals], then the part of the wall directly in line with the needle would be blocked from the visual faculty. Since, then, it must be perceived, but not directly, it is clear that is is perceived by means of refraction according to a form that is refracted at the surface of the eye to the sides of the needle. And this is also made evident if the experimenter replaces the needle with a wide object that is wider than the aperture in the uvea, for in that case he will seen none of the wall whatever, nor will he see that body [as if it were] transparent but [as completely] opaque.

[6.30] From the fact that the wall is perceived behind the needle because of its narrowness but is not perceived behind a wide object, then, we know that the perception is due to a form that reaches the eye's surface at [the sides of] the needle and is refracted in the eye's tunics. And because whatever is perceived by the visual faculty according to refraction is perceived straight along the perpendiculars, it follows that it perceives what it perceives according to refraction by means of a form on the [part of the object] directly in line with the needle along straight lines extending from the center of sight and connecting the center of sight with the portion of the wall facing the needle, and these lines are cut by the needle. And [so] the visual faculty perceives what lies behind the needle straight along these lines, and it perceives the needle as well along those straight lines, which is why it perceives the entire form as if through a transparent body in which there is a modicum of opacity.

[6.31] Moreover, if the experimenter writes something small on [a piece of] paper and attaches it to the wall, and if he stands far enough from the wall that he can read the writing and puts the needle directly in front of the middle of the eye, as he did before, then when he looks at the paper he can read the writing, and yet he will still see it as if through glass or through a transparent body in which there is some opacity. Hence, if the visual faculty did not perceive that portion of the paper behind the needle according to refraction, the writing would lack something in the way of visibility, for the needle ought to block the writing all the more in relation to the width of its [apparent] transparency than [the portion of the paper behind the needle] that he perceives in that case because of the distance of the paper

from the center of sight. But since none of the writing is hidden from the visual faculty, it is clear that it perceives what lies behind the needle, yet it cannot do so directly. It follows, therefore, that it must be done according to refraction.

[6.32] Furthermore, if the experimenter removes the needle, he will not eliminate the refraction that occurred previously, for the refraction was not due to the needle; instead, the refraction increases according to what is refracted from the area [uncovered] from the needle. And when the experimenter removes the needle, he will perceive what faces the eye more clearly, for he will perceive directly what was blocked by the needle, notwithstanding that he [also] perceives it by means of refraction, just as he perceived it when it was blocked, and according to this reinforcement [by the added orthogonal and oblique impingements] he perceives it more clearly than [he did] before he removed the needle, and from this experiment it is evident that what faces the eye within the cone of radiation is perceived both directly and by means of refraction.[147]

[6.33] From all these [observations], then, it is manifest that everything whose form reaches the center of sight directly, by reflection, or by refraction is perceived by the visual faculty according to refraction occurring at the eye's surface, and [it is also manifest] that some of the things perceived according to refraction at the surface of the eye are perceived [both] directly and by means of refraction at the same time. And so whatever faces the middle of the eye is clearer than what surrounds the middle, and when the visual faculty perceives something broad, it will perceive what is at the middle more clearly than what lies at the sides. This, moreover, was demonstrated in the second book, where we showed how this could be empirically tested and explained why this is so on the basis of radial lines, and this applies to those things that lie within the cone of radiation.[148] In the case of things outside [the cone], however, refraction is the cause. Thus, the overarching reason that what faces the middle of the eye is clearer than what surrounds [that point] is that what faces the middle of the eye is perceived both directly and by means of refraction at the same time. Furthermore, that whatever is perceived by the visual faculty is perceived by means of refraction was mentioned by none of the ancients.

CHAPTER SEVEN

On the visual misperceptions that arise according to refraction

[7.1] The misperceptions[149] that arise according to refraction are similar to those that arise in reflection, for what is perceived according to refraction

is not perceived in its [actual] place, since it is perceived at the image location, so the location of the form that is perceived will be different from the location of the visible object, and the same [holds for] their [relative] distance [from the center of sight]. Moreover, refraction weakens the refracted form, i.e., the form of the light and color in the visible object. This can be understood from the fact that, if you look at something in water and are far to the side of the normals dropped from the visible object to the water's surface, and if you inspect it properly, then change position, move your eye until you line it up on any normal dropped from the visible object to the surface of the water, and look [at the same object under water], you will see it more clearly than [you did] when you were off to the side. But there is no difference in the two situations except that in the first one the form that extends to the center of sight is refracted and sharply slanted [with respect to the refracting surface], whereas in the second [situation] the form, or some part of it, extends straight [through the refracting surface], while some [neighboring parts of the form extend] at a gentle slant or nearly straight [through the refracting surface]. From this experiment, then, it becomes clear that refraction weakens refracted forms.

[7.2] Furthermore, when they are refracted to the eye, [the forms of] things in water, or behind glass, or [behind] similar [media] convey the color of the body in which they lie.[150] Hence, in the case of things that are perceived according to refraction through transparent bodies, misperceptions arise on account of the refraction that do not arise in the case of things that are seen directly, i.e., a disparity in location and distance, as well as a weakening of light and color. In addition, [the misperceptions that] arise in the case of things that are seen directly also arise in those [things that are seen by means of refraction], for the forms of things perceived by means of refraction are perceived directly in front of the eye along straight radial lines. Consequently, what happens in the case of things that are seen along straight radial lines [also] happens in the case of things [seen according to refraction]. In the third [book], moreover, we explained all those misperceptions and their causes, and these are also the causes for [misperceptions] in the case of things [that are seen according to refraction].[151] In the latter case, however, [misperception] arises more frequently and quickly on account of the weakening of such [refracted] forms.

[7.3] In addition, specific misperceptions that arise on account of the shapes of the surfaces of transparent bodies are manifold, but they rarely occur in vision, for the things that are [normally] perceived through transparent bodies different from the air include the stars and things in water, whereas things that lie behind glass and transparent stones of various

shapes are rarely perceived by the visual faculty. And what applies in the case of these transparent bodies is not what applies to mirrors, for mirrors are often regarded by people so that they can see their images in them, and they are kept in homes. Likewise, when a person looks into any polished body, he will also see the forms of things [surrounding him] that face [that body], and by the same token, if he looks into water, he will see his own form in it, and he will see what faces [the water], but such is not the case for what is seen through glass and transparent stones, for people rarely look at what lies behind glass and transparent stones. And since this is the case, let us talk about specific misperceptions that arise from refraction all the time and without complication, i.e., the ones that arise in the case of things that are seen in the heavens and in water, and we will discuss [only] briefly those things that are seen through glass and stones.

[7.4] Accordingly, we maintain that the visual faculty is always deceived about things that are perceived through a transparent body different from air, and aside from [misperceptions] of their location, distance, colors, and light, [it is deceived about] their size and about the shapes of some of them, for things that are seen in water and through glass or transparent stones appear magnified. In addition, the stars and the intervals between stars sometimes appear magnified and sometimes diminished.

[7.5] **[PROPOSITION 12]** Accordingly, let A [in figure 7.7.61, p. 199] be a center of sight, and let BG lie behind a transparent body denser than air. I say that BG appears larger than it [actually] is.

[7.6] To start with, then, let the surface of the transparent body be plane. [Center of sight] A either lies on the normal dropped from the middle of BG to the surface of the [transparent] body, or [it lies] outside [that normal]. First, let it lie on it, and let that normal be AMZ [and let Z be the midpoint of BG]. Let us produce the plane containing lines AZ and BG, and let it form line DME on the surface of the transparent body. Thus, line AM is perpendicular to line DME, and the plane within which the two lines AZ and BG lie will be perpendicular to the surface of the transparent body.

[7.7] Moreover, the only plane perpendicular to the surface of the transparent body that passes through A and through any point [in addition to Z] on line BG is the one in which lines AZ and BG lie, for no plane passes through A perpendicular to the surface of the transparent body unless it passes along AZ, which is a line normal to the surface of the transparent body, and AZ is the only line extending from A that is normal to the surface of the transparent body. Hence, no plane passes through A perpendicular to the surface of the transparent body unless it passes along AZ, and only the plane containing the two lines AZ and BG passes through any point

on line BG as well as along line AZ. Thus, only the plane containing the two lines AZ and BG and perpendicular to the surface of the transparent body passes through A as well as through any point [in addition to Z] on line BG, so the form of any point on BG is refracted only at line DE [within that plane].

[7.8] Let us then drop two perpendiculars from B and G. Accordingly, let them fall at the two points D and E on line DE, [and let them be] BD and GE. Let [line] BG be parallel to line DE at the outset, and let the form of B be refracted to A from T, and [let] the form of G [be refracted] to A from H. Let us then connect lines BT, TA, GH, and HA, and let us extend AT to L and AH to K. Therefore, since Z was assumed to lie at the middle of line BG, the location of B with respect to A will be equivalent to the location of G with respect to A, so the distance of T from A will be equal to the distance of H from A, and so angle DTL = angle EHK. But the two angles [BDE and GED] at D and E are right, and line DT = line EH, since [line] TM = line MH. Thus, DL = EK.

[7.9] Let us connect LK. It will therefore be equal to line BG. And let us connect AB and AG. Hence, angle GAB < angle KAL, and line LK is the cross-section of the image of line BG, for [the form of] every point on line BG is refracted from some point on line TH because, if the form of B is refracted from T, the [form of a] point that lies between B and Z is refracted from some point between T and M. Let us [for instance] take [some] point N on line BZ. If, therefore, its form were refracted from some point [R] outside line MT on the side of D, the line along which the form of N extends would intersect line BT, and so the form of the point of intersection [at X] will be refracted to A from two points [i.e., T and R], which is impossible, as we said in the chapter on images [i.e., chapter 5, proposition 3]. Thus, [the form of] N is refracted to A only from some point between T and M, and likewise [the form of] every point on ZG will be refracted to A only from line MH. Line LK is therefore the cross-section of the image of line BG, so the form of BG will be seen on LK.

[7.10] Now we have already shown that a refracted form is weaker than [one seen in] direct [vision]. Hence, the form of BG, which is perceived by means of refraction, is weaker than the form of BG that is perceived directly, and because of the weakening of the object's form, the visual faculty matches it to the form of the object seen from a greater distance, for an increase in distance weakens the form. We have also shown in the second [book] that the visual faculty perceives the image of a visible object according to the size of the angle [it subtends at the center of sight] in relation to the distance and orientation of the visible object vis-à-vis the center of sight.[152] But angle KAL > angle GAB, the orientation of LK is equivalent to the orientation of

BG, BG appears at LK, and LK is perceived as if [it lay] at a greater distance than BG because of the weakening of the form. Therefore, the visual faculty perceives BG according to refraction by correlating an angle [KAL] greater than angle GAB to a distance greater than the distance of BG [i.e., according to the magnification of the image's apparent distance by the weakening of the form] as well as to an orientation equivalent to the orientation of BG, so BG is perceived according to refraction as magnified, and this for two reasons: i.e., the size of the [visual] angle and the weakness of the form. And the reason for the [increased] size of the angle is the proximity of the [vertex of the] angle to the center of sight, and the reason for its proximity is refraction.[153] Hence, refraction is the reason BG is perceived as magnified.

[7.11] To continue, let us redraw the diagram [in figure 7.7.61a, p. 199], but let BG not be parallel to line DE. Let us draw a line from endpoint [G] of BG away from DE parallel to line DE, and let it be GQ. Let us then extend AZ to O. O will therefore lie in the middle of GQ, since Z lies in the middle of BG because BQ is parallel to ZO. Hence, QO:OG = BZ:ZG. Let the form of Q be refracted to A from T, and [let] the form of G [be refracted] to A from H. Let us connect AT and let it pass to L, and let us connect AH and let it pass to K, and let us connect LK. LK will therefore be the cross-section of the image of QG. Let us connect AQ and AG. Angle KAL will therefore be larger than angle GAQ, so [the center of sight at] A perceives the image of QG as larger than QG, as we said earlier.

[7.12] Moreover, line QT will intersect line BG at R. [The form of] R will therefore be refracted to A from T, so [the form of] B will be refracted to A from a point between the two points T and D, for if it were refracted from a point between T and M, the previously mentioned impossibility would result [i.e., the form at the point of intersection would be refracted from two points]. Therefore, let [the form of] B be refracted to A from F. Let us connect AF, and let it pass to I. Then let us connect IK. IK will thus be the cross-section of the image of BG, and IK's orientation with respect to A is equivalent to the orientation of BG [with respect to A] because IK will either be parallel to BG, or there will not be [enough] difference between them to affect the orientation, for there is no significant difference between the distance of IK and the distance of BG from the center of sight, so the divergence of IK from a line extending from K parallel to BG will be quite small.[154] Therefore, angle IAK > angle BAG, the orientation of IK is equivalent to the orientation of BG, and IK is perceived as if [it were] more distant because of the weakening of its form. Thus, line IK appears larger than line BG, as we demonstrated in the previous [part of this] theorem. But IK is the image of BG, so BG will appear larger than it [actually] is, and this is what we wanted [to demonstrate].

[7.13] **[PROPOSITION 13]** Now let A [in figure 7.7.62, p. 200] be the center of sight and BG the visible object, and let us draw perpendiculars BD and GE and connect DE. Let BG be parallel to DE, and let A lie outside plane BDGE in a plane [AHZ] that joins it. Let us bisect BG at Z, let us draw perpendicular AH and connect AZ, and let AZ be presumed to be perpendicular to BZG. Thus, the location of B with respect to A is equivalent to the location of G with respect to A, and the distance of B from A is equal to the distance of G from A. Let [the form of] B be refracted to A from T [in plane of refraction AHBD], and [let the form of] G [be refracted] to A from K [in plane of refraction AHGE]. The location of T with respect to A is thus equivalent to the location of K with respect to A, and the distance of T from A is equal to the distance of K from A.

[7.14] Let us then connect lines BT, TA, GK, and KA. The plane in which the two lines AT and BT lie [i.e., ATBH] is therefore perpendicular to the surface of the transparent body, since it is the plane of refraction; so normal BD will lie in this plane, and [so will] the normal extending from T. Line AT therefore intersects [line] BD. Let AT be extended, then, let it intersect BD at L, and let AK be extended to intersect GE at O. Hence, AL = AO, and BL = GO. Let us connect LO, which is the cross-section of the image of BG, and [so] LO = BG. Let us connect AB and BG. Hence, both planes ALB and AOG are perpendicular to the surface of the transparent body, and the three planes [ADB, AZH, and AGE] that pass through points B, Z, and G perpendicular to the transparent body's surface intersect one another on the normal [AH] dropped from A to the surface of the transparent body.

[7.15] Angle BTL will be [equal to] the angle of refraction, and line BLD is perpendicular to the surface of the [transparent] body. Thus, line AL is oblique to it. Line AT therefore makes an acute angle with the normal dropped from T to the surface of the [transparent] body on the side of L. Let us then draw the normal, and let it be TC. TC will thus be parallel to LD, so angle TLD is acute. Hence, angle ALB is obtuse. Consequently, line AL < line AB, and it is demonstrated the same way that AO < AG. But lines AL and AO are equal, [lines] AB and AG are equal, and line LO = line BG. Therefore, angle OAL > angle GAB.

[7.16] Furthermore, the orientation of LO is equivalent to the orientation of BG, since the line extending from A to the middle of LO is perpendicular to line LO because LO is parallel to BG, and BG is perpendicular [to the planes] in which AZ and DB lie [i.e., BG is perpendicular to both AZ and DB]. Hence, line LO is perpendicular to line AZ.[155] Line LO is therefore perpendicular to the plane [AZH] that connects A with the middle of LO, so the orientation of LO with respect to A is equivalent to the orientation of BG with respect to A. But LO is perceived as farther [away than it would

normally be judged to be] because of the weakening of its form, so LO will appear larger than BG. But LO is the image of BG, so BG will appear larger than it [actually] is.

[7.17] Let us now recapitulate the diagram [in figure 7.7.62a, p. 200], let BG not be parallel to DE, and let us draw GF parallel to DE. Let us connect AF, and let T be the point from which [the form of] F is refracted to A, while [the form of] B is refracted to A from Q. Let us connect AQ and extend it to C. C will therefore be higher than L, for B lies beyond line AF, so line AC lies beyond line AL. Hence, C is higher than L.

[7.18] Let us connect CO. Thus, CO will be the cross-section of the image of BG, and CO > LO, and AC < AL. The two lines AC and AO lie in two intersecting planes, i.e., ACB and AOG, and the common section [AH] of these two planes passes through A. In addition, the two lines dropped orthogonally from A to this common section within these two planes [i.e., AX in plane ACB and AY in plane AOF] are higher than the two lines AC and AO. Therefore, angle CAO > angle BAG, and the distances of CO and BG from A are not different by much, and line CO will either be parallel to BG, or there will be no perceptible difference in orientation between the two.[156] Thus, the orientation of CO with respect to A is equivalent to the orientation of BG with respect to A, and there is no perceptible difference between the distances of CO and BG with respect to A, so CO will appear larger than BG. But CO is the image of BG, so BG appears larger than it [actually] is, and this [is what] we wanted [to demonstrate].

[7.19] **[PROPOSITION 14]** To continue, let us copy the first diagram for this chapter [i.e., figure 7.7.61, redrawn as figure 7.7.63, p. 201], and let AMOZ be a normal intersecting LK. LO will thus be half LK. But point Z will be seen at O because it is seen on normal ZM. Thus, BG will be seen on line LK. But BZ is half BG, LO is half LK, and LK appears larger than BG, so LO appears larger than BZ.

[7.20] The reason for BG's magnification is refraction, so the reason for BZ's magnification is [also] refraction. Moreover, A lies on normal AZ, which extends from the end of BZ to the surface of the transparent body. This same point obtains in the [demonstrations based on] the three figures following the first one, i.e., in the second, third, and fourth [figures] of this chapter [figures 7.7.61a-7.7.62a, pp. 199-200], that is, the visual faculty perceives the halves of the visible objects [in those three cases] as larger than they [actually] are. Moreover, the center of sight lies on the normal dropped from the end of the half-segment, or [it lies] in a plane that is perpendicular to the surface of the transparent body and that passes through the end of the half-segment, for the point that constitutes the

midpoint of the image lies on the normal dropped from the midpoint of the visible object, whether the visible object is or is not parallel to the surface of the transparent body.

[7.21] Now BN is a segment of line BZ. Let us draw normal NC. The image of N will thus lie on line NC. Accordingly, let C be the image of N. C will therefore lie either on line LC or near it, so [line] LC will either be equal to line BN or nearly so.[157] In the first theorem of this chapter [i.e., proposition 12], however, we demonstrated that BG is perceived as larger than it [actually] is, and the reason for this is refraction. Furthermore, the refractions of forms that lie farther from the normal dropped from the center of sight to the surface of the transparent body are more pronounced than the refractions of forms that lie nearer the normal. Hence, the refraction of the form of BN to A is more pronounced than the refraction of the form of line segment ZN to A. Thus, what causes the form of BZ to appear magnified causes BN to appear proportionately larger in relation to that very [form] than BZ does in relation to BN.[158] Consequently, LC, which is the image of BN, is perceived as larger than BN.

[7.22] Furthermore, if [the center of sight at] A does not perceive the image of BN as larger than BN [itself], it will not perceive the images of the remaining segments of line BN that are nearer to Z as larger than the segments themselves, for the forms of the remaining segments are refracted less than the form of BZ. But refraction causes the image [to be magnified], so [the visual faculty at] A would not perceive LO as larger than BZ, but in fact it does perceive LO as larger than BZ. Hence, it will perceive BN as larger than it [actually] is. Furthermore, A lies outside the normals dropped from BZ to the surface of the transparent body, and the line extending from A to the midpoint of BZ is not perpendicular to BZ, and this same situation follows in the [demonstrations based on the] three [previous] figures, i.e., in the second, third, and fourth of this chapter [figures 7.7.61a-7.7.62a, pp. 199-200].

[7.23] Accordingly, everything perceived by the visual faculty through any transparent body [that is] denser than air [and] whose surface is plane is perceived as larger than it [actually] is, whether the center of sight lies on any normal dropped from the object seen to the surface of the [transparent] body, or whether it lies outside [all such normals], and it is irrelevant whether the cross-section of the visible object is parallel or not parallel to the surface of the [transparent] body.

[7.24] **[PROPOSITION 15]** Now let the surface of the [transparent] body be spherical [figure 7.7.64, p. 201], let its convexity face the eye, and let it be denser than the air [in which the eye is located]. Let A be the

center of sight and BG the visible object, and let the center of the sphere lie beyond BG with respect to the center of sight. Let D be the center and Z the midpoint of BG, and let us connect DB, DZ, and DG and extend these lines until they intersect the surface of the sphere at E, M, and N. Then let us extend ZM in the direction of M.

[7.25] First, let the center of sight lie on ZM. AMZ will thus be a straight line. Let BD = GD for a start. Therefore, AZ will be perpendicular to BG, so the location of B with respect to A will be equivalent to the location of G with respect to A. Let us produce the plane in which DE and DN lie. It will therefore form an arc of a great circle on the spherical surface. Let EMN be that arc, then, and this [lies within a] plane [that] is perpendicular to the spherical surface, and no refraction occurs outside this plane, for AZ is perpendicular to the surface of the [transparent] body. Hence, the form of any portion of BG is refracted to A only from the circumference [of the great circle containing] EMN.

[7.26] Accordingly, let [the form of] B be refracted to A from H, and [let the form of] G [be refracted] to A from T. Consequently, the location of H with respect to A, as well as its distance [from A], is equivalent to the location and distance of T [with respect to A]. Let us connect BH, HA, GT, and TA, let us extend AH to K and AT to L, and let us connect KL. Therefore, AK = AL, LK will be the image of BG, and it will be parallel to BG, so it will be larger than BG. Let us connect AB and AG. Hence, angle KAL > angle BAG, and the orientation of KL will be equivalent to the orientation of BG. Between KL and GB, moreover, there is no [perceptible] difference in distance [with respect to center of sight A], as we claimed earlier, so KL will appear larger than BG. But KL is the image of BG, so BG will appear larger than it [actually] is because its image is larger than it. And this [is so] because its form is weaker than its true form [would be if seen from the same distance], and this is what we wanted [to demonstrate].[159]

[7.27] **[PROPOSITION 16]** Accordingly, if BD and GD are unequal, then AK and AL will be unequal, so BG and KL will be oblique to line AD. Consequently, as we claimed [on the basis of the demonstration tied to] the second figure of this chapter [i.e., figure 7.7.61a, p. 199], KL will be [perceived as] larger than BG in the visual faculty.

[7.28] Moreover, if A [in figure 7.7.65, p. 202] lies [in a plane] outside [of and oblique to] the [vertical] plane [containing] BZG [and DZM perpendicular to BG], it will be demonstrated, as [it was] in the proofs [based on] the third and fourth figures of this chapter [i.e., figures 7.7.62 and 7.7.62a, p. 200], that, whether BD and GD are equal or unequal, KL [seen under angle KAL] will appear larger than BG [seen under angle

BAG]. But let [AO] intersect KL at O in front of DM.[160] KO will therefore be the image of BZ. Furthermore, angle KAO > angle BAZ, the orientation of KO is equivalent to the orientation of BZ, and the distances of KO and BZ with respect to A are not much different, so KO will appear larger than BZ.

[7.29] And [if] A lies on normal ZM [in figure 7.7.65a, p. 203], which extends to the surface of the [transparent] body from the endpoint [Z] of BZ, let BC be a segment of BZ, and let KR be the image of BC. Hence, as we claimed in the [demonstration based on the] fifth figure of this chapter [i.e., figure 7.7.63, p. 201], it is evident that KR will appear larger than BC. Now A lies outside all the normals extending from BC to the surface of the [transparent] body, for the line that extends from A to the midpoint of BC is not perpendicular to BC. And since BG and KL are oblique to AZD or to the plane that passes through line MD, and since KO is the image of BZ and LO the image of ZG, and since the angle that KO subtends at the center of sight is greater than the angle that BZ subtends at the center of sight, and likewise, since the angle that OL subtends is greater than the angle that ZG subtends, KO will appear larger than BZ, and by the same token KR will appear larger than BC. And all these [conclusions] are established in the [demonstration based on the] fifth figure of this chapter [i.e., figure 7.7.63, p. 201]. But in this case there is something additional: namely, that KL, which is the image of BG, is actually larger than BG, and KO is [also actually] larger than BZ.

[7.30] In the first case, i.e., in [that of] a plane [refracting] surface, the two images are the same size as the two objects that are seen, so [in the case of the spherical surface] the image of KL and the image of KO are larger to the visual faculty than the objects themselves, and so they are in actuality. But it is clear that, when the center of sight lies outside the plane in which DE and DZ lie, as we claimed in the [demonstration based on the] fourth [figure] of this chapter [i.e., figure 7.7.62, p. 200], the angle that KL subtends at the center of sight is greater than the angle that BG subtends at the center of sight, and the angle that KO subtends at the center of sight is greater than the one BZ [subtends at the center of sight]. Therefore, if the visual faculty perceives something through a [transparent] body denser than air, if that body's surface is spherical and its convexity faces the center of sight, and if its center lies beyond the visible object as far as the center of sight is concerned, it will perceive that object as larger than it [actually] is, whether the center of sight lies on or outside the normal dropped from the visible object to the spherical surface, or whether the line extending from the center of sight to the middle of the visible object is perpendicular or oblique to the visible object, and this is what we wanted to demonstrate.

[7.31] This happens in [the case of] things seen in water, for the convex spherical surface of the water faces the eye, and the center [of curvature] of the water's surface lies beyond the things perceived in the water, and water is denser than air.[161] However, if the water is clear and shallow, the visual faculty may not perceive what is seen in the water as larger in the water than [it would appear to be] if it were in air, for in that case its size does not vary according to sense perception, i.e., its size in water and in air, because the magnification in the water will be slight, so the sense [of sight] will not discern that magnification.

[7.32] Nonetheless, it can be understood experimentally as follows. Take a cylindrical object not less than a cubit in length, and let it be somewhat thick and white, for white is quite clearly discerned in water. Let the surface at its base be plane, so that it can stand upright by itself on the ground. When this condition is met, take a capacious vessel, let its [bottom] surface be plane, and pour clear water into the vessel to a height less than the length of the cylindrical object. Then put that cylindrical object in the water, and place it in the middle of the vessel on its base. Consequently, some portion of this body will lie outside the water, for the height of the water is less than the length of this object. In that case, when the water calms down, you will see the portion of the object under water [as] thicker than the portion above the water. From this experiment it is evident that every visible object perceived in water is perceived as larger than it is in actuality.

[7.33] To continue, let there be a [transparent] spherical body with its convexity facing the eye, let the visible object lie beyond the center [of curvature] of the spherical surface, and let that [transparent] body be denser than air. Now in [the case] of customary visible objects there is no such thing to be seen beyond a transparent spherical body denser than air and lying beyond the sphere's center while the visible object will nonetheless lie inside the sphere, for this happens only if the spherical body is made of glass or stone and if the entire spherical body is solid and the visible object lies inside it, or if the spherical body constitutes a portion of the sphere greater than a hemisphere, while the visible object is attached to its base. But these two situations rarely occur. Hence, things of this sort are not among customary visible objects, so we need not deal with what happens with these sorts of visible objects.

[7.34] There are, however, some customary visible objects that are seen through a transparent spherical body that is denser than air and that has its convex [surface] facing the eye, [such as] when the visible object lies behind a crystalline or glass sphere, and the object lies in the air instead of inside the sphere; but the conditions under which such visible objects

[are seen] are manifold. Such situations are rarely perceived, however, and if they are perceived, they are rarely noticed. Consequently, it is not appropriate to differentiate all these situations, so we are satisfied [to deal with] one single situation, i.e., the one in which the center of sight and the visible object lie on the same normal [passing] through the surface of the [transparent] spherical body.[162]

[7.35] **[PROPOSITION 17]** Accordingly, let A [in figure 7.7.66, p. 204] be the center of sight and BGZD the [transparent] spherical body, and let E be its center. Let us connect AE and extend it straight, and let it intersect the surface of the sphere at the two points B and D. Then let us extend it to H on the side of D. Let us also produce a plane surface along line HBA to cut the sphere.[163] It will thus form [great] circle BGZD on the sphere's surface.[164]

[7.36] Now in the ninth proposition of chapter [five] on images [i.e., proposition 9, pp. 291-292], we said that there are several points on line BD whose forms are refracted to A from circumference BGZD and that the form of that entire line is refracted to A, if BGZD is continuous and not interrupted [by a refractive interface] on the side of D.[165] [Let us therefore start by assuming that the entire area below arc ZD is of the same density as the area within the sphere. Accordingly] let [the form of line segment] HL [on the extension of BD] be refracted to A from [arc BT on] circumference BGZD, and let [the form of] H be refracted to A from G and [the form of] L [be refracted] to A from T. Hence, the [entire] form of HL will be refracted to A from arc GT. Let us connect lines GMH, GA, LZT, and TA. [The form of] H therefore extends along GH and is refracted along GA, and [the form of] L extends along LT and is refracted along TA. Let us then connect lines EG, ET, EM, and EZ, and let us extend EM to C and EZ to F.

[7.37] Accordingly [if we switch the direction of radiation], the form [of point A] extending along AG is refracted along GH and reaches H, whereas the form [of point A] extending along AT is refracted along TL and reaches L. This [is the case] if the transparent body is continuous all the way up to G. If, therefore, the spherical body is demarcated [by a refractive interface] at the spherical surface [containing arc ZD], the form that extends along AG is refracted along GM toward normal EG, and when the form reaches M, it will be refracted again away from normal EMC. So let it be refracted to K. Hence, the form that extends along AT is refracted [toward normal ET] along TZ, and when it is refracted at Z, it will be refracted again away from normal EZF. Let the refraction of the form reaching Z be along line ZO, then.

[7.38] Consequently [if we again switch the direction of radiation], the form of K extends along KM, and it will be refracted along MG; then it is refracted again along GA. Likewise, the form of O extends along OZ, and it is refracted along ZT; then it is refracted again along TA. Hence, the form of the entire [line] KO is refracted to A from arc GT. And if line AK is [held] stationary, and if we imagine the figure [delineated by] AGMK to be rotated around AK, arc GT will form a circular figure, like a ring, from whose entire [surface] the form of KO will be refracted to A, and the image of KO will be [located at] the center of sight, which is A. Thus, the form of KO will appear on the entire circular [ringlike] surface that constitutes the area of refraction, which lies on [the endpoints of] the straight, radial lines [extending from A] and is shaped like a ring.[166] Hence, the form [TG] of KO will be larger than [KO] itself, and the shape of the form [which is convex with respect to A] will be different from the shape of KO [which is straight].[167]

[7.39] This, moreover, can be determined empirically as follows. Take a perfectly round crystalline or glass sphere, and take a small object or a small piece of wax the size of a chickpea, for the experiment will be clearer [when conducted] with a small object. It should be colored black, and the shape of the wax should be spherical. You will then put it on the point of a needle, place the crystalline sphere in front of either eye, and close the other eye. Then raise the needle behind the sphere, look toward the center of the sphere, and pose the wax [sphere] directly opposite the middle of its image, so that, as far as can be sensibly determined, it is opposite the center of the sphere in a straight line. Then look at the surface of the sphere, for in that case you will see a round black [area] in the shape of a ring on that surface. In case you will not see this, however, move the wax to and fro until you see the round black [image].[168] Remove the wax at that time, and the blackness will disappear; then the wax should be returned to its [original] place, and you will [again] see that round black [image].

[7.40] It will thus be manifest from this experiment that, if the visible object lies behind a transparent spherical body that is denser than air, and if the center of sight, the visible object, and the center of the spherical body lie on the same straight line, the visual faculty will perceive that visible object in the form of a ring.

[7.41] **[PROPOSITION 18]** On the other hand, if BGZD [in figure 7.7.66, p. 204] lies on a [transparent] cylindrical body, and if the body is denser than air, the form of KO will be seen on arc GT as well as on an equal arc corresponding to it on arc BD, but this form will not be circular because, when it is revolved about AK, the figure [delineated by] AHMG

will not carry arc GT through the entire surface of the cylinder along that line. Instead, the form may be refracted from some portions of the [surface of the] cylinder, but it will be straight and continuous, for the plane through LK that passes along the cylinder's axis forms a straight line [of longitude] on the surface of the cylinder facing A [and] that [plane] passes through B and [forms a line through B that] extends along the length of the cylinder. But the form of KO is not refracted from that straight line, for KB will be perpendicular to that straight line.[169] Therefore, if the [transparent] body is cylindrical, the form will not be round [and annular], but there will be two images, each one of them refracted to [one or] the other [side of B].[170] Hence, KO will appear double, both of [its images] larger than KO, and the shape of both will be different from the shape of KO, yet those two forms will be [located at] the same point, i.e., the center of sight.

[7.42] Now among customary visible objects there are none perceived by the visual faculty behind a spherical transparent body denser than air with its concave surface facing the eye, for if [that transparent body] were made of glass or some [transparent] stone, it follows that a portion of the sphere must be concave and that the visible object must lie inside [the transparent body containing the concave portion of] that sphere, or else the [back] surface [of the transparent body] beyond its concavity must be plane, and the visible object must be attached to it. But these two situations are not encountered, or only rarely, so we need not concern ourselves with such a case.

[7.43] Furthermore, no [transparent] body rarer than air with a plane or convex surface facing the eye is encountered, and the only body rarer than air through which anything is perceived is the body of the heavens and [the body of] fire. The body of air will not be subdivided by [any] surface that demarcates one part from another, but [instead], the closer the air approaches the heavens, the more it is purified until it becomes fire. Its rarity is therefore continuously graduated, not [defined] according to a determinate differentiation. Consequently, when they extend to the eye, the forms of objects in the heavens are not refracted at the concave surface of the sphere of fire [demarcating that sphere from the sphere of air below it], since there is no determinate concave surface there. Thus, there is found no body rarer than air, other than the celestial body, through which the forms of visible objects extend and at whose surface they are refracted to the eye, and the celestial body is spherical concave with respect to the eye. Hence, [the forms of] all the stars in the heavens extend through the body of the heavens, are refracted at the concave surface of the heavens, and extend straight through the body of fire and through the body of air until they reach the eye, and the center of the heavens's concave curvature is the

center of the Earth.[171]

[7.44] I say, then, that for the most part the stars are not[172] perceived in their [actual] places and that they are never perceived according to their [proper] sizes, yet nonetheless the size of each one of them varies according to how their location varies. A variation in location, on the one hand, is due to the orientation of the refracted rays, as we said earlier. A variation in size, on the other hand, is due to the distance [the stars lie from the center of sight], for according to [such] distance they will be perceived as smaller than they are in reality, as we said in the third book, i.e., those that lie extremely far away are perceived as smaller [than they actually are].[173] But a variation in size due to a variation in location occurs according to refraction, and we have [already] shown why here [in this chapter].[174] In the fourth chapter, moreover, we showed that the forms of the stars that are perceived by the visual faculty are refracted.[175]

[7.45] I say, therefore, that every star is perceived at every location in the sky through which it moves as smaller than it actually is according to [the size] dictated by its distance, i.e., smaller [than] if it were seen directly, as long as there are no clouds or thick vapor between it and the center of sight. Moreover, every star lying at the viewer's zenith appears smaller than [it does] in any other location in the sky, and the farther it lies from the zenith, the larger it appears, so that at horizon it appears larger [than it does] at any other location. And this applies universally to all heavenly bodies [whether they lie] at a great distance or [relatively] near [as do the sun and moon in respect to Saturn and the fixed stars].

[7.46] Furthermore, if there is thick vapor in the air behind which a star lies, it will be perceived as larger than [it would be] if it were [perceived] without that [intervening] vapor, and it frequently happens that thick vapor lies at the horizon, so for the most part stars appear larger at the horizon than [they do] in the middle of the sky. In [the case] of the intervals between stars this [effect] is [even] more apparent than in [the case of] the sizes of the stars themselves, for the size of a star is small as far as sight is concerned, whereas at the horizon the magnification of the intervals between stars is quite obvious to the sense [of sight], especially in [the case of] vast intervals, and especially if there is thick vapor at the horizon.

[7.47] **[PROPOSITION 19]** Accordingly, let BK [figure 7.7.67, p. 207] be the circle of the meridian in some horizon plane [XT], and let the common section of this circle and the concave surface of the heavens be circle MEZ. Let G be the center of the world and T the center of sight [GT thus being the radius of the Earth], and let us extend GT on the side of T. Let it intersect the circle of the meridian at B, and let it intersect the circle on the concave

surface of the heavens at E. B will thus be the zenith for center of sight T. Let [straight line] KL be the cross-section of some star or the [rectilinear] interval between two stars, and let line TB pass through the middle of KL; it will intersect it at C. Hence, arc KB = arc BL. Then let us connect the two lines TK and TL. Angle KTL will thus be the one under which [the center of sight at] T perceives KL, if it were perceived directly.

[7.48] Let [the form of] K be refracted to T from M, and [let the form of] L [be refracted] to T from Z.[176] Let us connect GM and GZ, and let them continue to F and O. Let us then connect lines KM, MT, LZ, and ZT. The form that extends from K along KM is refracted along MT, and GM is the normal dropped from M, which is the point of refraction, to the surface of the [transparent] body on the side of T, and since the [transparent] body [beyond] ZM is rarer than the [transparent] body [containing] GT, MT's refraction will occur toward normal MG. M will therefore lie between the two lines TB and TK, for if M lay beyond TK, the perpendicular dropped from G would lie beyond TK, and when it extended to that point, the form of K would be refracted toward the normal but would not reach the normal [itself], and [so] it would not reach T. Hence, M lies between the two lines TK and TB, and it will be demonstrated the same way that Z lies between the two lines TB and TL.

[7.49] Let us extend TM to Q and TZ to R. Therefore, arc QK = arc LR, and angle QTR < angle KTL. But angle QTR is the one under which [the center of sight at] T perceives KL according to refraction, and angle KTL is the one under which [the center of sight at] T would perceive KL if it were perceived directly. KL's distance from the center of sight is vast, however, so its size is not accurately determined, [and] so [the center of sight at] T estimates the distance of KL, as we explained in the second [book] of this treatise.[177] But when it perceives [things] according to refraction, the way it estimates [distance] does not differ from the way it estimates [distance] when it perceives [things] directly, except that [the visual faculty] judges that it perceives [it] directly when [in fact] it perceives it by means of refraction. According to refraction, then, [the center of sight at] T perceives KL under a smaller angle than the one under which it perceives it directly and [it perceives it] by correlating it to the same distance to which it would correlate it if it perceived it directly. But the visual faculty perceives size on the basis of the size of the angle with respect to distance, so according to refraction [the center of sight at] T perceives KL to be smaller than if it perceived it directly.

[7.50] If, moreover, we rotate figure KTL about TB [while holding TB] stationary, it will form a circle, and the angles that the two lines KT and TL, as well as their counterparts [during the revolution], form at T will be equal. Thus, according to refraction [the visual faculty at] T perceives

KL, when it lies at the zenith, to be smaller everywhere with respect to the circle of the meridian than if it perceived it directly. And if TB bisects KL, the two points Q and R will also lie between the two points K and L, and angle QTR < angle KTL, and every [counterpart of] angle [QTR within the circle of revolution and] issuing from point [T] to intersect the star [will be smaller than KTL], and a line passing from T on the surface of that circle will intersect the circle, and [so] it will be perceived as smaller than it is. And so the entire star will appear smaller than it is.

[7.51] Therefore, a star at the zenith is perceived as smaller than if it were perceived directly, and likewise the interval between two stars will be perceived in every situation as smaller than if it were perceived directly, when the zenith point lies between the two endpoints of the interval, and this is what we wanted [to demonstrate].

[7.52] **[PROPOSITION 20]** Likewise, if the star or the interval lies below the zenith and the horizon, on the horizon [itself], or between the horizon and the zenith [it will appear smaller according to refraction than if seen directly].[178]

[7.53] Let A [in figure 7.7.68, p. 208] be the center of sight and B the zenith, and let us connect AB. Let DE, the diameter of the star or the interval [between stars], be parallel to the horizon [XY], and let circle BD be the vertical circle passing through one endpoint of the diameter or interval and BE the [vertical] circle passing through the other endpoint. Let the two common sections of [these] circles and the concave surface of the [inner] sphere [defined by the heavens] be the two circles HG and GZ. Hence, the form of D is refracted to A in the plane of circle BD. Let us connect AD and AE. Accordingly, arc BD = arc BE because DE is parallel to the horizon [by construction], and [the form of] D is refracted to A from H, whereas [the form of] E [is refracted] to A from Z.

[7.54] Let us connect lines AH, HD, AZ, and ZE. Let M be the center of the world, and let us connect MH and MZ, and let them continue to F and N. MH will thus be the normal dropped from H to the surface of the transparent body, and [ray] HA will be refracted toward HM, so it will be refracted away from HF [i.e., toward A in the opposite direction from F]. H is therefore higher than AD, and it will be demonstrated in the same way that Z is higher than AE. Thus, the two points F and N lie between the two points D and E,[179] and the angle of refraction at H is equal to the angle of refraction at Z, for the two points D and E are equivalently situated with respect to A, so F lies as far from D as N [lies] from E.

[7.55] Let us extend AH to T and AZ to K. Accordingly, T will lie as far from D as K [lies] from E. Let us connect TK. It will thus be parallel to

DE, so it is smaller. But lines AT, AK, AD, and AE are equal because A is virtually the center of the two circles BD and BE.[180] The two lines AT and AK are thus [virtually] equal to the two lines AD and AE. But base TK [subtending angle TAK] is smaller than base DE [subtending angle DAE], so angle TAK < angle DAE, and angle TAK is the one under which DE is perceived according to refraction, and angle DAE is the one under which DE is perceived directly.

[7.56] Therefore, if the star lies on the horizon [itself] or between the horizon and the meridian circle, and if its diameter is parallel to the horizon, it will appear smaller than if it were seen directly, and the same applies to the interval between two stars, if the interval is parallel to the horizon, and this is what we wanted [to demonstrate].

[7.57] **[PROPOSITION 21]** To continue, let us recapitulate [the previous] diagram [in figure 7.7.69, p. 209], but let the diameter [of the star] or the interval [between stars] be upright, i.e., on the same vertical circle. Let that diameter or interval be line DE on vertical circle BDE, and let the common section of this circle and the concave surface of the [inner] sphere [defined by the heavens] be circle GHZ. Let us connect AD and AE, and let [the form of] D be refracted to A from H, and [let the form of] E [be refracted] to A from Z. As in the preceding figure, it is clear that H is higher than AD and that Z is higher than AE. Let us connect lines AH, HD, AZ, ZE, MH, and MZ, and let us extend MH to T and MZ to K. Angle AZM will thus be tiny, and angle of refraction [AZY] will [only] be a part of it, so [vertical] angle EZK [which = AZM + AZY] will be acute, and likewise [vertical angle] DHT [which = AHM + AHX, will be] acute, and [so] both angles AHD and AZE are obtuse.[181]

[7.58] Now Z will lie either on the horizon or above it, so it will lie at the endpoint of the normal [CA] dropped to AB from A [within the plane of the horizon], or [it will lie] above it, and H is higher than Z. Therefore, angle AHM < angle AZM, so angle DHT < angle EZK. Consequently, angle AHD > angle AZE. But the two lines MT and MK are diameters of circle BDE and [coincide with] diameters [MH and MZ] of circle GHZ, so MT = MK, and MH = MZ. Hence, HT = ZK, and angle DHT < angle EZK, so line HD < [line] EZ.

[7.59] Also, the two lines AD and AE are [virtually] equal, and A is virtually the center of circle BDE, so the circle that circumscribes triangle AHD is larger than the circle that circumscribes triangle AZE because angle AHD > angle AZE. However, as has been demonstrated [earlier], line HD < [line] ZE, so HD cuts off a smaller arc on the circle circumscribing triangle AHD than the equivalent arc that ZE cuts off on the circle circumscribing

triangle AEZ. Therefore, angle HAD < angle ZAE.[182]

[7.60] So let angle ZAD be common. Therefore, angle HAZ [which = ZAD + HAD] < angle DAE [which = ZAD + ZAE], and angle HAZ is the one under which [the center of sight at] A perceives DE according to refraction, whereas angle DAE is the one under which it perceives DE, if it were to perceive it directly. Consequently, according to refraction [the center of sight at] A perceives DE as smaller [than it would if it perceived it] directly, and this demonstration will follow if circle BDE is the circle of the meridian [as in proposition 19 above].

[7.61] When it is upright and faces the eye directly, then, the diameter of a star or the [rectilinear] interval between two stars is perceived as smaller according to refraction than [it would be if seen] directly, and this is what we wanted [to demonstrate].

[7.62] Every star in the sky is perceived as round, so its diameters are perceived [to be] equal.[183] And since it is evident that both its upright and transverse diameters across the width are perceived as smaller than if they were perceived directly, it follows that each one of its inclined diameters is perceived as smaller than if it were perceived directly. By the same token, the intervals between stars are perceived everywhere and in every case as smaller than if they were perceived directly. Furthermore, we have said that every star lying at the zenith is perceived as smaller than anywhere else in the sky, and [we have said that] the farther it lies from the zenith, the larger it will be perceived and that it is perceived as largest when it is perceived at the horizon. This fact, i.e., that the star appears smaller in the middle of the sky than when it lies at an intervening part of the sky, is certainly attested to be [the case], and the same [holds] for intervals [between stars], both the star and the intervals being likewise largest at horizon. So it remains to explain why this is so.

[7.63] I maintain that, when we dealt with [the perception of] size in the second [book] of this treatise, we showed that, if the visual faculty perceives the sizes of visible objects, it will perceive them according to the sizes of the angles that the visible objects subtend at the center of sight as well as [according to their] distances [from the center of sight], and [the final judgment of size will be based] on correlating the angles to the distances. We also showed that the visual faculty never perceives the sizes of visible objects [properly] unless the distances they span lie in line with [a] continuous [succession of] neighboring objects and that, if the visual faculty does not determine the distances of visible objects accurately, it will not determine the sizes of [those] visible objects accurately. At that same point we also showed that, if the visual faculty does not determine the distance of a visible object accurately, it can assess its distance [by estimation] and

match it to the distances of customary visible objects according to which such a visible object is perceived in such a form and shape, and then it perceives its size according to the size of the angle that the visible object subtends at the center of sight with respect to the distance it assesses [by estimation].[184]

[7.64] But the distances of the stars do not lie in line with [a continuous succession of] neighboring bodies, so the visual faculty does not perceive their sizes [properly], nor does the visual faculty determine the distances of the stars accurately, so the visual faculty assesses the distance of the stars [by estimation] and matches their [estimated] distances to vast terrestrial distances that are perceived, and it assesses their sizes [accordingly]. Moreover, that the body of the heavens is spherical and that its concave [outer] surface faces the eye is not discernible to the sense [of sight], nor does the visual faculty sense the bodily mass of the heavens,[185] nor does the visual faculty sense anything about the heavens except its bluish color.[186] Indeed, its bodily mass, its extension in three dimensions, its roundness, and its concavity can in no way be perceived. And when the visual faculty does not perceive something accurately, it will match it to those things corresponding to it among customary objects, so it perceives the sun and moon as flat, and at a vast distance [it perceives] convex and concave objects as flat, and it will perceive arcs whose convexity or concavity faces the eye as straight, for if it does not perceive the nearness of the middle and the remoteness of the edges [of a convex surface] or the remoteness of the middle and the nearness of the edges of a concave [surface], it will match [remote] convex and concave surfaces to flat surfaces, and it will match [remote] arcs to straight lines because customary visible objects are generally flat and straight.[187]

[7.65] Also, when the form of a star reaches it, the visual faculty does not sense that the form is refracted or that it will be refracted at the concave [inner] surface [of the heavens], and [it does not sense] that the [transparent] body [of aither] in which the star lies is rarer than the [transparent] body [of air] in which the eye lies. On the contrary, the form of the star is perceived in the same way as the forms of other things that are perceived directly in the air, and as far as the visual faculty is concerned, the forms of visible objects are not refracted when they encounter a [transparent] body different from air, nor does the visual faculty sense their refraction, and [it does not sense] the surface at which the forms are refracted in bodies whose transparency is different on account of the natural tendency of the form of light and color to extend through transparent bodies [along straight lines]. Consequently, [when their light is] refracted, the forms of the stars reach the eye in the same way as the forms of objects in the air reach the eye, and

they are perceived in the same way they are perceived in air.

[7.66] Furthermore, the visual faculty perceives the color of the sky, but yet it does not accurately determine its shape by brute sensation.[188] And when the visual faculty perceives some color extending the length and breadth of something whose shape and form it will thereby perceive, it will perceive it as flat, for it will match it to various customary surfaces, such as walls and other [flat surfaces], and this is how it will perceive convex and concave surfaces at a vast distance. The visual faculty also perceives the face of the Earth as entirely flat, and it does not sense its convexity unless there are mountains and valleys there. Hence, the visual faculty perceives the surface of the heavens as flat, and it will perceive the stars in the same way it perceives customary visible objects scattered over vast areas [on a flat surface]. And when the visual faculty perceives any visible objects scattered over a vast area, and when it perceives them under the same angles and perceives the distances of those visible objects, it will perceive what lies farther away as larger, for the extent of the distances will be perceived according to a correlation of the angle that the distance subtends at the center of sight to a remote distance, and it will perceive how far away a near object lies according to a correlation of the angle that the near object subtends, which corresponds to the angle that the distance subtends to a near distance.[189]

[7.67] This is clear and is attested to by the fact that, when two visible objects are perceived by the visual faculty under two equal angles, and when their distances are perceptibly different, the farther [distance] will appear longer, and the farther [object] will appear larger. For if a person faces a broad wall, raises his hand until it is in front of the eye, closes the other eye, looks with the remaining one [facing the hand], and poses his hand in between his eye and that wall, his hand will block a portion or the [entire] width of that wall, and he will perceive the wall and his hand at the same time. Hence, he will perceive his hand under an acute angle, and in this case he will perceive that the wall is many times wider than his hand. Then, if he moves his hand so that the portion of the wall that his hand blocked is uncovered, and if he looks at what has been uncovered of the wall and [also] looks at his hand, he will see that the uncovered portion of the wall is many times larger than his hand. But he will perceive his hand, and he will perceive the wall under two equal angles, from which it is evident that the visual faculty perceives size by correlating the angle to the distance.

[7.68] Accordingly, the visual faculty will perceive the surface of the heavens as flat and will not sense its concavity, and it will perceive the stars scattered on it. It will therefore perceive the stars scattered on it, [which are

all] the same size, as unequal [in size], for it correlates the angle that a star nearest the horizon subtends at the center of sight to a far distance, and it correlates the angle that a star at or near the middle of the sky subtends to a near distance. By the same token, it perceives a star at or near the horizon as larger than one at or near the middle of the sky. Hence, it perceives the same star or interval [between stars] to be different sizes at different places in the sky, so it perceives the same star or interval at or near the horizon as larger than [when it lies] at or near the middle of the sky, for when the star is at the horizon, [the visual faculty] correlates the angle that the star subtends at the center of sight to a far distance, and when the star is in the middle of the sky, it correlates the angle the star subtends at the center of sight to a near distance. But there is hardly any difference between the angle the star subtends at the center of sight when the star lies in the middle of the sky and the angle it subtends at the center of sight when the star lies at the horizon; on the contrary, although different, the two angles are close [to equal], and the same holds for the intervals between stars.[190] And when the sense [of sight] correlates two angles nearly equal in size to two [noticeably] different distances, it perceives the farther one as larger.

[7.69] What substantiates this reasoning is that, since the lines that form the angles subtended by the same star at the center of sight anywhere in the heavens are refracted because the center of sight is at the center of the heavens, and since the refracted forms of the stars are not much reduced in size according to those angles, and since those reductions in size are not significant, the difference between the refracted angles under which the star is perceived or the difference among intervals between stars at various places in the sky will not be significant. And since the difference between those angles is not significant, the size of the star will not be perceived as significantly different. The demonstration that the angles after refraction are not significantly smaller than the angles formed by the straight lines and that [the reductions in size of those refracted angles] are quite small follows from what was claimed in the earlier experiment in chapter [4] on refraction, where we showed that the visual faculty perceives a star according to refraction and that it observes a fixed star [according to its angular distance] from the celestial pole, and its distance is [measured] from that [pole] during one revolution, for the difference [between its distance from the pole at one point in the sky and its distance from the pole at another point in the sky] turns out to be slight, from which it is evident that the angles of refraction are slight.[191] Hence, according to the difference between them the angles under which the star is perceived at various places in the sky do not differ very greatly.

[7.70] Nevertheless, the [apparent] size of a star and the [apparent size

of an] interval between stars differ considerably when they are at horizon and when they are in the middle of the sky, so the difference in size of the star or the interval at various places in the sky is not due to a difference in the angles of refraction. Moreover, we have already shown that the visual faculty perceives size by correlating angles to distances, so if the difference between the angles is slight but [the difference] between distances is great, an object will appear larger at a longer distance. Hence, the reason the distances of the stars at horizon appear longer than [they do] at or near the middle of the sky is because the sense [of sight] estimates that at the horizon they lie farther away than [when they lie] in the middle of the sky, and the visual faculty's perceiving the stars as different sizes at various places in the sky is a constant error because its cause is invariant, and it is due to the visual faculty's perceiving the surface of the heavens as flat, and it [therefore] does not sense its concavity or the equality of the distances [of all points] on it from the center of sight. But it is established in the soul that on a flat surface that extends in every direction the distances on it differ in [relation to] the center of sight and that what lies closer is what lies nearer the head. Accordingly, it perceives what lies at the horizon as farther away than what lies in the middle of the sky, and [it also perceives] that the angles the same star subtends at the center of sight anywhere in the sky do not differ by much. Moreover, because the visual faculty perceives the size of an object by correlating the angle the object subtends to the distance of that object from the center of sight, it perceives the size of the star or the size of the interval between stars by correlating the angle [subtended by the star or interval] to a remote distance, when the star is at or near the horizon, or by correlating the angle, [which is] equal or nearly equal to the first [angle], to a near distance, when the [star and interval] are at or near the middle of the sky, and between that distance and the distance to the horizon there appears to be a vast difference.

[7.71] This, then, is why the visual faculty errs in [perceiving] a difference in the size of stars or the intervals [between stars], and the cause for this [error] is fixed, constant, and immutable. But the visual faculty perceives the stars as small because of their remoteness, for they subtend tiny angles at the center of sight. Yet the sense [of sight] does not determine the distance of the star accurately; rather, it estimates [the distance] and correlates the [estimated] distances of the stars to the distances of customary visible objects on Earth so that it assumes that the distance of the star is like the distance of something extremely far away on Earth. Hence, it correlates the angle the star subtends at the center of sight, which is small, to a distance that is like the distance of things that are on Earth, and so it perceives the star as small according to this correlation. But if the visual faculty were certain

about the size of the star's distance, it would perceive it as large, and in the same vein, if they are perceived as small, all objects that lie at an extremely remote distance on Earth are [so perceived] because their distance is not accurately determined, and this has already been demonstrated completely in the third book of this [treatise].[192] And just as the visual faculty errs in [judging] the size of the stâr's distance because it is not certain about it, and [just as it] matches it to distances on Earth, so it errs in [judging] that the distances of things at various places in the sky are different, when they are [in fact] equal, because it also matches them to different distances to the right or left or in the facing direction on the Earth's surface, there being no doubt that they are different. And just as an error about the distance and size of the star is constant, an error in [perceiving] a difference in the distances and size of the stars at various places in the sky is constant, for the forms of these distances do not differ according to the visual faculty at various times but are always of the same sort, and the visual faculty matches them to the distances of customary objects that lie extremely far from the center of sight on the face of the Earth.[193]

[7.72] There also happens [to be] another factor that causes things in the heavens to appear for the most part larger at the horizon, namely, thick vapors that lie between the eye and the stars. When the vapor lies at or near the horizon but does not rise uninterrupted to the middle of the sky, it will form a segment of a sphere whose center will be the center of the world, which includes the Earth, and so it will be truncated on the side of the sky's middle, and the surface on it that faces the eye will be plane, so the form [of a star] or the intervals [between stars] that lie beyond that [wall of] vapor will appear larger [than they would] without that vapor, for the form of the star lies at the spot on the [inner] concave surface of the heavens from which the star's form is refracted to the eye, and it extends straight to the center of sight from that point if there is no thick vapor at the horizon.

[7.73] On the other hand, if thick vapor is present, this form will extend to the surface of the vapor facing the heavens and will lie on that surface, and so the visual faculty will perceive it as it will perceive objects that lie in vapor, that is, the form extends straight through the thick vapor and is refracted away from the normal dropped to the vapor's surface at the surface of the vapor on the side of the eye (that surface being plane), for the air on the side of the eye is rarer than the vapor; so it follows from this that the form should appear larger than [it would] if it were seen directly, just as we claimed in the first proposition of this chapter [i.e., proposition 12, pp. 309-311]. And since the rarer body lies on the side of the eye and the denser one on the side of the visible object, the surface of the denser

body will be flat. Hence, the form that reaches the surface of the vapor on the side of the heavens constitutes the visible object, and the body through which this form extends is the thick vapor, whereas the air in which the eye lies is rarer than it.[194]

[7.74] So the main reason that the stars and the intervals between stars appear larger at the horizon than in the middle of the sky is the one described before, and it is fixed and invariant. If it happens that there is thick vapor, however, their [apparent] size increases. But this factor applies invariably in some places and [only] occasionally in others.[195] Thus, all the misperceptions befalling the visual faculty according to refraction that we have discussed in this chapter are ones that always occur, or [at least] for the most part, and they suffice for what we need [to show] concerning the misperceptions caused by refraction.

[7.75] Let us now conclude this book, which is the end of the treatise, etc.

NOTES TO BOOK SEVEN

[1]The Latin term here is *reflectitur*. Although *reflectere* and *reflexio* refer primarily to reflection in the context of Alhacen's analysis of mirrors in books 4-6, particularly 4 and 5, in the context of his analysis of transparent media in this book they refer almost exclusively to refraction; the single exception is found in paragraph 2.82, p. 34, line 15. This exception aside, the translator of book 7 uses *conversio* and *convertere* for "reflection" and "to reflect," and in that regard he is wholly consistent with the translator responsible for chapters 6-9 of book 6; see A. Mark Smith, ed. and trans., *Alhacen on Image-Formation and Distortion in Mirrors* (Transactions of the American Philosophical Society 98.1. Philadelphia: American Philosophical Society, 2008), pp. xlv-xlvi.

[2]As will become clear in due course, Alhacen assumes that the more transparent a body is, the less refractive it is because it allows light to pass through more easily. However, the difference in transparency (*diaffonitas*) that causes refraction is not a function of the clarity with which light shines through the transparent medium but of the resistance the medium poses to the passage of light because of its "density." Accordingly, clear glass will still be more refractive than murky water even though it is more transparent.

[3]The Latin term *fantasma*, used here to signify "image," is unique to this passage, the standard term used throughout the *De aspectibus* being *ymago*. Both terms, however, convey the all-important notion that, as products of the imagination (*imaginatio* or *fantasia*), images are psychological rather than physical constructs.

[4]The Latin term *visibilia*, which I have translated here as "visible objects," can be taken in two ways. On the one hand, it can refer to individual physical bodies, and on the other it can refer to the particular visible characteristics of those bodies, such as color, shape, size, and so forth, which constitute the visible intentions.

[5]This discussion actually occurs in the penultimate chapter (i.e., chapter 3) rather than in the fourth and final chapter of book 2. For Alhacen's overall account of how the twenty-two visible characteristics (or "intentions" in Alhacen's parlance) are perceived, see Smith, *Alhacen's Theory*, pp. 429-512.

[6]Alhacen's efforts to demonstrate the rectilinear propagation of light occupy chapter 3 of the Arabic original of the *De aspectibus*. Unfortunately, this and the initial two chapters of that version are missing in the Latin version. For an English translation of the relevant portion of the Arabic text, see Sabra, *Optics*, vol. 1, pp. 13-51.

[7]As units of measurement, the digit and cubit vary rather widely in length according to place and time. Generally speaking, the digit measures a finger-width, and a cubit measures the length from the elbow to the tip of the fingers

when the hand is held straight out from the wrist. The most common measures of this length range from around 46 cm (the so-called short cubit) to around 53 cm (the so-called royal cubit). Alhacen's first mention of digits and cubits occurs in book 1, chapter 3 of the *De aspectibus*, where he describes the fashioning of a board four digits wide and one cubit long (see Smith, *Alhacen's Theory*, p. 573). The width of this board, he goes on to tell us, is equal to the distance between the midpoints of the outer surfaces of the two eyes (i.e., between the two pupils), which I took to be around 3 inches (i.e., 7.6 cm), from which I concluded that Alhacen's digit was nearly 1.9 cm long. Unfortunately, we have no equivalent way of determining what he had in mind for the length of the cubit, although we are probably safe in assuming that it consists of some multiple of digits between 24 and 30. For various reasons that will become apparent in due course, we can plausibly assume that Alhacen's cubit contains 26 digits. Accordingly, the register plate measures "no more than" roughly 50 cm in diameter, and the rim stands roughly 3.8 cm high. As it turns out, the exactitude of these measures has little or no bearing on the actual use of the apparatus or the experiments based on it.

[8]Like the digit and cubit, a grain of barley is somewhat variable as a unit of measurement. In the previous edition of books 4 and 5, I chose .85 cm as the measure of a grain of barley (see Smith, *Alhacen on the Principles*, p. 352, note 30). It is not unlikely that, just as Alhacen's digit represents an increment of the cubit, his grain of barley represents an increment of the digit, the most obvious choice being two grains of barley per digit. This would make his grain of barley around .95 cm according to the measure of 1.9 cm for the digit. Both values are probably somewhat high, .9 and 1.8 perhaps being nearer the mark. Accordingly, half a grain of barley is about 4.5 mm. Whatever the case, these rough measurements give us a fairly good idea of the scale of the apparatus.

[9]The construction of the apparatus to this point can be illustrated as follows. Let circle ABDE in the top diagram of figure 7.2.1, p. 147, represent the register plate with its center at C and its diameter BCE equal to one cubit, which translates to roughly 50 cm. As seen from a three-quarter view in the bottom diagram, a rim is attached around its circumference to a height of two digits (i.e., somewhat less than 4 cm), as measured by perpendicular FK extending from the base of the register plate to the rim's lip. A relatively thin cylinder three digits long (i.e., somewhat less than 5.5 cm) is attached to the back side of the register plate at centerpoint C, as represented in figure 7.2.2, p. 148. This cylinder will function as an axle later on in the experiment.

Returning to figure 7.2.1, we are to draw two diameters, AD and BE on the face of the register plate so that they intersect orthogonally at centerpoint C. At a distance of one digit to the side of endpoint A on diameter ACD we are to mark point F at the base of the rim, and from that point we are to draw diameter FCG on the face of the register plate. From that same point F we are to draw line FK on the inner wall of the rim perpendicular to the face of the register plate, as shown in the inset to the lower diagram, which represents a blown-up view of that portion of the apparatus's rim seen face-on. Likewise, from opposite endpoint G of diameter FCG we are to draw line GL perpendicular to the register plate on the inner wall

of the rim. Hence, FK, GL, and diameter FCG will lie in the same plane, which is perpendicular to the register plate.

Now from point F we are to draw line FN on perpendicular FK equal to half a grain of barley in length, as illustrated in the inset. Line NM of the same length is then added to FN along the same perpendicular, and to that is added another line MP of the same length. Thus, the distance from F to N is half a grain of barley, the distance from F to M is one grain of barley, the distance from N to P is also one grain of barley, and the distance from F to P is 1.5 grains of barley.

Once these points are marked, we are to draw circles on the inner wall of the rim through points N, M, and F, all of them parallel to the circle at the circumference of the register plate and, therefore, parallel to one another. Consequently, the planes formed by all three circles are parallel to the plane of the register plate, and since perpendicular GL is directly opposite perpendicular FK on diameter FG, it will be cut by the three circles at points corresponding to N, M, and P on FK. As represented in the bottom diagram of figure 7.2.1, then, diameter MM′ connecting corresponding points within the middle circle will be parallel to diameter FG and will lie in the same plane as KF and GL, that plane being perpendicular to the plane of the register plate.

Starting from point M, where the middle circle intersects line FK, we are to subdivide the middle circle into 360° and then into minutes if feasible. This subdivision is represented in full 1:1 scale in figure 7.2.3, p. 148, where the zero point is at M and the arc to the right is divided successively into degrees. The first degree is in turn subdivided into quarters, consisting of 15′ each, the second into fifths, consisting of 12′ each, and the third into sixths, consisting of 10′ each. Given the small size of the arcs subtending each degree, further subdivision would be possible but not particularly useful for the purposes of later experimentation. The division of the middle circle into degrees and minutes does not come into play until the next chapter, where refraction is subjected to a quantitative analysis.

The last phase in this stage of the construction involves boring a hole precisely one grain of barley in diameter through point M on the middle circle. Accordingly, as represented in the bottom diagram of figure 7.2.1, the circumference of the hole will be tangent to the highest of the three circles at point P and to the lowest at point N. The distance from point F at the base of the register plate to point M at the center of the hole will thus be precisely one grain of barley.

[10]The construction of the panel is illustrated in figure 7.2.4, p. 148. We start with square chunk ABCD of unspecified material, probably bronze, since the panel, like the register plate, is designated by the Latin term *lamina* ("sheet" or "plate"), each of its sides being 2 digits long and its thickness sufficient to allow it to remain stable when stood upright. At midpoint E of its bottom edge DC perpendicular EF is drawn on its face. From endpoint E on perpendicular EF a line EG is drawn to a length of half a grain of barley, and to that is added a second line GH of the same length, and to that, finally, a third line HK of the same length. A hole one grain of barley in diameter and centered on point H is drilled through the panel perpendicular to its face. Thus, the distance from point E at the bottom of the panel to midpoint H of the hole is one grain of barley, and the distance from that same point E to the top of the hole at point K is 1.5 grains of barley. It therefore

follows that the hole in the panel is situated in precisely the same location on its perpendicular EF as the hole in the rim on its perpendicular FK in figure 7.2.1, p. 147, which is to say that the holes on their respective perpendiculars are perfectly congruent in both size and location.

[11]The attachment of the panel to the register plate is illustrated in figure 7.2.5, p. 149. First, radius CF on the register plate is bisected at point R, and a line is drawn through that point perpendicular to diameter FCG. The panel is then stood upright with its front edge flush with the line through R so that midpoint E on its lower edge coincides with point R. Once it is set up according to these strictures, the panel is attached permanently to the register plate, presumably with glue. In that position, the hole in the panel is perfectly aligned with the hole in the apparatus's rim so that diameter MM′ of the middle circle scribed on the inner wall of the rim passes through the midpoints of the holes in the rim and panel. This configuration, Alhacen observes, is equivalent to that used to establish a line of sight in an astrolabe (i.e., the alidade), which is in turn based on the sighting system in the surveying instrument called *dioptra* in Greek. The system of holes in this case is designed not for sighting but for channeling a beam of sunlight into the instrument for experimental purposes. Accordingly, the hole in the apparatus's rim will be pointed toward the sun so that the light streaming through it will be channeled toward the opposite side of the apparatus through the second hole in the panel. Together, these two holes serve the same function as the individual holes drilled through the wall of the wooden ring for testing the reflection of light in book 4, and the equivalency extends to the size of the holes, those in the aforementioned ring also being one grain of barley in diameter (see Smith, *Alhacen on the Principles*, p. 303).

As was pointed out in the edition of books 4 and 5 just cited, there are certain clear affinities between Alhacen's and Ptolemy's experimental apparatuses for testing the equal-angles law of reflection; indeed, all indications are that Alhacen took his inspiration from Ptolemy in both the design and application of his apparatus (see ibid., pp. lxvii-lxviii). It was also pointed out that there are marked differences in design between the two but that these are mainly due to a fundamental difference in what was being tested. Ptolemy, on the one hand, was testing the reflection of visual rays and could therefore rely on fairly rudimentary instrumentation. Alhacen, on the other, was testing the reflection of light rays and thus had to rely on more elaborate instrumentation. Much the same can be said about Alhacen's apparatus for testing refraction. It, too, has certain affinities in both design and application with the apparatus Ptolemy constructed in book 5 of his *Optics* for the same purpose, and it too has marked differences in design that are due to Alhacen's focus on light rays rather than visual rays. For further elaboration on these points, see pp. xlvi-xlix of the Introduction.

[12]Thus, as represented in figure 7.2.6, p. 150, the quadrant in the rim bounded by AO and BP is cut out. The reason for excising this section is to open a direct view through it to the inner wall of the rim on the other side. It bears noting that, according to the measure of 50 cm for its diameter, the middle circle on the inner wall of the apparatus's rim has a circumference of almost exactly 157 cm, so its division into 360° means that each degree subtends an arc of just under .44

cm. This is remarkably close to the length of half a grain of barley based on the value of .9 cm for the length of a full grain of barley tentatively offered in note 8, p. 334 above. Furthermore, as we saw earlier, Alhacen located point F one digit to the side of point A, which marks one limit of the excised quadrant. If in fact Alhacen's digit equals precisely two grains of barley, and if he meant for each degree to subtend exactly half a grain of barley on the rim, it would follow that he intended point F to be located precisly four degrees to the side of point A. That being the case, it would have made sense for him to finish the register plate and its rim separately before attaching them to one another. Accordingly, he could have started by forming the register plate with a circumference of precisely 180 grains of barley and then cut the rim from a flat sheet of bronze to form a strip of that length with a width of two digits. Better yet, he could have cut it to a length of 135 grains of barley, which is three-quarters of the register plate's circumference. Then he could have scribed three lines on its face parallel to its lengthwise bottom edge, the first one lying half a grain of barley above that edge, and the other two at intervals of half a grain of barley above one another. Next he could have subdivided the middle of the three lines into 270 units, each one half a grain of barley long, drawn a line crosswise through the point marking the fourth unit from the leftward edge of the strip, drilled a hole one grain of barley in diameter with its center at that point, and drawn another line crosswise through the point lying 180 units to the right of that point. With all the points and lines preset on the strip, Alhacen could have then attached it upright to the register plate with its left-hand edge poised at point A, the line through the center of the hole posed at F, the line 180 units to its right posed at G, and the right-hand edge posed at B. That way, the open quadrant would already have been in place, and point F would lie precisely four degrees to the right of point A.

[13]As illustrated in figure 7.2.7, p. 150, we open this phase of the construction by selecting the bronze bar to the left. It should be square in cross-section, each side being two digits across, and it should be longer than the diameter of the register plate (i.e., longer than 50 cm). In this case, it is represented at around 60 cm in length according to the scale at which the register plate is represented. A hole should be drilled orthogonally through the center of one of its faces so as to pass through the opposite face. This hole should be barely large enough to accommodate the cylindrical axle attached to the back of the register plate. The bar should then be attached to the back of the apparatus by slipping it onto the axle, as represented at the right of figure 7.2.7. When the bar is pushed down against the back surface of the register plate, the tip of the axle will protrude one digit beyond the top surface of the bar, since the bar is two digits thick and the axle three digits long. Once properly attached, the bar will protrude beyond the outer rim of the apparatus by some amount on each side, 5 cm in this case. This excess amount is to be cut off, leaving main segment A of the bar flush at both ends with the apparatus's rim. The remaining pieces B and C cut from each side are then attached to the central segment so that they overlap it by a digit. In order to further secure the attachment of these excess pieces, Alhacen suggests drilling a hole through the overlapping portion into the central segment and fastening the two with a copper pin.

Taken directly from folio 89v of manuscript *O*, figure 7.2.7a, p. 151, shows the various elements of the apparatus discussed to this point. The large figure on the upper left represents the register plate and the rim. The middle circle is the face of the register plate divided into quadrants by diameters AB and EH, with the sighting diameter RF oblique to them according to the placement of R one digit to the side of A. RL is the perpendicular drawn on the inner wall of the rim, FG its counterpart on the opposite side of the rim. The successive circles concentric with circumference REBH of the register plate represent the three circles scribed on the rim at successive distances of half a grain of barley, and the circle centered on K represents the hole in the rim, which is one grain of barley in diameter. The circle passing through K is the middle circle, which is shown roughly subdivided according to degrees. The line with the numbers 5 and 14 at its endpoints is presumably the line on FG along which the panel with the second hole is to be applied, the panel itself being represented in the lower left. Finally, the holding strip with the hole to accommodate the cylinder on the back of the register plate and the two overhanging end strips is represented to the lower right.

[14]The ruler just described is formed from a thin strip of copper at least half a cubit long, although for the sake of functional convenience it could be somewhat longer. Thus, as represented in figure 7.2.8, p. 152, each of the edges AC, BD, EH, and FK along the length measures no less than half a cubit. Each of the edges AH, BK, CE, and DF along the width measures two grains of barley, and edges AB, CD, EF, and HK are one grain of barley high. The ruler is cut along line AG so as to form acute angle CAG, edges AB, GL, CD, and EF all being parallel. Finally, a line is drawn parallel to edges AC and EG on the top face so as to bisect it along its length. Accordingly, when the ruler is laid upon the register plate along its wider face BDFL, it will stand one grain of barley high, which means that its top face ACEG will lie in the plane of the middle circle scribed on the inner wall of the rim. On the other hand, if the ruler is stood up edgewise on its narrower face ABDC, the line passing through the middle of the top face will lie in that plane, since it too lies a grain of barley above edge AC.

[15]The vessel Alhacen has in mind here is illustrated in figure 7.2.9, p. 152. According to his instructions, the top lip should be smoothed so as to form a plane, and the diameter of the brim at its top must be longer than the diameter of the apparatus so that the apparatus will fit into it. It must also be deeper than the radius of the apparatus so that, when the vessel is filled with water and the apparatus is inserted into it orthogonally (i.e., with the register plate perpendicular to the water's surface) and hung from the lip by the bar attached to the axle at the back, the water can reach somewhat above the center of the register plate. The test with the object placed at the bottom of the water-filled vessel presupposes something that has yet to be empirically demonstrated: namely, that when it strikes a refractive interface (in this case the interface between water and air) orthogonally, light will pass straight through that interface unrefracted. Here the light passes from the object in the water into the air, and since it passes straight through the interface, we can conclude by extension that light passing through that interface in the opposite direction will also pass straight through. As we will see shortly, confirming this supposition with Alhacen's experimental apparatus is

problematic. The primary purpose of the test suggested here is to establish that, when the object at B is looked at along perpendicular line of sight AB in figure 7.2.9, it will appear undistorted by refraction, provided that it is small; otherwise the rays from its outer edges will be refracted enough to cause noticeable distortion. Being appropriately small, the object will therefore appear more or less precisely as it would appear if the vessel were empty, all other things being equal. On the basis of this simple test, we are henceforth advised to make all of our experimental observations from directly above so that what we see is undistorted by refraction. The secondary purpose of this test is to confirm, albeit indirectly, that the rays of luminous color emitted by the object pass straight to our center of sight through the interface between water and air, just as they would if that interface were not there.

[16]The reason Alhacen is so solicitous about having the lip of the vessel perfectly horizontal and, therefore, having its plane perfectly parallel to the surface of the water is to ensure that when the experimental apparatus is hung in the vessel, the top surface of the register plate will be perfectly perpendicular to the water's surface. As will become clear in short order, the surface of the register plate will be parallel to the plane of refraction when sunlight is passed into the water through the hole in the apparatus's rim, and that plane must be perpendicular to the water's surface.

[17]As illustrated in the top diagram of figure 7.2.10, p. 153, the apparatus is inserted into the vessel so that it hangs on the vessel's lip by the small pieces attached to the central segment of the bronze bar attached to the back of the apparatus on its axle. Accordingly, centerpoint C of the apparatus lies one digit below the vessel's rim, since the bar attached to its axle at C is two digits thick. Water is then poured into the vessel until it reaches the level of centerpoint C of the apparatus, which leaves the waterline precisely one digit below the vessel's brim. When brought into sunlight, the apparatus is adjusted until the rim in the quadrant between A and the brim of the vessel to its right shades the water and the opposite wall of the rim below water. The experimenter then rotates the apparatus on its axle until a beam of sunlight passes through the hole in the rim at F and continues to the hole in the panel at R, passing thence to the water. The sunlight passing by edge A of the rim will be curtailed along line AL so that what lies to the right of AL will lie in shadow. In order to certify that the light is passing through the holes properly, the experimenter is to take the copper ruler EK in his free hand (i.e., the one not used for rotating the apparatus) and place it edgewise on the surface of the register plate beyond the hole at R to ensure that the light cast on the ruler's wider face is "even." By this I assume Alhacen means that the resulting light should form a circle on the ruler's face, in which case the experimenter must hold the ruler perpendicular to diameter FCG. When the apparatus is properly set up and the ruler removed, the experimenter will observe that the light not only reaches the water's surface but is also cast on the rim below the waterline to form a circle centered on point D, and he will be aided in this observation by the fact that the light cast on both the water's surface and the rim below lies in the shaded area of the apparatus.

Although Alhacen could have channeled the light through a tube extending from F to R, such an expedient would have been impractical for at least two interrelated reasons. For a start, given the tube's considerable length (c. 12.5 cm), it would have to be perfectly aligned with the sun so that the sun's axial ray would coincide with the axis of the tube. The minutest deviation from this alignment would cause the entering sunlight to reflect off the walls of the tube, dampening its light to the point that very little, if any, would emerge from the tube. Moreover, the entire apparatus, including the vessel and its content of water (which alone weighs at least 50 kg) is awkward to manipulate. The slightest adjustment of the inserted apparatus either around the brim of the vessel or by rotation around its own axle will disturb the water, but in order for the experiment to work properly, the water must be perfectly still. Consequently, each adjustment made for the purpose of properly orienting the holes toward the sun requires time to make the adjustment itself and to allow the water to calm, and during that time the sun will have shifted its position in the sky by some amount. The virtue of Alhacen's two-hole arrangement, therefore, is that it allows some leeway in aligning the holes with the sun so that it need not be perfect in order for the experiment to work. That, in turn, allows time for the apparatus to be adjusted and the water to calm.

[18]This observation simply confirms that the beams of incident and refracted light lie in the same plane, which follows from the fact that the circle of light on the rim, which is centered on point D in the upper diagram of figure 7.2.10, p. 153, is more or less bounded at the top and bottom by the lowest and highest of the three circles scribed on the rim's inner wall. From this it follows that the axial ray parallel to line FC on the register plate and the axial ray following line of refraction CD both lie in the plane of the middle circle on the inner wall of the rim. When dealing with the reflection experiment in book 4, Alhacen observes that light has a tendency to disperse in the form of a narrow cone after being channeled through a hole (see Smith, *Alhacen on the Principles*, pp. 309-310), and he takes this phenomenon into account here as well by pointing out that the circle of light cast on the apparatus's rim may exceed the limits of the lowest and highest of the three circles by some moderate amount but that it will do so symmetrically.

[19]The Latin term *acus* occurring at this point in the text literally means "needle" or "pin." I have chosen to render it henceforth as "stylus" on the assumption that *acus* refers to the stylus's needle-like shape rather than to an actual needle apart from the stylus itself.

[20]The Latin text implies that the stylus is to be put back up to the hole in the rim, but later repetitions of this technique under different conditions indicate that this second test should be made at the hole in the panel. Otherwise, this second test would be needlessly repetitive.

[21]This experiment with the stylus is meant to demonstrate that, no matter where the stylus is posed at the hole in the rim or the hole in the panel, its shadow will appear at precisely the same place in the circle of light cast on the water's surface or on the inner wall of the rim below water. Consequently, whether the stylus is placed with its point at the center of the hole or across the hole, as illustrated in the bottom-left diagram of figure 7.2.10, p. 153, its shadow will appear at precisely the same location in the circle of light cast by the light passing through that hole,

as illustrated in the diagram at the bottom right. This experiment also confirms that the axial ray passing through the center of the hole in the rim passes through the center of the hole in the panel, then through the center of the circle of light cast on the water's surface, and finally to the center of the circle of light cast on the apparatus's rim. Since, moreover, the center of the circle of light cast on the rim falls on the middle circle scribed on the inner wall of the rim, it follows that the incident and refracted light passing along the axial ray lies in the plane of that circle.

[22]I am assuming that *acus longa* at this point in the text refers not to the stylus used in the previous test but to an actual needle longer than that stylus.

[23]In this test, which is illustrated in figure 7.2.11, p. 154, the experimenter is to lay the ruler on the register plate along its wider face and place the pointed edge in the circle of light so that it bisects that circle. With its pointed edge fixed in this position, the ruler is then to be pivoted until the lengthwise edge on its bottom, which lies on the register plate's surface, touches centerpoint C of the register plate. Accordingly, the lengthwise edge on its top will intersect the water's surface directly above centerpoint C at a distance of a grain of barley, which is the thickness of the ruler according to construction and which is also the height of the middle circle scribed on the inner wall of the apparatus's rim. The top surface of the ruler will therefore lie on the midline of the circle of light at the water's surface within the plane of the middle circle. Moreover, since the leading edge of the ruler lies on the centerpoint of the register plate, it too will bisect the circle of light on the water's surface within the plane perpendicular to the plane of the middle circle. As a result, it will block a quarter of the light passing through the water at centerpoint C, and it will cast a shadow within the same quarter of the circle of light that is projected on the rim below water by the light passing through the water's surface. This point is illustrated in the bottom set of diagrams in the figure. With the ruler set up in the way just described, the experimenter is to take the long needle and apply its point to the top edge of the ruler precisely where it meets the water, as in the diagram at the lower left. He will then observe that the point of the resulting shadow lies at the vertex of the shadow cast by the ruler within the circle of light on the rim, as represented at the lower right. Having made that determination, he is to place the needle at various points on that same edge below the water, and he will observe that the point on its shadow continues to lie at the vertex of the ruler's shadow. On this basis he will confirm that the axial light refracted by the water continues all the way through the water in a straight line along that edge. He will also confirm that after refraction the axial ray lies in the same plane as the axial ray before refraction—i.e., the plane of the middle circle inscribed on the inner wall of the apparatus's rim.

[24]Let the arc in figure 7.2.12, p. 155, represent the arc of the middle circle scribed on the apparatus's rim in the quadrant directly opposite the one that contains the hole. Let F'C' be the axial ray of light within the plane of this circle. As already confirmed, this ray follows a path parallel to the diameter of the register plate and thus passes through the centers of the two holes to the center of the circle of light on the water's surface. If it were to continue in a straight line after reaching that surface, then it would eventually reach point G', where the middle circle intersects

the line drawn perpendicular to the endpoint of the diameter on the inner wall of the rim. Instead of continuing straight, however, axial ray F′C′ is refracted at point C′ and subsequently reaches point D. Accordingly, when the experimenter looks directly down at point D along line of sight AD, he will observe a noticeable discrepancy between points G′ and D, and when he poses the ruler perpendicular to the surface of the water with its leading edge EK passing through centerpoint C′ of the middle circle, he will notice that D lies between G′ and endpoint K of normal EK. From this it follows that the axial ray has refracted toward the normal dropped through point of refraction C′.

[25]Here we see why confirming that light striking the air-water interface orthogonally passes straight through it is problematic (see note 15, p. 338 above). In order for that confirmation to be made with the apparatus Alhacen describes, the beam of sunlight must pass through the holes to the water along the orthogonal, and that can happen only if the sun lies at the experimenter's zenith (i.e., directly over his head). The farthest points north or south of the equator at which this happens lie on the tropics of Cancer and Capricorn, roughly 23.5° north or south of the equator, so the test must be conducted on a latitude at or between the tropics. Alhacen, for instance, could not have conducted the test in Cairo, where he presumably settled in the 1020s, because Cairo lies slightly more than 30° north of the equator. At the Tropic of Cancer the sun reaches its zenith during summer solstice only, and at the Tropic of Capricorn during winter solstice only, so at these latitudes the test can be conducted once in any given year. At the equator, on the other hand, the sun reaches its zenith twice, at the autumnal and vernal equinoxes, so the test can be conducted twice in any given year. Likewise, at every latitude between the equator and the tropics the sun reaches its zenith twice, but it does so at symmetrical points between solstice and the two equinoxes, depending on how far north or south of the equator it lies.

[26]Presumably this should be done with some sort of gum rather than with a glue that hardens.

[27]The arrangement of the apparatus described to this point is as follows, according to figure 7.2.13, p. 155, which provides both a face-on and three-quarter view. Several glass cubes are formed with each edge two grains of barley in length, and they are carefully ground and polished so that all of the angles at their vertices are precisely 90°. A line is scribed through centerpoint C on the face of the register plate perpendicular to diameter FC, and the first cube is applied to this line with the bottom edge of its front face flush with the line and the midpoint of that edge on point C. Once it is properly in place, it is attached firmly to the surface of the register plate with gum or some other temporary adhesive that will allow it to be conveniently removed. The second glass cube is then applied face-to-face with the first one and attached in place to the register plate the same way, and so on *seriatim* with the remaining glass cubes until the succession of cubes reaches as closely as possible to the panel with its hole at R. Ideally, the succession of glass cubes will form a long, perfectly rectangular glass rod extending from C to R, or thereabouts. Suffice to say, the formation of such a rod requires that the glass cubes be perfectly aligned, which in turn requires that the six faces of every glass cube intersect one

another at precisely 90°. Any deviation will cause a skew in the alignment, and that skew will be multiplied by the number of glass cubes composing the rod.

When the glass cubes are in proper position, the copper ruler is applied to the register plate below point C on its narrower side so that its wider side with the midline scribed through it faces the first glass cube. It is then brought up to a point near C but far enough away that the experimenter can see the portion of its face opposite the face of the first glass cube when he looks straight down at it. Once there, it is adjusted until it is perpendicular to diameter FC on the register plate and, therefore, parallel to the glass cube's face. It is then attached in that position with a temporary adhesive. With everything properly positioned in this way, the apparatus is inserted back into the vessel, which is empty of water, and the apparatus is adjusted so that a beam of sunlight passes through the two holes into the glass and thence to the ruler. The experimenter will then observe that this beam of light creates a circle of light on the face of the ruler and that its midpoint lies on the midline passing through the ruler's face. From this he can infer that the beam has passed straight through the glass. Moreover, since the light may have spread out during its passage through the glass, Alhacen assures us that all the light projected onto the ruler's face will have passed through the glass because the edges of the cubes along both height and width extend beyond the circumference of the holes by no less than half a grain of barley on all sides, which is more than enough to accommodate the spreading light.

[28]Here we seem to have reverted back to the pointed wooden stylus in place of the long needle mandated in the previous experiment carried out under water.

[29]It is not clear from the text which of the glass cubes constitutes the "first," "second," and so forth in this phase of the experiment. In the initial construction, the "first" glass cube was the one attached at centerpoint C on the register plate. If, therefore, Alhacen had the same designation in mind here, the first cube to be removed would be that one, and so on down the line toward R. On the other hand, it is equally likely that the first cube to be removed in this experiment is the one closest to R and so on down the line toward C. Whichever the case, the results will be the same. No matter how many of the glass cubes are removed, the beam of light passing through the remaining cubes (or cube) will form the same circle of light on the ruler's face. Presumably, this experiment is meant to demonstrate that, when it extends through all, some, or only one of the cubes, the beam of light follows a perfectly straight line rather than being shunted from cube to cube in zig-zag fashion according to refraction at the interfaces between cubes.

[30]The section of a sphere that has been excised from the hemisphere is actually somewhat larger than a quarter-sphere, since one of its faces consists of a semicircle plus the excess due to the grain of barley added to its radius. For the sake of convenience, however, I will henceforth refer to this spherical section as a quarter-sphere.

[31]Let the circle to the left of figure 7.2.14, p. 156, represent the base of the glass hemisphere, let AB and HK be two diameters on that base intersecting each other orthogonally at centerpoint D, and let those diameters be somewhat less than half a cubit in length, which leaves the radius of the base less than half the radius of the diameter on the register plate. Measure off distance DC′ equal to one grain

of barley, and pass line LM through point C′ parallel to diameter HDK. Point C′ will thus be the midpoint of line LM, and arc LAM will be slightly more than a semicircle. Make a perpendicular cut through the base circle along line LC′M so as to excise the portion of the hemisphere based on arc LBM, which is slightly less than a semicircle. What will remain amounts to slightly more than a quarter-sphere, whose radius AD is somewhat less than half a cubit. As represented in the diagram to the right of figure 7.2.14, this quarter-sphere stands on its semicircular base with the face circumscribed by arc LAM perpendicular to it. That face contains centerpoint D of the sphere encompassing the excised quarter-sphere. Line HK, which is parallel to common section LC′M of the two plane faces and which passes through centerpoint D of the sphere, lies one grain of barley above the quarter-sphere's semicircular base. So disposed, the quarter-sphere is applied to the register plate so that point C′ on it coincides with centerpoint C of the register plate and so that common section LC′M is perpendicular to diameter FCG of the register plate. The resulting arrangement is illustrated in figure 7.2.15, p. 156, which provides a bird's-eye view. According to this disposition, then, line HDK on the quarter-sphere's face lies in the plane of the middle circle scribed on the inner wall of the rim, since both lie one grain of barley above the surface of the register plate. Therefore, point D on that line, which lies directly above centerpoint C of the register plate, lies on the straight line passing through the centers of the two holes parallel to diameter FC.

[32]When the ruler EK is attached to the register plate as illustrated in figure 7.2.16, p. 157, the apparatus is reinserted into the empty vessel so as to hang by the excess pieces attached to the bar that holds the register plate by its axle. Thus, when the apparatus is adjusted to face the sun so that a beam of light shines through the two holes to the convex surface of the quarter-sphere and thence through it to the ruler, the experimenter is ready to conduct the standard tests that are described in the next few paragraphs.

[33]In other words, the experimenter is to put the stylus's point at point D on the upright face of the quarter-sphere represented in the diagram at the upper right of figure 7.2.14, p. 156.

[34]Thus, with the apparatus already inserted into the empty vessel according to the disposition in figure 7.2.16, p. 157, the experimenter is to remove the ruler and fill the vessel with water, bringing the water's surface up beyond centerpoint C, as represented in figure 7.2.17, p. 157. Under these conditions, the light beam will pass through the water to the rim on the opposite side of F, and it will cast a circle of light on the perpendicular erected from endpoint G of diameter FCG. This, of course, shows that the light is unrefracted because it passes through the interface between water and glass along the perpendicular. For all practical purposes, this experiment and the ones following could have been conducted with a glass semicylinder, whose formation would have been considerably simpler than that of the glass quarter-sphere. Why Alhacen chose this latter, more complicated expedient is probably explained by his desire to make the physical conditions of his experiments as mathematically precise as possible, even when such precision yields imperceptible results. Accordingly, if he used a glass semicylinder, the narrow beam of light passing out of its convex face

would be somewhat distended along the vertical, whereas no such distension will occur when the light passes through a spherical section. It is worth noting, however, that later in chapter 3, when he describes an experiment to determine angles of refraction when light passes through a concave rather than a convex glass surface, Alhacen uses (or at least recommends the use of) a semicylindrical section; see paragraphs 3.29-31, pp. 257-258.

[35]In this experiment the position of the glass quarter-sphere is reversed so that its flat face is pointed toward the holes and its convexity faces point G on the lower rim of the apparatus, as illustrated in figures 7.2.18 and 7.2.19, p. 158. If the vessel is empty of water, as in figure 7.2.18, then the light will pass straight through to the perpendicular erected on the inner wall of the rim from endpoint G of diameter FCG, its axial ray thus forming a diameter of the middle circle parallel to diameter FCG on the register plate. If the vessel is filled with water to a sufficient level below centerpoint C of the register plate, as in figure 7.2.19, then the light will also pass in a straight line to the perpendicular erected on the inner wall of the rim from endpoint G. In both cases, the light will not refract at the flat face erected at centerpoint C of the register plate because it will strike that face along the perpendicular. Likewise, when it strikes the glass at its convex surface it will pass straight through because it travels through the glass along a radius, and all such radii are perpendicular to the quarter-sphere's convex surface.

[36]Up to this point in book 7 Alhacen has tied refraction to the relative transparency of air, water, and glass—air being more transparent than water and water more transparent than glass. Here for the first time he adverts to the physical cause of transparency—and therefore of refraction—in terms of how dense, compact, or crass (Lat., *grossus*) or how rare (*subtilis*) a body's material substrate is; the denser that substrate, the less transparent the body will be, and the less transparent the body, the more refractive it will be. Toward the very end of this chapter, Alhacen will use this correlation between material structure and transparency to ground his physical explanation of refraction, which is discussed in detail in note 50, pp. 348-351.

[37]At this point Alhacen has established that the refraction occurs at a particular surface in its transit through the air and glass, but he has also established that it occurs *only* at that surface, and at a particular point on it. Nowhere else in its transit does it suffer such deflection. This point is crucial, because it means that, as far as Alhacen is concerned, the process of refraction is virtually, if not literally, instantaneous insofar as it has both its beginning and completion at a particular spot on the interface between the two media.

[38]In this experiment the glass quarter-sphere is to be rotated counterclockwise on its semicircular base while the midpoint on its bottom edge remains on centerpoint C of the register plate, as illustrated in figure 7.2.20, p. 159. The upright face will therefore be aslant to both diameter FCG and the line drawn perpendicular to that diameter through C on the register plate. Being oblique to that face, the beam of light passing through the holes at F in the rim and R in the panel will be refracted at centerpoint D on the flat face of the quarter-sphere directly above centerpoint C of the register plate. The resulting beam will pass straight through the glass to its convex surface and, reaching that surface orthogonally, will continue unrefracted

to the wall of the rim below. The experimenter will then notice that the circle cast there does not lie in line with diameter FCG but inclines to some point K between point G and point L on normal ACL, dropped through the point of refraction on the upright face of the quarter-sphere. From this the experimenter will be drawn to conclude that the light was refracted at that face and nowhere else during its passage from F to K. He will also conclude that refraction toward the normal is characteristic of the passage of light from a rarer to a denser medium.

[39]This experiment is illustrated in figure 7.2.21, p. 159. As in the previous experiment, so in this one, the refraction occurs at centerpoint D on the flat face of the glass quarter-sphere directly above centerpoint C of the register plate. In this case, however, the light passes from glass into air—i.e., from a denser to a rarer medium—so the refracted ray CK inclines away from normal ACL on the other side of the perpendicular drawn from endpoint G of diameter FCG on the wall of the rim.

[40]For this experiment the glass quarter-sphere is disposed as illustrated in figure 7.2.22, p. 160. Again, the light will be refracted at centerpoint D on the flat face of the glass quarter-sphere directly above centerpoint C of the register plate, and it will be refracted to some point K on the inner wall of the rim beyond G, thus inclining away from normal ACL. However, since water, though rarer than glass, is still denser than air, the light will not be refracted as severely as it was in the passsage from glass to air.

[41]The setup for this experiment is illustrated in figure 7.2.23, p. 160, where the straight edge of the quarter-sphere is placed on the line drawn perpendicular to diameter FCG at point C but in such a way that centerpoint D on its flat face lies to the side of C. Accordingly, the beam of light passing through the holes at F and R will reach the convex surface of the quarter-sphere at some point through which normal AD is drawn. Clearly, then, the beam of light strikes that surface obliquely, since it makes an angle with that normal, so it follows that it will be refracted at that surface. Moreover, since glass is denser than the air through which the light originally passed to reach the quarter-sphere, it will be refracted toward the normal and, therefore, toward centerpoint D of the sphere out of which the quarter-sphere is formed. In order to observe this clearly, the experimenter is advised to attach the copper ruler EK to the register plate just beyond point C and observe the circle of light cast on it. He will then see that this circle lies between diameter FC and normal AD. He will also see that its midpoint lies on the midline drawn through the ruler's face. As Alhacen points out, the light will refract away from normal SP dropped through the flat surface of the quarter-sphere at refraction-point P when it passes into the air between that surface and the face of the ruler, but since the ruler lies so close to the glass, this refraction will not affect the basic observation that the light inclines toward normal AD as it passes through the glass.

[42]Thus, as illustrated in figure 7.2.24, p. 161, the light beam passing through the two holes meets the flat face of the glass quarter-sphere at a point directly above centerpoint C of the register plate and passes through that face orthogonally, but its entry point lies beyond centerpoint D of the sphere encompassing the quarter-sphere. Subsequently, when it meets the convex surface, it does so obliquely, so it will be refracted as it passes through that surface into the air below it, and it

will be refracted away from normal AD so as to form a circle of light at K on the apparatus's rim, which lies on the other side of point G from point A.

[43]This experiment is illustrated in figure 7.2.25, p. 161. In qualitative terms, the very same thing happens in this experiment as in the previous one testing the passage of light from glass into air, except that, because it passes into water, which is denser than air, the light is not as intensely refracted.

[44]As discussed in the previous edition of books 1-3, essential light (*lux*) for Alhacen is the light inherent in self-luminous objects, whereas accidental light (*lumen*) is the physical effect of essential light in transparent or opaque objects, where it is manifested as radiation or illumination (see, e.g., Smith, *Alhacen's Theory*, p. 395, n. 2).

[45]According to this distinction between sunlight and daylight, the first is essential and the second accidental. Strictly speaking, though, as tested in the apparatus, sunlight is accidental, not essential, insofar as it is the immediate physical effect of the sun's inherent light on the transparent media through which it passes as well as on various physical surfaces it illuminates. Daylight, for its part, is accidental to sunlight insofar as it is a mediate physical effect of sunlight's having illuminated various opaque bodies, such as particles in the surrounding air or neighboring walls. Hence, sunlight is taken here as "essential" in terms of its immediacy, and daylight is taken as "accidental" in terms of its mediacy.

[46]Book 2, chapter 3, paragraphs 3.60-62, in Smith, *Alhacen's Theory*, pp. 445-447; see also book 4, chapter 3, paragraph 3.99, in Smith, *Alhacen on the Principles*, p. 321.

[47]Tortured though this passage may be in both its Latin and English incarnations, the sense of the passage is clear enough: by virtue of being physical, all physical bodies are necessarily material. Consequently, however rare we may imagine their material substrate to be, bodies must have such a substrate if they are to remain physical. Since all matter has some density or compactness, and since physical bodies impede the passage of light in proportion to their density or compactness, the rarest imaginable physical bodies will still impede that passage to some extent, minimal though it may be.

[48]The distinction here between essential and accidental motion parallels the distinction Alhacen draws earlier in book 4 between natural and accidental motion (see Smith, *Alhacen on the Principles*, pp. 321-322). Accordingly, natural (or essential) motion is intrinsic to a given physical object by virtue of its heaviness or lightness, the free fall of a heavy body toward its natural place at the center of the universe being the paradigmatic case. Accidental motion, on the other hand, is due to an extrinsic cause, which compels the object to move unnaturally or violently, a case in point being the projection of a heavy body straight upward or horizontally. In drawing the analogy between light and a projectile, Alhacen is reducing light to quanta and the light ray to a trajectory followed by such quanta. These quanta presumably consist of *lux minima*, or "least light," which is a concept Alhacen introduces in book 4, chapter 3, paragraph 3.97 (in Smith, *Alhacen on the Principles*, p. 320). According to that account, however, *lux minima* is not atomic. It is the smallest amount of *effective* light and, as such, is contingent on both the intensity of the luminous source and the capacity of whatever receives its light to be affected by it. All other things being equal, then, the more intense the source—for instance,

the sun as opposed to the moon—the smaller the spot of *lux minima* will be on it. The illuminating effect of this spot, however, will diminish according to distance: the farther an opaque body lies from this spot, the weaker its effect, until finally it fails to illuminate the body entirely. On the other hand, at that same distance a larger spot of *lux minima* will illuminate the same body. Furthermore, the measure of the effectiveness of *lux minima* seems to lie in its visibility, which means that the size of a spot of *lux minima* on any given luminous source, whether self-luminous or illuminated, depends upon how close a viewer with a healthy optic system is to that source.

[49]The Latin verb translated here as "will be reflected back" is *reflectetur*, the future passive of *reflectere*. To this point in book 7, *reflectere* has been consistently used to mean "to refract," but in this instance, reflection, not refraction is clearly intended (see note 1, p. 333). What Alhacen is getting at in this passage is that, when they meet such stiff resistance that they cannot penetrate a given surface, moving physical bodies (*mota naturalia*), light included, have a tendency to bounce back up either directly along the path of incidence, if they strike orthogonally, or at an equal adjacent angle, if they strike obliquely.

[50]Alhacen has already had recourse to the analogy between light and a tiny, swiftly moving ball in order to explain why light reflects at equal angles. That explanation, which occurs in the third chapter of book 4 (Smith, *Alhacen on the Principles*, pp. 322-323), is based on assuming that the surfaces of reflective bodies are perfectly, or almost perfectly, compact—i.e., that they have few or no pores into which the light can be diverted and trapped—and perfectly, or almost perfectly smooth (see ibid., p. 300). Unable to penetrate such surfaces, light is forced to rebound from them because of the overwhelming upward resistance they pose. The limiting case for such rebound is that in which light strikes a reflective surface orthogonally so as to bounce straight back up along its original path, leaving the angle of reflection equal to the angle of incidence, both being 90°. In this case, of course, the resistance posed by the reflective surface is equal and opposite to the force exerted by the incoming light, which is to say that it is exerted along the vertical and only along the vertical. By the same token, when light strikes such a surface obliquely, it is resisted along the vertical in proportion to its downward force, so its rebound will be proportional to the vertical resistance of the reflecting surface. Since, however, the light meets no resistance along the horizontal, its motion in that direction is unchanged, so it will rebound symmetrically from the reflective surface. This analysis can be understood graphically by recourse to figure 7.2.26, p. 162. Let AR in both the top and bottom diagrams represent equal trajectories along which a "ball" of light reaches reflective surface DRC in some given time according to its velocity, and let the force of its motion over that time be proportional to that velocity and thus to the length of the trajectory (i.e., distance/time). If the trajectory is perpendicular, as in the top diagram, and if the reflective body were not there to impede it, the light would continue straight on at the same velocity and force, so in the same amount of time it took to traverse AR it would traverse equal trajectory RB′. When it encounters reflective surface DRC, however, it is fully resisted by force B′R equal and opposite to its incident force AR, so it will bounce straight back up along AR in the same amount of time it took

to pass down it. Likewise, if AR is oblique, as in the bottom diagram, the light will arrive at R with the same velocity and force it had in the previous instance, and if the reflective surface were not there to impede it, it would continue straight to B′ along RB′, which it would traverse in the same amount of time it traversed AR. When it encounters reflective surface DRC, though, it will meet resistance, but only along the vertical. Hence, if we break the light's overall motion along AR into vertical component AC and horizontal component AN, the incoming light will exert downward force along component AC and will be resisted by an equal and opposite force EC that will drive it back upward at the same velocity and force. Thus, the motion downward along AC will be matched by motion upward along DB. Meantime, having encountered no resistance along the horizontal, the light will maintain its motion along AN, continuing to B as it bounces back upward along DB. The equality of the angles of incidence ARN and reflection NRB follows automatically from the model.

As illustrated in figure 7.2.27, p. 163, Alhacen's analysis of refraction is based on the same model, except that the interface DRC between the rarer medium above and the denser, less transparent medium below is permeable. Thus, when the light strikes that interface orthogonally, as in the top diagram, it will lose some of its velocity and force when it encounters the denser medium according to Alhacen's supposition that light moves more quickly in rarer media. This loss of velocity is due to the resistance posed by the medium, which in this case is limited to the vertical resistance. Let CE represent the force of that resistance, just as AR represents the force of impact. Accordingly, when it enters the denser medium, the light loses velocity and force in proportion to the medium's resistance along CE, the result being that in the same amount of time it took for it to traverse AR it will traverse a commensurately shorter trajectory RB in the medium. Now let the light strike the denser medium obliquely, as represented in the bottom diagram. In that case, it will be resisted along both the vertical and horizontal, the vertical resistance CE being proportional to the downward force it exerts along AC and the horizontal resistance FN being proportional to the horizontal force it exerts along AN. Therefore, if it were to continue in a straight line toward B′, it would lose velocity and force according to both resistances, so the resulting trajectory RB′ would be commensurately shorter than the trajectory resulting from an orthogonal impingement. In other words, the velocity and force of the light passing through the denser medium would be dependent on the relative obliquity of its path through that medium once it has entered it; the more oblique that path, the more slowly and less forcefully it will move along it. But why should its motion in the medium be affected by the direction it takes in passing through it *after* entry and the initial impedance it suffers at interface DRC? In response to this issue Alhacen adduces what amounts to a principle of conservation of composite motion, by which he presumably means that the overall motion of the light in the medium must remain the same, regardless of its direction. In order that this composite motion be conserved, though, the light must be shunted toward an easier passage, and such a passage necessarily inclines toward the normal. Therefore, after being refracted at point R in the lower diagram of figure 7.2.27, the light is forced by the medium's horizontal resistance along FN to follow a particular path RB that

allows it to conserve its composite motion insofar as possible, according to the "natural" loss of velocity and force it suffers from the medium's vertical resistance. Theoretically, then, the light will travel along RB in the lower diagram of figure 7.2.27 at the same velocity as it travels along normal RB in the upper diagram, and it will follow that path because it represents the path of least resistance according to the conservation of composite motion.

As sophisticated as it is, this vectorial analysis and dynamic explanation of reflection and refraction is fraught with difficulties at a variety of levels. For one thing, tying relative transparency to the density or compactness of the medium is problematic, as is tying refractivity to relative transparency. After all, water is denser than most kinds of wood, yet wood is clearly less transparent than water. For another thing, certain transparent media are more refractive yet less dense than others, oil, for instance, being both less dense and more refractive than water. It is only fair to note, however, that the three media Alhacen investigates empirically, i.e., air, water, and glass, fit the correlation between density and refractivity. Nonetheless, one can easily imagine glass that is so exquisitely clear and air that is so badly adulterated that the glass is more transparent than the air while still being more dense. Furthermore, Alhacen's definition of reflectivity in terms of the smoothness, compactness, and lack of porosity of the reflecting surface is problematic, because in those terms such a surface is as perfectly opaque and impermeable as possible. Yet polished glass, while eminently transparent, is also eminently reflective, two qualities that should be mutually exclusive if Alhacen's conception of transparency and its physical grounds are to be taken seriously. In addition, glass is roughly 2.5 times as dense as water, so it should be roughly 2.5 times as refractive, a conclusion that Alhacen must have disproved in his quantitative experiments with refraction in the next chapter. Here we seem to be caught in a dilemma: either reflectivity and transparency are due to the inherently dense nature of the *matter* composing the medium and defining its surface, or else it is due to the physical *structure* of that matter according to its porosity or the looseness of its constituent particles. Perhaps it is a combination of both. Whatever the case, Alhacen neither acknowledges nor attempts to resolve this dilemma.

Problematic as well is Alhacen's principle of conservation because it depends on having the vertical and horizontal resistances posed by the denser medium act in fundamentally different ways. This point can be easily understood if we imagine the diagram at the bottom of figure 7.2.27, p. 163, rotated 90° counterclockwise so that interface DRC between the rarer and denser media stands vertical, as represented in figure 7.2.28, p. 164. Let us further imagine that the denser medium extends beyond point R toward point N to form interface RN between the two media. Accordingly, when the light traveling along AR strikes that interface at R, it will be resisted along the vertical by force FN and along the horizontal by force EC, force FN slowing it down and force EC shunting it toward the normal so as to follow refracted path RB″. In the previous analysis, though, force EC did the slowing and FN the shunting. Furthermore, if we imagine the denser medium above and to the right of interface NR evacuated to leave nothing but the rarer medium, the dynamic situation will be unchanged because the impedance occurs at point R,

which is shared by both interfaces DRC and NR. Yet the resulting paths RB and RB″ are radically different in direction and supposed ease of passage. How, then, do the two resistive forces know which role to play—slowing or shunting—in any given situation, the choice of roles contingent on whether NR or CR is taken as the relevant normal? In addition, if the refraction itself is instantaneous (see note 37, p. 345), then the exertion of those forces must be instantaneous as well, which is to say that they will disappear as soon as they are generated.

It should be clear by now that Alhacen's dynamic explanation of refraction is problematic to the point of incoherence, yet despite its manifold shortcomings, it was accepted without demur by his Latin Perspectivist followers and remained more or less canonical throughout the later Middle Ages and Renaissance. Even Johannes Kepler's trenchant critique of that explanation and its implications in the fourth chapter of his *Ad Vitellionem paralipomena* (see Donahue, *Kepler*, pp. 95-104, esp. 100) failed to discredit it, as is evident from Descartes's having drawn liberally from it in constructing his own model of refraction in the *Dioptrique*. For further discussion of the genesis and fate of Alhacen's refraction model, see A. I. Sabra, *Theories of Light from Descartes to Newton* (1967, rpt; Cambridge: Cambridge University Press, 1981); David C. Lindberg, "The Cause of Refraction in Medieval Optics," *The British Journal for the History of Science*, 4 (1968): 23-38; and A. Mark Smith, *Descartes's Theory of Light and Refraction*, Transactions of the American Philosophical Society, 77.3 (Philadelphia: American Philosophical Society Press, 1987).

[51]Alhacen's causal account of refraction from a denser into a rarer medium and the consequent shunting of light away from the normal appears to be the obverse of his account of refraction from a rarer to a denser medium, insofar as the same vertical and horizontal forces are at play. However, a close look reveals that it is based on inconsistent dynamic principles. Let AR in figure 7.2.29, p. 164, be the path along which light travels in some given time through the denser medium, let it pass orthogonally into the rarer medium at R, and let it continue along RB in that rarer medium in the same amount of time. In that case, when it encounters the rarer, less resistive medium, it will be unaffected by horizontal force FN but impelled by vertical force CE to gain a commensurate amount of velocity and force. Since the light strikes refractive interface DRC orthogonally, the light's motion along RB will be the easiest and most powerful possible according to Alhacen's dynamic model. Furthermore, in the previous analysis we were led to conclude that all composite motion within any given transparent medium will occur at the same velocity, regardless of the direction it takes in that medium. Now let the light follow oblique path A'R. If there were no refractive interface because the denser medium occupied the space above DRC, the light would continue straight along RB″ at the same speed as before, so in the same amount of time it took to traverse A'R it would traverse an equal distance RB″. Because the space above DRC consists of a rarer medium, though, the light will be subject to both vertical impulse EC and horizontal impulse FN, the two conspiring to accelerate it in its passage along RB″. But by how much? There are two possibilities. On the one hand, if orthogonal RB represents the easiest trajectory possible, then the acceleration will not be enough to conserve the composite motion along RB″,

which means that the motion along RB″ will be too slow and, therefore, that RB″ will be shorter than RB. Consequently, in order to recoup the lost motion, the light will have to be shunted along a path between RB″ and RB equal in length to RB in order to find an appropriately easy passage. On the other hand, the light could be accelerated to the point that it either conserves its composite motion or surpasses it. If it conserves that motion, then RB″ will be equal to RB, so there will be no reason for refraction. If it exceeds that motion, then RB″ will be an easier and more forceful path than RB, in which case RB is not optimally efficient in contradiction to Alhacen's model of orthogonal impingement. Why, then, is the light shunted along RB′? If we assume that RB′ is the path along which the light's composite motion is conserved, and if we assume that in this capacity it represents a path of easier motion, then it follows that motion along any trajectory between RB′ and RB must be more difficult, the difficulty increasing as the trajectory approaches orthogonal RB. That being the case, then either RB represents the path of greatest difficulty or there is a quantum leap between it, as the easiest possible trajectory, and its closest neighbor to the right, which is the most difficult possible trajectory. On the other hand, if we assume that the light has been shunted along RB′ because it is a more difficult path, then any trajectory between it and RB must be less difficult. But RB′ = RB by supposition, so all paths between RB′ and RB will be longer and easier than RB. Hence, although logic dictates that refraction from the denser medium to the rarer and from the rarer to the denser be formally (i.e., geometrically) symmetrical, the two are clearly not dynamically symmetrical according to Alhacen's analytic principles.

[52]The Latin text of the subtitle, *De qualitate reflexionis lucis in corporibus diaffonis*, is misleading insofar as what follows is not a qualitative bur rather a quantitative analysis of refraction based on the division of the middle circle on the experimental apparatus's rim into 360 degrees.

[53]Translated here as "world," the Latin term *mundus* applies to both the terrestrial sphere and the celestial sphere with all its contents: in short, the universe. By Aristotelian and Ptolemaic lights both spheres share the same centerpoint, the sublunar realm consisting of four nesting spheres. At the bottom is the sphere of earth clustered around the center of the universe. It is enveloped by a sphere of water, which is enveloped by a sphere of air, which in turn is enveloped by a sphere of fire. Alhacen is thus arguing that, being concentric with the universe, the water's surface in the experiment is perfectly horizontal and therefore somewhat convex rather than truly flat. Since the plane containing the middle circle in the apparatus is orthogonal to that surface, it follows that the normal dropped from the center of that circle to the water's surface will be a segment of the radius not only of the spheres of water and earth but also of the sphere of the universe. Since, moreover, the middle circle defines the plane of refraction, the refraction of light from air to water will necessarily occur in a plane normal to the plane of the water, which is concentric with the universe as a whole.

[54]While Alhacen's effort to relate the geometry of his experimental apparatus to the geometry of the universe may seem excessive, even unnecessary, it bespeaks the penchant for absolute systematic rigor and comprehensiveness that informs his analysis throughout the *De aspectibus*.

[55]The gist of this explanation seems to be that, since they share the same normal and are thus in the same plane with it, the incident and refracted rays must also be in the same plane. This in fact need not be the case because the incident and refracted rays could lie in separate planes intersecting along the normal.

[56]What Alhacen means by the qualification "at different times or in different places" is unclear, but I take him to be drawing a distinction between two separate instances of refraction through the same media, the angle of incidence being different in each instance. The main point of this paragraph is that, if light passes in a given direction but at different angles of incidence from one medium to the other, the smaller the angle of incidence becomes, the greater the relative difference between the angles of incidence and their resulting angles of refraction becomes. Hence, if angle of incidence i yields angle of refraction r in one case, and if angle of incidence i' yields angle of refraction r' in another, and if $i > i'$, it follows that $r':i' < r:i$, which is to say that the relative difference between r and i is less than the relative difference between r' and i'. This generalization is in fact false if the angles of incidence and refraction are both measured with respect to the normal. On that basis, for instance, we know from current theory that $i = 80°$ yields $r = 47.8°$ in refraction from air to water, whereas $i' = 10°$ yields $r' = 7.5°$, so these two cases flout Alhacen's generalization, given that r' (7.5°):i' (10°) > r (47.8°):i (80°), and the same holds for any other set of angles we may choose in the quadrant of incidence. As will become clear in fairly short order, however, Alhacen measures the angle of refraction with respect not to the normal but to the continuation of the line of incidence. Thus, in modern parlance Alhacen's angle of refraction constitutes the angle of deviation, which = $i - r$ (henceforth symbolized by r/d). By Alhacen's account, therefore, when $i = 80°$, $r/d = 32.2°$ (i.e., 80° – 47.6°), and when $i = 10°$, $r/d = 2.5°$ (i.e., 10° – 7.5°), so it does follow that r/d' (2.5°):i' (10°) < r/d (32.2°):i (80°). At the end of the chapter, Alhacen reformulates this rule in reverse order, claiming that $r/d{:}i > r/d'{:}i'$ (see note 69, p. 361).

[57]The construction to this point is illustrated in figure 7.3.30, p. 165, which provides a bird's-eye and three-quarter view of the register plate, its rim with the middle circle inscribed on its inner wall, the hole in the rim, and the panel with the hole in it on diameter FCG. The experimenter is to start by measuring off arc FA of ten degrees to the right of the hole's centerpoint on the middle circle, which is already subdivided into 360°, and then mark point A at the end of the arc. Through that point he is to draw a line on the inner wall of the rim perpendicular to the register plate. The point where that line intersects the middle circle constitutes the "first mark." From the bottom of that perpendicular, he is next to extend diameter ACA" across the register plate. Starting at point A, he is then to measure off arc AA' = 90° and mark the point at A', where the corresponding perpendicular on the rim's inner wall intersects the middle circle. This constitutes the "second mark." Finally, he is to mark point G, which lies at the endpoint of diameter FCG passing through the two holes to the opposite wall of the rim. This constitutes the "third mark." Therefore, arc FA from F to the first mark = 10°; arc AA' from A to the second mark = 90°, which means that CA' is normal to ACA"; arc A'G from A' to the third mark = 80°; and arc GA" = arc FA = 10°. Copied directly from folio 95r of ms *O*, figure 7.3.30a, p. 166, is meant to illustrate this construction. Accordingly,

arc RK from centerpoint R of the hole (to the left rather than the right of it) = 10°, and R is the first mark. From R diameter RVM is projected to the other side of the rim through centerpoint V of the middle circle and is parallel to line LQ passing through the center of the register plate. Q should therefore lie directly below V, although the figure clearly misrepresents this fact. KVZ is the diameter along which the light passes through the two holes to the other side of the rim. Point O lies 90° to the left of R, and from it OV is dropped perpendicular to RM. Thus, R is the first mark, O the second, and Z the third, so arc OZ = 80°.

[58]The insertion of the apparatus into the vessel and the experimental determination of the angle of refraction described to this point is illustrated in figure 7.3.31, p. 167. Once inserted into the vessel so that it hangs from the vessel's lip by the excess pieces at the end of the bronze bar attached to the register plate through the axle at its back, the apparatus is turned on its axle until line ACA″ is perfectly horizontal. Water is poured into the vessel until it reaches precisely to centerpoint C of the register plate and thus coincides with line ACA″ as well as with the diameter in the middle circle that is parallel to it. The experimenter must arrange this fairly soon after sunrise or not long before sunset, because the sun must lie 10° above the horizon if its light is to shine directly through the holes. When the apparatus is properly adjusted so that the sunlight shines through the holes to the water and thence to the rim under water, the experimenter is to mark centerpoint D of the circle of light cast on the rim, that point lying on the middle circle inscribed on the apparatus's rim. According to Alhacen, the angle of refraction will subtend the resulting arc GD (i.e., the angle of deviation), not arc DA′, as it would according to modern canons or those followed by Ptolemy. This angle can thus be compared to the so-called angle of inclination, which is 10° as subtended by arc FA, as well as to the 80° subtended by arc A′G, which is equal to the angle of incidence formed by FC and the continuation of normal A′C above the water's surface.

[59]According to Alhacen's description of this stage in the experiment, the experimenter is to erase the original lines AA″ and CA′ in figure 7.3.31, p. 167, and find a new point B that is 20° to the right of the hole's center, as in figure 7.3.30, p. 165. He is then to draw diameter BCB″ and measure off arc BB′ = 90°, which leaves CB′ normal to diameter BCB″ and also leaves angle B′G = 70° = the angle of incidence that will be formed by FC and the extension of normal CB′ beyond C. He will then insert the apparatus in the vessel so that diameter BCB″ is horizontal so as to coincide with the waterline, and he will adjust the apparatus until the sunlight shines through the two holes at an angle of 20° above the horizon. That light will be refracted to some new point D on the apparatus's rim under water, the resulting arc GD measuring the angle of refraction when the angle of incidence is 70°.

[60]The values for the angles of refraction at intervals of 10° predicted by the modern sine law, based on an index of refraction of 1.33 for air to water and slightly rounded off, are given in figure 7.3.32, p. 167. The angles of incidence with respect to the normal are listed in the column labeled *Incid.*; the angles of inclination with respect to the water's surface are listed in the column labeled *Inclin.*; the angles of refraction with respect to the normal are listed in the column labeled *R(Mod.)*;

and the angles of refraction according to Alhacen's definition (i.e., the angles of deviation) are listed in the last column labeled *R(Alh.)*. Assuming that Alhacen determined the angles as precisely as possible within the limits of instrumental accuracy—perhaps to the nearest 10′ or 12′ of arc (see note 9, pp. 334-335)—Alhacen's values might have diverged slightly, but only slightly, from the ones listed, assuming, of course, that he could pinpoint the center of the circle of light cast on the apparatus's rim fairly accurately. Notice that in every case the ratio of any smaller angle of refraction/deviation in the last column to its corresponding angle of incidence is less than the ratio of any larger angle of refraction/deviation in the last column to its corresponding angle of incidence, just as is mandated by the generalization discussed in note 56, p. 353.

Despite the detailed instructions Alhacen provides for determining the angles of refraction/deviation from air to water for the eight angles of incidence ranging from 10° to 80°, he does not give his own tabulations. Nor does he give any tabulation for the subsequent experiments testing refraction from air to glass, glass to air, or glass to water. This is in marked contrast to Ptolemy, who provides tabulations at ten-degree increments of incidence for the refraction of visual rays from air to water, air to glass, and water to glass (see Smith, *Ptolemy's Theory*, pp. 233-236). Suffice to say, were these tabulations based on light rays, they would measure refraction from water to air, glass to air, and glass to water according to the principle of reciprocity, of which Alhacen was well aware (see note 63, p. 358). Had he translated his values for the angles of refraction/deviation for air to water into angles of refraction according to the normal and then compared them to the values given by Ptolemy, Alhacen would have noticed that for angles of incidence between 10° and 70° his and Ptolemy's values are discrepant by less than 1°, considerably less in some cases: i.e., 7.5 vs 8.0, 14.9 vs 15.5, 22.5 vs 22.0, 28.0 vs 28.8, 35.0 vs 35.0, 40.2 vs 40.5, and 44.8 vs 45.5 (see Smith, *Ptolemy's Theory*, p. 233). But he would also have noticed that at $i = 80°$ his and Ptolemy's values for r (47.8° vs 50°) diverge by at least two degrees.

Because Alhacen failed to provide any tabulations, it is impossible to determine from his analysis in the *De aspectibus* whether he agreed with Ptolemy's results. We do, however, have a clear indication in a later work entitled "Treatise on the Burning Sphere" (which was never translated into Latin) that he did. In the fifth proposition of that work, which is devoted to the focusing properties of solid and water-filled glass spheres, Alhacen adverts specifically to book 5 of Ptolemy's *Optics* and cites two values provided there for refraction from air to glass, those values being 25° and 30° for the angles of refraction at angles of incidence of 40° and 50°, respectively (see Smith, *Ptolemy's Theory*, p. 236). These angles, Alhacen continues, translate respectively to 15° and 20° according to his measure of the angles of refraction/deviation. Not only does Alhacen cite those values, but he actually applies them in his mathematical analysis, which indicates that he accepted them as correct. This point may shed some light on the feasibility of the experimental determinations Alhacen describes in chapter 3. According to the sine law, based on an index of refraction of 1.5 for air to glass, the correct values of r for $i = 40°$ and $i = 50°$ are roughly 25.4° (= 14.6° angle of deviation) and 30.7° (= 19.3° angle of deviation). These values are fairly close to Ptolemy's, but if Alhacen

was actually able to determine the angles of refraction/deviation to within even a quarter of a degree of accuracy within the limits of the apparatus described in chapter 3, he could not have failed to notice a perceptible discrepancy between his results and those given by Ptolemy—enough of a discrepancy that he should have at least adjusted Ptolemy's figures to 25.5° and 30.5°, which would yield angles of refraction/deviation of 14.5° and 19.5°. That he in fact did not do so in the "Treatise on the Burning Mirror" suggests either that he never actually made the experimental determinations detailed in the *De aspectibus* or that the experimental conditions were such that he could not make those determinations with the accuracy implied in his account (see, for instance, Simon, "L'Expérimentation," pp. 355-375). For an Arabic text and French translation of Alhacen's "Treatise on the Burning Mirror" see Rashed, *Géométrie et dioptrique*, pp. 110-133; esp. 124 for the citation of Ptolemy.

Although simple enough in concept, Alhacen's method for determining the angles for refraction/deviation from air into water is actually fairly complicated and cumbersome in practice. First of all, if the full range of angles is to be taken into account, the experiment must be done at a latitude no higher than ten degrees to the north of the Tropic of Cancer, Cairo being well within that limit. Otherwise, the sun will never reach high enough to shine through the holes at an angle of incidence of 10°, and even then the experiment will have to be carried out at, or very nearly at, summer solstice. In addition, the full test will occupy an entire morning or afternoon, as the experimenter waits for the sun to rise in ten-degree increments from dawn to noon or from noon to twilight. Add to this that the sun follows an arc in its passage through the sky, so the experimenter must have some idea of where and when to aim the holes in order to catch the sun at the right moment. Every adjustment of the apparatus in the water-filled vessel will disturb the water, forcing the experimenter to let it calm down, as all the while the sun continues to move through the sky, shifting both its altitude and its longitude.

[61]Apparently Alhacen has in mind the arrangement in figure 7.3.33, p. 168, where the glass quarter-sphere is placed flush with line ACA″ so that its flat face points directly up and the midpoint of its common section lies at centerpoint C of the register plate. Accordingly, when the apparatus is inserted in the vessel, which is presumably empty of water, and when it is adjusted so that the sunlight shines through the two holes, the incoming light will strike the centerpoint of the sphere encompassing the quarter-sphere on the flat face in line with point C and will be refracted along line CD to the rim after passing straight through the convex surface. Once he has marked the centerpoint of the circle of light cast on the rim at D, the experimenter will remove the apparatus from the vessel and determine the size of arc GD. This, of course, will be the measure of the angle of refraction/deviation according to Alhacen's definition. The experimenter is then to remove the glass quarter-sphere, erase line ACA″, and move on to the next ten-degree increment at B, marking line BCB″ on the register plate and affixing the glass quarter-sphere flush with that line so that the midpoint of its common section lies at C. Reinserting the apparatus in the vessel, he will repeat the experiment he did before, but this time at an angle of incidence of 70°. When he has gone through all the angles of incidence in this way, he will have all the respective angles of

refraction. This experiment can also be done with the water from the previous test of refraction left in the vessel, provided that the water's surface does not overrun the flat face of the glass quarter-sphere. Under those conditions, the refracted light will pass straight through the convex interface between the glass and the water, so the water will have no effect on the direction of its passage to the rim.

[62]The arrangement of the apparatus for this test of refraction from glass to air is illustrated in figure 7.3.34, p. 168. As before, the experimenter measures off arc FA of ten degrees and draws line ACA". He then affixes the glass quarter-sphere to the register plate so that the common section of its two flat faces is flush with that line and the midpoint of that common section lies at C. This time, however, the convex surface will face the two holes. With everything set up in this way, the experimenter will insert the apparatus in the empty vessel, adjust it so that the sunlight shines through the two holes, and mark the centerpoint of the circle of light refracted to point D on the rim, that light having been refracted away from normal CA', as expected. Once he has determined the angle of refraction/deviation according to the measure of arc GD, he will repeat the process for the remaining ten-degree increments so as to generate the full list of angles of refraction/deviation keyed to their respective angles of incidence and inclination. As Alhacen suggests at the beginning of the paragraph, the same test can be done for refraction from glass to water, in which case the apparatus can be left as is in the vessel and water poured in until it reaches just to or slightly beyond the flat face of the glass quarter-sphere.

If one were actually to conduct this experiment, though, he would immediately encounter a snag of significant proportions. As it turns out, when light passes through any transparent medium to a refractive interface, its action at the interface depends upon the so-called critical angle, which is the angle of incidence at which the light will suffer total internal reflection because it is barred from penetrating. It is this angle, for instance, that plays a crucial role in the formation of rainbows. So far none of the experiments has been affected by the critical angle because all of them have involved the passage of light from air into a denser medium, and in such cases the air has virtually no critical angle, which is to say that the denser medium can be penetrated at any angle between just over 0° and 90°. Glass, however, has a critical angle of around 42°, so in the experiment illustrated in figure 7.3.34 the light reaching the flat face of the glass through the convex surface cannot penetrate that face at an angle of incidence equal to or larger than 42°. The case illustrated in that figure is therefore false: the light reaching along line FC at an angle of 80° will not be refracted to D but, rather, will be fully reflected to a point on the rim above A" at an equal angle of 80°. The same holds for the cases in which the angle of incidence is 70°, 60°, and 50°, which leaves only the four remaining angles between 40° and 10° that can be appropriately tested.

Alhacen's failure to note this rather startling fact suggests either that he thought it not worth mentioning, which seems highly unlikely in view of his overall punctilio in providing relevant details, or that he never conducted the experiment, at least not as described. The same holds for the experiment suggested at the beginning of the paragraph to test refraction from glass to water. In that case there will still be a critical angle for the light passing to C through the glass, although

its size (around 61°) will be affected by the fact that the light passes into water rather than air. Accordingly, the tests for angles of incidence of 80° and 70° will be fruitless because no refraction will occur. The critical angle also has a bearing on certain experiments with the glass quarter-sphere described in chapter 2. For instance, in the experiments illustrated in figure 7.2.21, p. 159, the experimenter is instructed to place the quarter-sphere with its common section at a slant to diameter FCG, but he is given no restrictions on the amount of slant. Clearly, if the slant is severe enough, angle FCA of incidence can reach or exceed the critical angle of around 42°, in which case there will be no refraction, and the same holds for the experiments illustrated in figures 7.2.22 and 7.2.23, p. 160. Likewise, in the experiments illustrated in figures 7.2.24 and 7.2.25, p. 161, the eccentric placement of the quarter-sphere will determine the size of the angle of incidence formed by FCG and normal AD. In this case, the experimenter is to apply the quarter-sphere so that its common section is perpendicular to diameter FCG, but so that the centerpoint on its upright flat face is eccentric to centerpoint C of the register plate. Again, he is given no restrictions on this eccentricity, and it is obvious that it can be large enough to make the angle of incidence reach or exceed the critical angle.

[63]Here we have an explicit articulation of the principle of reciprocity that underlies Alhacen's dynamic account of refraction into and out of denser transparent media at the end of chapter 2. Accordingly, if the light passes through the rarer medium along line AC in figure 7.3.35, p. 168—i.e., "the line along which the light extends *to* the point of refraction"—and if it is refracted along line CD, then when the path is reversed so that the light passes from D to C—i.e., "the line along which the light extends *from* the point of refraction"—it will be refracted along line CA. In the first case, then, the angle of incidence will be ACE and in the second DCF, but in both cases the respective angles of refraction/deviation DCA" and ACD' will be equal, since they are vertical angles. The symmetry of such refraction is logically intuitive if one supposes, as Alhacen apparently does, that the rules governing natural processes are perfectly consistent. Demonstrating this symmetry empirically, however, is not as easy in practice as it may seem in theory because it demands perfect results. Anything less will fall short of a true demonstration because it will yield at best a close approximation, as is the case with the empirical demonstration of the equal-angles law in reflection (see Smith, *Alhacen on the Principles*, pp. xxiv-xxvi).

[64]There is obviously a lacuna at this point either in the text or in Alhacen's exposition. Normally, after having explained precisely how the glass should be affixed to the register plate, Alhacen would go on to instruct the experimenter to insert the apparatus into the vessel, pose the vessel in sunlight, adjust the apparatus so that the sunlight shines through the two holes, take note of where the refracted light strikes the inner wall of the rim, and mark its centerpoint. Only then would the experimenter be instructed (as he is right after this bracketed portion of the translation) to examine that mark in order to determine where it lies with respect to the continuation of the line passing through the two holes.

[65]The arrangement described in this experiment is illustrated in figure 7.3.36, p. 169. As represented in the top diagram, ten-degree arc FA is measured off, and line ACA" is drawn on the register plate. Distance CX equal to the radius of the

glass quarter-sphere is marked off on line ACA″, and from point X a perpendicular is dropped to diameter FCG. The glass quarter-sphere is then affixed to the register plate so that the common section of its two flat faces is flush with the perpendicular dropped through point X and so that the midpoint of that common section lies at X. The semicircle at the base of the quarter-sphere will therefore touch centerpoint C of the register plate. According to this disposition, then, diameter FCG will be perpendicular to the flat face of the quarter-sphere, and so the light passing through the two holes will pass through that face orthogonally. However, since the point at which it enters that face is eccentric to the center of the sphere encompassing the quarter-sphere, which lies directly over point X, the light passing parallel to diameter FCG will strike the convex face of the quarter-sphere obliquely. It will therefore be refracted away from the normal as it enters the air below point C, the normal being ACA″ in this case. So disposed, the apparatus is inserted in the vessel, as represented in the lower diagram, where the angle of incidence is 10°, as measured from normal ACA″, and the angle of inclination is 80°, as measured with respect to the extension of line CA′ above point C. When everything is adjusted so that two holes face the sun directly, the sunlight will enter the glass orthogonally, as predicted, and it will be refracted to point D on the inner wall of the rim, which lies beyond both point G and point A″. As also predicted, it will be refracted away from normal ACA″. According to the sine law, the resulting angle of refraction measured by arc A″D will be 15.1°, which means that arc GD measuring the angle of refraction/deviation according to Alhacen's definition will be 5.1°.

In this experiment, just as in the previous one from glass to air, it will be pointless to test the angles of incidence from 50° to 80°, since they exceed the critical angle of 42°. As Alhacen points out, this test of refraction from glass to air should yield the same results as those obtained in the tests of refraction from air to glass according to Alhacen's designation of the angles of refraction as angles of deviation and according to the principle of reciprocity. The same procedure can be followed for refraction from the convex face of the glass quarter-sphere into water, if water is poured to a level slightly above line ACA″. In that case, of course, the angles of refraction/deviation will be somewhat larger than those for refraction from glass to air, and the experiment will work for angles of incidence from 30° to 80° because the critical angle for refraction from glass to water is around 61°. Figure 7.3.36a, p. 170, which is copied directly from folio 295r of ms *P3*, represents this particular experimental setup. I have added the lettering in conformance with figure 7.3.36. Accordingly, the glass quarter-sphere is placed with its flat face flush with the line through X, which is perpendicular to diameter FCG that passes in line with the two holes to the other side of the rim. Diameter AXCA′, which passes through midpoint X of the common section of the two flat faces of the quarter-sphere—obviously misrepresented in this diagram—intersects diameter FCG at midpoint C of the register plate and is thus meant to be normal to the convex face of the quarter-sphere at that point. When the beam of light passes through the two holes in line with F and R, it strikes the convex face obliquely so as to refract away from normal ACA′ when it enters the air below the glass, thus following refracted ray CD.

[66]Although Alhacen's description of the apparatus and its disposition for this experiment is somewhat difficult to follow because of its painstaking, sometimes confusing detail, it is easily understood when graphically represented, as it is in figure 7.3.37, p. 171. The first step is to produce the hollow glass piece illustrated in the upper diagram. The hollow itself is semicylindrical, and the semicircles at its bases have a radius EN equal to the radius of the glass quarter-sphere used in previous experiments to test refraction in glass at a convex surface. Parallel edges HEM and KL are equal to the diameter at the base of the semicylindrical hollow plus a grain of barley. Thus, the two end pieces at H and M on the top surface of the glass extend beyond that cirumference by half a grain of barley each. Parallel edges HK and LM are one grain of barley longer than radius EN of the semicircle, and the entire piece with its hollow is two grains of barley high. Once this piece is properly formed—Alhacen suggests doing it in a mold—and finished by polishing, the experimenter is to turn to the main apparatus represented in the lower diagram. First, he is to measure off the arc FA he wants to test, draw a perpendicular at A on the inner wall of the apparatus's rim, and then draw line ACA′ from the endpoint of that perpendicular on the register plate. From centerpoint C of the register plate he is then to measure off distance CE on line ACA′ equal to radius EN of the semicylindrical hollow in the glass piece, and through point E he is to drop a perpendicular to diameter FCG, extending it on both sides. Then he is to apply the glass piece to the register plate so that the bottom edge parallel to line HM on the glass piece is flush with the perpendicular just drawn, and so that centerpoint E of the diameter on the semicircular base of the hollow lies directly above the intersection of that perpendicular and line ACA′. Therefore, line ACA′ will pass along a radius of the semicircle at the base of the hollow. Since C lies on that line, and since CE is equal to the radius of the semicircle at the hollow's base, CE will be a radius of that semicircle, from which it follows that C lies on the circumference of that semicircle.

Now the middle circle on the apparatus's rim lies one grain of barley above the surface of the register plate, and the center of the hole in the rim lies on that circle, so it too lies one grain of barley above that surface. But the glass piece is two grains of barley high, so it is more than high enough to accommodate a beam of light reaching it from the two holes at F and R, and the axial ray in that beam will lie in the plane of the middle circle and will be parallel to diameter FCG. Accordingly, when the beam reaches flat face KL of the glass piece, it will pass straight through because it strikes that face orthogonally. When it reaches point C on the concave hollow, though, it will strike it at an angle of ten degrees, i.e., angle FCA subtended by arc FA, so it will be refracted into the air at that point and will be deflected along path CD inclining away from normal ACEA′. Since the light is refracted at an internal surface in the glass, this test is limited by the critical angle of 42°, so it can only be conducted at ten-degree increments starting with an angle of incidence of 0° and ending with an angle of 40°. Figure 7.3.37a, p. 172, shows how this experimental setup is represented on folio 296r of ms *P3*, the letters having been added by me to make it conform to figure 7.3.37. Note that the refracted ray is not shown in this diagram. For the sake of further discussion, I

have tabulated the results predicted by the sine law for refraction from air to glass, glass to air, and glass to water in figure 7.3.38, p. 173. Like the tabulations for refraction from air to water in figure 7.3.32, p. 167, these tabulations are extremely close to the ones Alhacen should have obtained had he conducted the experiments as precisely as possible within the limits of instrumental accuracy.

[67]Presumably, Alhacen is saying here that, if the situation is reversed, so that the light is incident to the concave surface of the glass along ray DC, whose continuation to the other side is CD′, as represented in figure 7.3.37, p. 171, the light will be refracted along CF toward normal AC, in which case the angle of refraction/deviation will be D′CF, which is equal to angle of refraction/deviation DCG when the light passes to C along ray FC.

[68]Alhacen makes three claims in this sentence, all of them meant to be universally true. The first and most straightforward is that, for light passing through two given transparent media, the same angle of incidence will invariably yield the same angle of refraction/deviation. The second claim is that a larger angle of incidence will yield a larger angle of refraction/deviation both when the light passes from a rarer to a denser medium and when it passes from a denser into a rarer medium—hence the reference to "two angles of refraction" in the case of the smaller angle of incidence. This claim is borne out by all the tabulations in figures 7.3.32 and 7.3.38, pp. 167 and 173. The third claim is that, if angle of incidence $i >$ angle of incidence i', and if angle of incidence i yields angle of refraction/deviation r/d, whereas angle of incidence i' yields angle of refraction/deviation r/d', then $r/d - r/d' < i - i'$. While this rule holds generally for the tabulations in figures 7.3.32 and 7.3.38, we find that it does not hold for refraction from glass to air at angles of incidence of 30° and 40°, where $r/d - r/d' = 16°$, which is significantly greater than $i - i' = 10°$.

[69]In other words, if angle of incidence $i >$ angle of incidence i', and if angle of incidence i yields angle of refraction/deviation r/d, whereas angle of incidence i' yields angle of refraction/deviation r/d', then $r/d : i > r/d' : i'$. At the beginning of the chapter, Alhacen articulates this rule in reverse order according to the proportion $r/d' : i' < r/d : i$ (see note 56, p. 353).

[70]Rule 3 states that, if angle of incidence $i >$ angle of incidence i', and if angle of incidence i yields angle of refraction/deviation r/d, whereas angle of incidence i' yields angle of refraction/deviation r/d', then $i - r/d > i' - r/d'$. This claim is universally borne out by the tabulations listed in figures 7.3.32 and 7.3.38, pp. 167 and 173. Rule 4, which also applies universally to the given tabulations, asserts that, when light passes from a rarer into a denser medium, the angle of refraction/deviation is invariably smaller than the angle of incidence. This of course is self-evident because if $r/d > i$, then the light would have to refract through and beyond the normal. According to both Omar and Rashed, the Arabic version of this rule is that, when light passes from a rarer to a denser medium, r/d is invariably less than one-half i; see Omar, *Ibn al-Haytham's Optics*, p. 145, and Rashed, *Géométrie et dioptrique*, p. xliv. Although this rule applies to all the tabulations for air to water and air to glass for angles of incidence up to 80°—presumably the angles actually tested by Alhacen (or at least by Ptolemy)—it does not hold for values of i for air to glass (at an index of refraction of 1.5), starting at slightly less than 83°, at which

point the angle of refraction/deviation > half *i*. The same holds *ceteris paribus* for media with an index of refraction higher than 1.5, the angle at which *r/d* > *i*, decreasing as the index of refraction increases.

[71]The Latin text of this passage, *et si lux exiverit a corpore grossiori ad subtilius, tunc angulus reflexionis erit medietas coniuncti duorum angulorum*, could be interpreted to say, "on the other hand, if the light passes from a denser into a rarer body the angle of refraction will be half the sum of the two angles." Taken literally, this rule claims that for any refraction from a denser to a rarer body, angle of refraction/deviation *r/d* = .5(*i* – *r/d*). This rule is contravened by all the cases presumably tested by Alhacen for the passage of light from glass to air or glass to water. For instance, when light passes from air to glass at *i* = 40°, it yields *r/d* = 14.6°, which is far less than .5(40° + 14.6°), which = 24.3°. By Omar's and Rashed's account the Arabic version of this rule is that, when light passes from a denser to a rarer medium, the angle of refraction/deviation is always less than one-half the angle of incidence plus the angle of refraction/deviation—i.e., *r/d* < .5(*i* + *r/d*); see Omar, *Ibn al-Haytham's Optics*, p. 145; Rashed, *Géométrie et dioptrique*, p. xliv. This, of course, is tantamount to saying that in such refraction the angle of incidence is always greater than the angle of refraction/deviation. Again, this rule does not apply universally to refraction from glass to air or from any other medium denser than glass. In the case of glass, for instance, if the index of refraction is 1.5, the angle of refraction/deviation will begin to oustrip the angle of incidence when *i* reaches a value of slightly over 41.4°, and it will continue to outstrip it until *i* reaches the critical angle of just over 41.8°.

[72]The gist of this extraordinarily cumbersome sentence is simple enough: if light passes from a given rare medium into two dense media of different densities at an equal angle of incidence, it will be more sharply diverted in the denser of the two dense media, which means that the resulting angle of refraction/deviation will be greater. For instance, if the light ray strikes water and glass at an angle of incidence of 80°, the angle of refraction/deviation for the water will be 32.4°, whereas it will be 39° for the glass, so the angle of refraction/deviation (39°) for the denser medium is greater than that (32.4°) for the less dense one. It therefore follows that 39:80 > 32.4:80, as Alhacen claims in the middle of the sentence when he adverts to the comparison of ratios.

[73]Here it is apparent that rarity is not to be thought of in terms of the physical density of the water but to its opacity relative to the glass. Thus, milky water is less dense yet less transparent than clear glass. Notice that Alhacen's claim that light passes (and thus refracts) from water to air and, as he observes in the next sentence, from water to glass, is based on deduction rather than induction because his experimental apparatus is not designed to have light pass from water into any other medium.

[74]I take this to mean that, if the form strikes an opaque spot on the otherwise transparent body in contact with the transparent body through which it reaches that spot, it will be blocked from continuing, whereas all the rest of the forms will continue through along refracted lines.

[75]Alhacen is making several points here. First, if a luminous, colored object lies in a transparent medium, then each point on that object's surface will radiate its

form, which consists of light mingled with color (i.e., luminous color), throughout the medium along straight lines to the medium's surface. If we take a spot of any size on that surface, the form of the point will be radiated to every point on that spot and will thus be distributed continuously throughout it. Thus, as Alhacen points out somewhat later, that spot will form the base of a cone of radiation with its vertex at the point from which the form originates. If the transparent medium in which the luminous, colored object lies is in contact with another transparent medium of different density, then each point on the continuous form at the given spot on the first medium's surface will radiate its light and color throughout the second transparent medium in the same way that each point on the original luminous, colored object radiated its light and color through the first medium.

[76]Lurking under the tangled verbiage of this paragraph is a simple point. Light radiating from a given point in one transparent medium and refracted to a given point in another transparent medium will follow the same path if the direction of radiation is reversed.

[77]Alhacen uses the same sort of analysis based on a cone of radiation in book 4 to explain the reflection of light to the eye from mirrors (see Smith, *Alhacen on the Principles*, pp. 325-327). In this case, Alhacen is inviting us to imagine a luminous, colored spot at point A in the air, as in the top diagram of figure 7.4.39, p. 174. That point will radiate its luminous color rectilinearly in all directions through every facing point in the air along rays AB, AC, AD, and so on to AX within the single plane represented. Now imagine that the luminous color passing along these lines encounters a denser medium at points B, C, D, and so on. There they will be refracted toward the normal, except at point N, which lies on perpendicular AN. The luminous color will thus extend through the denser medium along refracted lines BB′, CC′, DD′, and so on. If all possible such lines between AB and AX are taken into account, the luminous color at A will shine on every point between B and the point where ray AX encounters the medium to create a continuous, coherent form of A on the refracting surface that consists of all those points, the result within the represented plane being a triangle of radiation with its vertex at A and its base forming a continuous line on the surface of the denser medium. If we imagine this triangle rotated about axis AN, it will form a cone of radiation based on the circle formed by the luminous color shining from A to the denser medium's surface. This base constitutes the *locus* (translated as "spot") of A's form on that surface. The composite form on this circle will then continue to all points in the denser medium between B′ and Q′, so those points will form a base for the interrupted continuation of cone ABX. Now imagine the process reversed, with points B′, C′, D′, and so on to X′ lying on an illuminated and colored object, and imagine them to radiate their luminous color to points B, C, D, and so on at the interface between the denser medium below and the air above. All the rays at those points, except N, will thus be refracted away from the normal according to the principle of reciprocity so as to converge at point A in the air. If, therefore, a center of sight is posed at A, all the forms of the points on the object facing the eye through the denser medium will be seen at A, only one point on it, i.e., N′, reaching the eye unrefracted. And the obverse holds as well, the center of

sight being placed in the denser medium and the object in the rarer medium, as represented in the lower diagram.

[78]The experiment Alhacen describes here is the obverse of the air-to-water experiment illustrated in figure 7.3.31, p. 167, because in this case the light source is at point D under water and the object illuminated by that light is not the rim but the eye posed at F, which sees it through the narrow sighting-hole in the hollow of the reed that has been pushed through the two wax-filled holes at F and R. Accordingly, the first test described in paragraphs 4.15-16 requires placing the apparatus into the vessel so that diameter FCG is perpendicular to the water's surface and putting the stylus's endpoint at G. In that case, of course, the experimenter will see the endpoint of the stylus at point G, which lies on the normal, thus demonstrating that the ray from that point to the center of sight is unrefracted. The second test requires that diameter FCG be oblique to the water's surface, as actually represented in figure 7.3.31. When the apparatus is set up in this way, the experimenter is to pose the stylus's endpoint at G and observe that it cannot be seen. He is then to move that endpoint in a clockwise direction, i.e., in the direction away from the eye, and observe that it still cannot be seen. When he slides it along the rim in a counterclockwise direction, however, he will eventually see it when it reaches point D, and he will find that this point lies between point G and point A' on normal CA'. The underlying point of this experiment is to show that the image of the stylus's endpoint at D is seen straight along the refracted ray FG, just as an image in a mirror is seen along the reflected ray.

[79]The Latin text for this passage reads *deinde precipiat alii ut mittat in vas perpendicularem neque grossam nec gracilem*. The problem is that *perpendicularem* is adjectival and must therefore have a nominal referent, whether unstated or missing. Furthermore, given that *grossam* and *gracilem* are feminine and modify the unstated or missing referent, that referent must be feminine as well. Three of the manuscripts and the Risner edition offer specifications either by gloss or by replacement: *C1* glosses *perpendicularem* with *id est lingnum aliquod vel acum vel huiusmodi(?)—lingnum* presumably being a misspelling of *lignum*; *E* glosses *perpendicularem* with *lignum aliquod vel acum vel huiusmodi aliud*; *O* replaces it with *lignum*; and *R* interpolates *lignum aliquod vel acum* before it. Being neuter, neither *lignum* by itself nor *lignum aliquod vel acum vel huiusmodi aliud* will do because *lignum* and *aliud* disagree in gender with *perpendicularem*, *grossam*, and *gracilem*, all of which are feminine, so I have followed *R* by inserting "a wooden rod of some kind or a needle" (*lignum aliquod vel acum*) into the English translation as the unstated or missing referent, since *acus* ("needle") is feminine.

[80]This exercise is apparently designed to demonstrate that the refracted image is seen directly and exclusively through the point of refraction on the water's surface. Consequently, when that point is blocked by the rod, the image cannot be seen. That, in turn, demonstrates that the image is only seen along the refracted ray that passes from the point of refraction through the centers of the two holes to the center of sight.

[81]This situation is the same as the one illustrated in figure 7.2.12, p. 155, where F'C'G' is the experimenter's line of sight and D the actual location of the stylus's point according to its apparent location at G'. When the experimenter puts ruler

EK in place so that its leading edge on the register plate passes through the center of the register plate along the orthogonal, point K, where the sharp end of the ruler touches the rim will be the endpoint of the normal. Accordingly, when the experimenter pulls the apparatus out of the vessel and looks at all three points G,′ D, and K, he will see that D lies between G′ and K. It is somewhat puzzling that Alhacen did not follow this experiment out by testing the values he determined in his experiments for refraction from air to water by matching them to the values he could have derived here for refraction from water to air. All he needed to do was adjust the apparatus in the vessel so that the line of sight along diameter FCG formed angles at ten-degree increments from the normal, place the point of the stylus on the rim where he could see it along the axis of the sighting tube at each increment, and mark its position. Then he could have compared that position to the one marked on the rim when sunlight passed through the holes along FCG at the same angle. Assuming that the principle of reciprocity holds universally true, the two marks should have coincided, thus confirming the validity of his air-to-water experiments. In doing so, of course, he could also have confirmed, or disconfirmed, Ptolemy's tabulations for the refraction of visual rays from air to water, which is based on a reversal of the direction of radiation according to Alhacen's focus on light as the cause of vision.

[82]The arrangement described here is essentially the same as that illustrated in figure 7.2.19, p. 158, with diameter FCG perpendicular to the flat face of the glass quarter-sphere so that the line of sight through the two holes is also perpendicular to that surface as well as to the convex surface, since the continuation of that line of sight from C through the glass forms a radius.

[83]The arrangement for this experiment is like the one illustrated in figure 7.2.16, p. 157, with ruler EK removed.

[84]The arrangement for this experiment is illustrated in figure 7.4.40, p. 175. Line MCN is drawn aslant to diameter FCG on the register plate, and line ACD is drawn normal to MCN. The glass quarter-sphere is affixed with its flat surface facing the two holes, its common section flush with line MCN, and the midpoint of that common section directly on centerpoint C of the register plate. Diameter FCG will therefore pass obliquely through the flat surface of the glass at C and then straight through the convex surface along CG, which coincides with a radius of the quarter-sphere. Line ACD, for its part, is normal to both the flat and convex surfaces of the glass. Therefore, when the endpoint of the stylus is placed at point K, its form will pass from the air below into the glass straight along ray KC, be refracted away from the normal at point C, and follow the refracted path CF in the air above the glass. Having done the tests with the stylus's endpoint, the experimenter will eventually determine that point K, which is the only point at which he can see that endpoint, lies between endpoint G of diameter FCG, which is parallel to the line of sight, and endpoint D of normal ACD. Since the refraction is from the denser to the rarer medium, this experiment will not work if angle of incidence KCD reaches or exceeds the critical angle. This experiment is essentially a reprise of the experiment illustrated in figure 7.2.20, p. 159, with the direction of radiation reversed. The exercise with the rod placed at the point of refraction

is clearly identical in intent to that described in paragraph 4.18; see note 80, p. 364.

[85]The arrangement for this experiment is like the one illustrated in figure 7.2.16, p. 157, with ruler EK removed. Note that in this case diameter FCG is always perpendicular to the convex and flat faces of the glass, so it can be posed at any angle to the horizon.

[86]As we saw in note 53, p. 352, Alhacen accepts Aristotle's cosmological model, which consists of nesting spheres of earth, water, air, and fire between the center of the universe and the moon. Everything from the moon to the outer sphere of the fixed stars constitutes the heavenly body, which is composed of *aither*, a marvelously refined element that is subject to no change or corruption beyond the natural tendency to move forever in circles about the center of the universe. As such, it is virtually without density and, therefore, more transparent than even the clearest air or fire. The shell of fire and air surrounding the Earth below the heavens constitutes the "atmosphere," through which the light from heavenly bodies will be refracted on its way to the Earth, so those bodies are always seen by means of refraction. Although the Latin term *stella* is narrowly construed here as "star," it can also refer to heavenly bodies in general, including the moon, sun, and planets, all of them composed of *aither*. Throughout the rest of this chapter Alhacen's analysis of atmospheric refraction is based on supposing that it occurs at the interface beween the heavens (i.e., *aither*) and air, but later in chapter 7, paragraph 7.43, pp. 320-321, he is more specific in defining that interface according to the separation between *aither* and pure fire at the very top of the atmospheric shell. Here and in the remainder of this chapter, Alhacen is taking air generically, as the stuff composing the Earth's atmospheric shell.

[87]A detailed description of the armillary is given by Ptolemy in *Almagest* 5.1, where it is called an *astrolabon*, a zodiacal armillary for the measurement of ecliptic coordinates, longitude, and latitude. Directed to the celestial pole, the axis of the instrument is inclined to the north point of the horizon by the observer's geographical latitude, and it has fixed rings in the planes of the meridian and celestial equator. Within the fixed rings are two rotating rings graduated in degrees and some fraction of degrees, one to be set in the plane of the ecliptic for measuring longitude and one perpendicular to the ecliptic for measuring latitude, with the sights located on the latitude ring. The instrument described by Alhacen, an equatorial armillary, is simpler. As illustrated in figure 7.4.42, p. 176, it consists of three rings concentric with point E. The meridian ring NASB and the equatorial ring AB are fixed at right angles to one another, whereas the declination ring inside these fixed rings pivots on the north-south axis while remaining perpendicular to the equatorial ring. The equatorial ring is graduated in degrees and some fraction of degrees for the measurement of hour angle, i.e., distance from the meridian, from which right ascension may be found, and the declination ring is graduated in the same units and armed with adjustable sights for the measurement of either declination or polar distance, the complement of declination. According to Alhacen's instructions, the armillary is to be set up, as illustrated in the lower diagram of figure 7.4.42, with the north-south axis NS of the armillary aligned

with the celestial pole and the meridian ring NASB coincident with the observer's meridian. The equatorial ring AB will therefore be in the plane of the celestial equator, and the plane of the observer's horizon will be inclined to the plane of the equator according to the observer's latitude. By turning the declination ring toward a star or other body and viewing it through the sights, the observer can read the hour angle on the equatorial ring at the intersection with the declination ring, and he can also read the declination or polar distance on the declination ring according to his line of sight with the body. In the lower diagram of figure 7.4.42, for instance, the declination along line of sight EO will be 30°, and the polar distance will be 60°. For the purposes of the observations Alhacen has in mind, the line of the horizon must be flat with no topographical features, such as mountains, obtruding above it, which is why he suggests making the observation from a high point so as to make the plane of the horizon as extensive as possible in order to stay above any topographical irregularity at the edge of the plane.

[88]The phenomenon to which Alhacen is alluding in this paragraph can be explained by recourse to figure 7.4.43, p. 177. Let the smaller circle in the top diagram represent the Earth centered on E, let the larger circle DGSB concentric with it represent the celestial sphere containing the fixed stars, and let both circles lie in the plane of the observer's meridian. Let D be the celestial north pole, S the celestial south pole, and PEQ the common section of the plane containing the meridian circle DGSB and the plane of the celestial equator; and let Z be the location of an observer on the Earth at a latitude around 30° north of the equator, i.e., roughly the latitude of Cairo, where Alhacen might have conducted this set of observations. Let OZR be the common section of the plane containing the meridian circle DGSB and the plane of the observer's horizon. Let B be the observer's zenith, which lies on line EZB perpendicular to the plane of his horizon, and let BHG be the common section of the plane containing the meridian circle DGSB and the plane of the circle on which some fixed star rotates through the observer's zenith at midnight. The direction of its motion along that track is determined by the clockwise rotation of the celestial sphere around axis DS, as viewed from above the celestial north pole, one full rotation taking precisely one day. Accordingly, when it crosses the plane of the observer's horizon at point H to his northeast, the star will make its initial entry into the observer's night sky. The very same situation is represented to appropriate scale in the bottom diagram, where the Earth has shrunk to a virtual point in relation to the celestial sphere so that Z and E coincide. Under these conditions it is clear that, being fixed on the celestial sphere, the star that is carried along its track in plane BHG will always lie the same angular distance from the north pole, which means that arc BD = arc GD, and so on throughout the star's diurnal revolution. Hence, the angular distance from H, where the star appears at the horizon, and D should be equal to BD. In order to test this supposition, Alhacen instructs the experimenter to set up the armillary sphere with its north-south axis pointing directly at the celestial north pole and its meridian ring in the plane of his longitude, which passes through his zenith. With the armillary sphere set up in this way, the experimenter is to note the star's rising at point H and pivot the movable ring until it lines up perfectly with the star at that exact point. He will then measure the arc between H and D as projected onto that ring

(in the figure arc HD appears straight because it is represented as seen straight on). Subsequently at midnight, when the star reaches its zenith at point B on his meridian, the experimenter is to repeat the process, measuring arc BD. Instead of the expected equality, he will find that arc HD is slightly smaller than arc BD, which would suggest that the star is following a slightly smaller, more northerly circular path at twilight than it is at midnight. In the next paragraph Alhacen will explain this discrepancy between the two observed positions of the star on the basis of atmospheric refraction.

[89]In other words, since the stars have maintained the same positions on the celestial sphere throughout the history of recorded observations, then even if we were to concede that they could shift their positions, we would also have to concede that such a shift would necessarily occur over a longer time period than the one spanned by that history and certainly not in the relative eye-blink of a single night.

[90]Alhacen makes this point explicitly in paragraph 1.1 at the beginning of book 4; see Smith, *Alhacen on the Principles*, p. 3 (Latin) and p. 295 (English).

[91]The points made in this paragraph are illustrated in figure 7.4.44, p. 178. Let K be the star on its actual path KS′B, which is a segment of the arc through which the star passes over the viewer's zenith B during the course of a night. S′ is the point where that path crosses horizon OR. Because of atmospheric refraction the light from K is bent in such a way that the star appears to lie at H, where it coincides with point S on the horizon. Its apparent path at that time will therefore be HSB′. As a result, the angular distance between S and north pole D will be slightly less than the angular distance between S′ and D. At zenith-point B, on the other hand, the light from the star passes unrefracted through the atmosphere, since it lies directly overhead, so real path KS′B and apparent path HSB′ will coincide at that point, which means that the angular measure between B and D will give the true distance of the star from the celestial north pole on its true path KS′B, whose full arc passes directly over the observer's head as it moves to the point opposite S on the horizon. The plane within which the star's light is refracted will be defined by KHB, which is perpendicular to the horizon and which constitutes a segment of an arc that also passes directly over the observer's head at B. Under normal atmospheric conditions, the angle of refraction at the horizon is slightly higher than 30′. Hence, if angle BKH = 30° (i.e., the latitude of Cairo), and if HK in figure 7.4.44 represents an arcal distance of 30′, the arcal distance between S′ and S on the horizon will be just over 15′, from which it follows that the arcal distance between apparent orbit HSB′ and actual orbit KS′B will be just over 13′. By no means negligible, this discrepancy is still small enough that not only detecting it but also being confident that it actually exists requires a fairly sizeable and precise measuring instrument.

[92]Unlike the previous observational test with a star, this one is not based on distance from the north pole because the moon's orbit is not parallel to the equator but is oblique to both the equatorial circle and the ecliptic, to which it inclines at around 5°. As a result, its altitude, or polar distance, varies throughout its orbit, and the highest latitude at which it can reach true zenith is around 28.5° north, which is fairly close to the latitude of Cairo. Accordingly, this observational test

is based on the lunar orbit and the horizon-to-zenith distance on it. Let the large circle in figure 7.4.44a, p. 178, represent that orbit, which is concentric at E with the smaller circle representing the Earth. Let the observer stand at O on the Earth's surface, let his zenith be Z, and let the common section of his horizon and the lunar orbit be the line marked "horizon." The observer is then to mark the exact time at which the moon appears to peep over the horizon at M1′, and he is to determine the arc between that point and the zenith, which subtends angle M1′EZ. Knowing the lunar period with appropriate exactitude, he can then calculate the time it will take for the moon to pass over that arc. Aligning the sighting holes of the armillary sphere with point Z, he is to look through those holes with the expectation of seeing the leading edge of the moon at M2′ at the calculated time. In fact, he will not see the moon then but will have to wait a short time until it does appear. As in the case of the star, the cause is atmospheric refraction. Thus, when the observer sees the moon at M1′, it actually lies at M1, so it will pass over arc M1M2 rather than M1′M2′ during the calculated time, which is to say that it will fall somewhat behind its expected position. This experiment requires not only exceptional precision in the measure of angles and time, but also an exceptionally accurate tabulation of lunar motion.

The atmospheric refraction of moonlight raises interesting issues about the height and composition of the atmosphere. As mentioned earlier in note 86, p. 366, Alhacen insists that, as far as its optical properties are concerned, the atmosphere comprises both the sphere of air and the sphere of fire enveloping it and, therefore, that atmospheric refraction occurs at the interface between fire and aither. The space below that interface constitutes the sublunary realm, where elemental change and corruption occur, everything above it being exempt from all change other than circular motion, which is natural only to aither. Being incorruptible, moreover, aither is almost perfectly transparent and therefore almost perfectly rare. As far as the atmospheric refraction of moonlight is concerned, how close this interface lies to the moon is a crucial issue, because the closer that interface lies to the moon, the greater the density differential between the aither and the atmospheric medium below it must be. For instance, in figure 7.7.44b, p. 179, let E be the center of the Earth, O an observer on the Earth's surface, OH the plane of his horizon, and EM the center-to-center distance between Earth and moon. Assume that point T represents the topmost point on the moon and that at the time represented in the figure its light is seen by O right on the horizon. Angle TOH will therefore represent the amount of upward skew caused by refraction from O's perpective.

Now let AB and CD represent arcs on the outer surface of two possible atmospheric shells, CD lying quite close to the moon and AB lying much nearer the Earth. No matter which of the two surfaces represents the refractive interface, the refracted ray must follow line OH, so points R and R′ of refraction for both surfaces will lie on OH. TRN will thus be the necessary angle of incidence for ray TR striking interface CD, and ORE will be the resulting angle of refraction. Likewise, TR′N′ will be the necessary angle of refraction for ray TR′, and OR′E will be the resulting angle of refraction. Clearly, then, angle of refraction ORE is far smaller in proportion to its angle of incidence TRN than is angle of refraction OR′E

in proportion to its angle of incidence TR'N', which is to say that the light reaching R is far more severely refracted toward the normal than the light reaching R'. This means that the density differential at interface CD must be significantly greater than the density differential at AB. So we are left with two options. Either fire is enormously dense in comparison to aither, or else the aither-fire interface must lie quite close to the Earth. Alhacen never addresses this issue in the *De aspectibus*, although he gives a clue in chapter 7 (see note 180, p. 392). In a work long misattributed to Alhacen but actually written by his much younger contemporary Ibn Muʿādh, the atmosphere is shown by mathematical reasoning to extend to a height of around 85 kilometers above the Earth's surface. Ibn Muʿādh's argument, however, was based not on refraction but on the tinging of the topmost vapors in the atmosphere by the first rays of the sun at dawn, so there is no way of telling whether his atmospheric model was meant to account for the refraction of light from celestial objects as well. For the details of his analysis, see A. Mark Smith, ed. and trans., "The Latin Version of Ibn Muʿādh's Treatise 'On Twilight and the Rising of Clouds,'" *Arabic Sciences and Philosophy*, 2 (1992): 38-88.

[93]In other words, circle BHK is the circle passing through point H on the observer's horizon, and through point K below it in a plane perpendicular to that of the horizon. This would be the circle passing through line KHB in figure 7.4.44, p. 178, and, as mentioned in note 91, p. 368, it defines the plane within which the light from the star at K will be refracted to the center of sight so as to be seen at the horizon.

[94]Although the Latin text is unequivocal in instructing us to extend line KM to Z, that is clearly impossible. What we are apparently meant to do is draw line KZ straight from K to Z.

[95]The preceding explanation of atmospheric refraction and its effect on celestial observation is similar in most respects to that provided by Ptolemy in book 5, paragraphs 23-29, of his *Optics* (see Smith, *Ptolemy's Theory*, pp. 238-242). The main difference between the two is that, unlike Alhacen, Ptolemy gives no details about the observational basis of his account, alluding only vaguely to measurements that can be derived from "an instrument for measuring the stars." This instrument is presumably like the one described in *Almagest*, 5.1, if not that very one itself. For a general comparison of Ptolemy's and Alhacen's analyses of atmospheric refraction, see A. Mark Smith, "Ptolemy, Alhacen, and Ibn Muʿādh and the Problem of Atmospheric Refraction," *Centaurus*, 45 (2003): 100-115.

[96]In book 5, chapter 1, Alhacen defines an image simply as "the form perceived in a polished body" (see Smith, *Alhacen on the Principles*, p. 385). Here he elaborates on what makes an image an image: namely, that it is displaced from its object and that it does not represent the "true form" of its object. In reflection, of course, the image is displaced by appearing behind or in front of the mirror, and its true form is distorted according to shape, size, and orientation. In refraction, the image displacement is contingent on the disparity between the angles of incidence and refraction, and the distortion of its true form involves its apparent size as well as its orientation.

[97]Both the *obolus* and *denarius* are small silver coins. Harking back to Golden-Age Athens, the *obolus* remained in use as a denomination well into the Byzantine

Middle Ages and was circulated in the Latin West as well. The *denarius* has its origins in the later Roman Republic. By the Middle Ages it was a standard denomination for coinage of relatively low value, as compared to the *solidus* (12 *denarii*) and libra (240 *denarii*). The *denarius* was in fact the basis for the later English penny in the system of pounds (£), shillings (s), and pence (d). According to Na īf's account of this experiment from the *Kitāb al-Manāẓir*, Alhacen suggests using not a coin but a ring or an egg; see Mu afā Na īf, *al- asan ibn al-Haytham, bu ūthuhu wa kushūfuhu al-ba ariyya* (Cairo: Fouad I University, 1942-1943), p. 749.

[98]This experiment with the "floating" coin was well known in Antiquity and is described by Ptolemy in book 5, paragraph 5 of his *Optics* (see Smith, *Ptolemy's Theory*, pp. 230-231). It also forms the basis for definition 6 of Euclid's *Catoptrics*. Alhacen was of course intimately familiar with the first of these works and may well have known the second.

[99]As in reflection, then, so in refraction, the image is located at the intersection of the continuation of the line of sight and the cathetus of incidence. In reflection, the line of sight coincides with the reflected ray, so in refraction it coincides with the refracted ray.

[100]Used since Antiquity as a pigment, white lead (i.e., lead carbonate), or *cerusa*, is described in Theophilus's *De diversis artibus* (c. 1150) as a fundamental whitening agent in painting, often with the addition of ground bone ash, lime, or calcium carbonate. The binder suggested by Theophilus for this particular pigment is glair, a by-product of beaten egg whites, but other binders, such as gum arabic, will serve as well or even better. In his *Libro dell'arte* (c. 1400), Cennino Cennini observes that white lead is particularly appropriate for painting on wood panels. Unfortunately, Alhacen gives no indication of the binder to be used in painting the white lines on the wooden circle, and what is meant by the additive "snow-white milk" (*lac nivius*) is anyone's guess. Cennino does mention a pigment called *lac* or *lacca* ["lake"}, but according to his account it is red not white, much less snowy white. Evidently, then, *lac nivius* is meant literally, as especially white milk mixed in with the white lead, or it is the technical designation for some sort of white pigment. In fact, milk has been commonly used as a white pigment for millennia, so the literal version is probably what is intended. Whatever recipe Alhacen had in mind, it seems clear that it was intended to make the white as brilliant and conspicuous as possible. For Theophilus's and Cennino's discussions of white lead and its uses, see John G. Hawthorne and Cyril Stanley Smith, trans., *On Divers Arts: The Treatise of Theophilus* (Chicago: University of Chicago Press, 1963) and Daniel V. Thompson, Jr., trans., *Cennino d'Andrea Cennini,* The Craftsman's Handbook (New Haven: Yale University Press, 1933).

[101]This experiment is illustrated in figure 7.5.46, p. 180, where the original circle with two intersecting diameters and the centerpoint in black is represented in the top diagram and that same circle as it looks when sunk into the water-filled vessel in the bottom diagram. The breaking and upward tilt of the portion of the slanted diameter under water is quite striking, and Alhacen is right in suggesting that the experimenter lower his line of sight as much as possible in order to amplify the effect by making both the angles of incidence and refraction larger. From that vantage, morever, the entire circumference of the circle under water is grossly compressed.

[102]From the fact that the image of the centerpoint shared by both diameters lies on the perpendicular diameter Alhacen concludes that the image of every other point on the slanted diameter lies on the perpendicular or cathetus dropped from that point to the water's surface, hence the apparent straightness of the refracted portion of that diameter seen under water. Thus, as illustrated in figure 7.5.46a, p. 181, if E is the center of sight, AB the actual continuation of the slanted diameter under water, and AP′ the portion of its refracted image visible over the lip of the vessel, then, according to the cathetus rule, the image of centerpoint C on continuation AB of the slanted diameter will be at C′, where the extension of refracted ray RE intersects cathetus CC′, which lies on the perpendicular diameter. In this case, of course, the incident ray is CR. Likewise, the image of point P will lie at P′, where the extension of refracted ray R′E meets cathetus PP′, and point A will be seen where it is at the surface of the water. Altogether, then, the image of all the points on straight segment AP of the slanted diameter will be straight line AP′ composed of all the images of those points..

[103]The arrangement described to this point is represented in figure 7.5.47, p. 182. Chord KL is marked off on the wooden circle to a length of ten digits and bisected at M. Through M perpendicular MX is dropped, extended to the circumference of the circle on both sides, and colored white. This line will thus form a diameter in the circle. At endpoint L of the chord another diameter LX is drawn through the circle to cross the white diameter at centerpoint X. This diameter is tinted with some color other than white. Glass brick ABCHEF is formed so that its corresponding faces are parallel and so that it is eight digits long on edge AD, four digits wide on edge CD, and four digits deep on edge CH. This brick is then attached to the circle with edge FE flush with chord KL and with one end extending one digit beyond the white diameter and the other extending two digits beyond endpoint L of diameter LX.

[104]The Latin term translated here as "paper" is *bombax*. Presumably, the Arabic term rendered by *bombax* is *qirtas*, which derives from the Greek term *charthes* for "papyrus," although by Alhacen's time it was commonly used to designate "paper" rather than papyrus. For its part, the Latin word *bombax* seems to derive from *bambuxinon*, *bombuxinon*, *bambaxeron* (later *bambaxeros kartis*), the Byzantine terms for "paper" based on the place-name Bambyke. This was the Greek designation for the northern Syrian city of Manbij, a major producer of paper during the Middle Ages. For further details and elaboration, see Jonathan M. Bloom, *Paper Before Print: The History and Impact of Paper in the Islamic World* (New Haven: Yale U. Press, 2001), esp. pp. 27, 42-45, and 56-57.

The use of *bombax* at this point in the Latin translation of the *Kitāb al-Manāẓir* raises two intriguing issues. First, up to now historians have traced the earliest use of *bombax* and its variants for designating "paper" to Italy in the second half of the thirteenth century (see Bloom, *Paper Before Print*, pp. 211-212, and André Blum, *On the Origin of Paper* [New York: R. R. Bowker, 1934], p. 46). We have every reason, however, to suppose that the *Kitāb al-Manāẓir* was translated in Spain toward the end of the twelfth century. Furthermore, we know that paper was manufactured in Spain well before that time, so it is reasonable to suppose that the use of *bombax* to designate "paper" traces back at latest to twelfth-century Spain.

The second issue has to do with the translation of *qirtas* throughout the *De aspectibus*. The earliest appearance of this term occurs in book 3, chapter 2, paragraph 2.55, where it is rendered *pargamenum* ("parchment") in the Latin version (see Smith, *Alhacen's Theory*, p. 273; see also pp. 277-281 for its subsequent use in the same chapter), and it is rendered that way when it crops up again in book 4, chapter 3, paragraph 3.42 (see Smith, *Alhacen on the Principles*, see also p. 23 for its subsequent use in the same chapter). So far in my analysis of the Latin text of the *De aspectibus*, I have identified two clear points where a shift of translators occurred: book 3, chapter 3 (see Smith, *Alhacen's Theory*, pp. cxlxviii-clxix) and book 6, chapter 6 (see Smith, *Alhacen on Image-Formation*, pp. xlv-xlvii). I have also concluded that the translator responsible for the rest of book 6 after the shift in chapter 6 was also responsible for book 7. In my edition of book 6 I left open the question of whether the same translator was responsible for the initial portion of the text from book 1 through book 3, chapter 2, and the concluding portion of the text from book 6, chapter 6, to the end of the treatise. However, the shift from *pargamenum* to *bombax* between the opening and concluding portions of the text strongly suggests that it was not the same translator and, therefore, that there were at least three translators at work on the treatise.

[105]As I understand it, the procedure Alhacen has in mind here can be described as follows on the basis of figure 7.5.47a, p. 183. The experimenter is to stand the circle with its glass brick in an upright, facing position and place his right eye directly in line with endpoint L of the slanted diameter so as to establish line of sight EL. He is then to bring his right eye as close as possible to point E on that line so that he cannot see any of the glass brick's top surface or, for that matter, any other outer surface on it with that eye. For all practical purposes, then, his eye touches the glass and is therefore "in" it, which means that, although the rays reaching his eye through the front surface of the glass brick are refracted before they actually get to it, the effect of such refraction is minimal. With his right eye properly positioned, the experimenter is to to tilt his head somewhat so that with his left eye he can see some of the top surface of the glass brick, as well as the continuation of the white diameter above it. After covering the end of the glass brick to the right of E in order to block his view through the back surface of the glass brick of everything but segment LR of the slanted diameter, he is to look down through the bottom surface of the glass brick toward intersection-point X of the two diameters. With his right eye he will see that, rather than continue straight along RX, the slanted diameter LRX bends away along RX' at point R on the back edge of the bottom surface of the glass brick to form obtuse angle LRX'. He will also see that the perpendicular diameter passes straight through that same edge without any breaking whatsoever. Meantime, with his left eye he will see that all segments of the perpendicular line below the bottom edge of the glass brick, in the glass brick itself, and above it continue in a straight line. The main point of this experiment is to establish empirically that, when the form of any point on the slanted diameter is refracted from a rarer to a denser medium, the image of that point will lie below the point itself and will be found at the intersection of the refracted ray and the normal dropped from the point. Thus, as illustrated in the right-hand diagram of figure 7.5.47a, the form of centerpoint X radiates along XR'

to point R′ on the bottom surface of the glass and is refracted thence toward the normal along R′E. As seen from center of sight E, therefore, its image X′ will lie where extension ER′X′ of refracted ray R′E intersects the normal, which happens to be the perpendicular diameter, and the same holds for any other point along segment RX of the diameter below the bottom surface of the glass, so the composite image RX′ of that segment will be straight.

[106]This point harks back to Alhacen's model of visual selectivity based on a cone of radiation with its base on a given visible object and its vertex at the center of sight. According to that model, the form of any point on the visible object is seen only along the straight line connecting that point and the center of sight, and every such line is orthogonal to the cornea and the anterior surface of the crystalline lens, both of which are concentric with the eyeball as a whole. Since the crystalline lens is where visual sensation is initiated, it is peculiarly apt to be affected by (*patitur*) such orthogonal impingements and by no others. Hence, by its very anatomical and physiological structure the eye is constrained to see refracted forms along refracted rays, because those and those alone are perpendicular to the anterior surface of the crystalline lens.

[107]Alhacen's rationale in paragraphs 5.21-24 is best explained graphically. Suppose that O in figure 7.5.48, p. 184, is an object point in a denser medium and that, when it reaches point R at the interface with the rarer medium, its form is refracted along RE to a center of sight at E. According to Alhacen's earlier analysis of refraction, the velocity of the form's motion is inversely proportional to the density of the medium through which it passes. Therefore, as it passes through the denser medium, the form's motion is composed of the motion along vertical component OA and the motion along horizontal component AR, and when it passes into the rarer medium both its vertical and horizontal motions increase along AB and CE. Had the medium been uniformly dense according to the density of the lower medium, the form of O would have passed straight through toward E′ at a constant velocity, and in the same amount of time it took to reach E along refracted path RE, it would reach E′, its overall motion measured by vertical component AB′ and horizontal component B′E′. On the other hand, had the medium been uniformly rare according to the rarity of the upper medium, and had E been a visible point, its form would pass straight through R to O′ in the same amount of time that O's form passed to E′ in the uniformly dense medium, which is the same time it took for it to pass from O to E along OR and RE. Therefore, since the eye is constrained to see the image along refracted ray RE, it senses the form of O as if it had actually arrived at a uniform velocity along line O′E. But the eye is also constrained by the cathetus rule to locate the form on normal OA, so it senses the form of O as if it had arrived from I, which is of course the image point.

[108]Although it is possible that Alhacen is drawing a distinction here between normal transparent bodies, such as glass and water, and anomalous ones, such as doubly refracting crystals, which yield double images, the discussion that follows indicates that the "common" transparent bodies to which he is referring are those whose surfaces are spherical or plane.

[109]As we saw earlier, Alhacen follows Aristotle in supposing that water naturally forms a spherical shell concentric with the Earth and therefore with the

universe as a whole. Consequently, like the Earth's surface itself, the surface of any body of water on Earth will be convex spherical, even though its convexity is imperceptible because of the extreme slightness of its curvature. This, of course, is why the face of the Earth within the plane of the horizon appears flat, as does the surface of any body of water within that plane.

[110]In other words, since the larger angle of incidence yields the larger angle of refraction according to rule 1 in paragraph 3.34, p. 259, the sum of the two larger angles ought to be greater than the sum of the two smaller angles, i.e., angle of incidence BEZ (= TEH) + angle of refraction AET = angle AEH, should be greater than angle of incidence BOQ (= LOF) + angle of refraction LOA = angle AOF, which is clearly not the case.

[111]From Euclid, III.18, we know that, since HZ is tangent to the circle at B, a diameter drawn to point B will be perpendicular to that tangent, so it will also be perpendicular to AG, which is parallel to tangent HZ by construction. But we also know from Euclid, III.3, that a diameter perpendicular to any line in a circle bisects that line. Hence, diameter BC in figure 7.5.52a, p. 187, passes through line AG to form two right angles BCA and BCG, both of them subtended by equal arcs BA and BG.

[112]Euclid, III.20, is a special case of this lemma. In that theorem Euclid proves that, for any circle, if an angle with its vertex at the center and an angle with its vertex on the circumference are subtended by equal arcs, the angle at the center will be double the angle at the circumference. Accordingly, if the two chords AB and BD are intersecting diameters, point E of intersection will be at the circle's center, and so angle AEB at the center will be equal to the angle subtended by arcs AB and DG, which are equal, whereas angle DBG at the circumference will be equal to arc DG alone.

[113]This follows from the fact that angle ZMC (= NML) in triangle ZMC < angle CEB (= HET) in triangle ECB. Since angle MCZ = angle ECB, and since ZMC < CEB (because NML < HET), the remaining angle MZC (i.e., MZT) > the remaining angle CBE (i.e., MBE).

[114]In other words, since angle MCZ in triangle MZC = angle ECB in triangle ECB, angle MZC (i.e., MZE) + angle ZMC (i.e., ZMB) = angle CEB (i.e., ZEB) + angle CBE (i.e., MBE). Hence, since MZE + ZMB = ZEB + MBE, it follows that MZE – MBE = ZEB – ZMB.

[115]For the Arabic version and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 83-89.

[116]See book 5, paragraph 2.313, in Smith, *Alhacen on the Principles*, p. 448.

[117]For the Arabic text and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 89-91.

[118]Except for manuscript *C1*, which designates the line of refraction as "EH," the rest of the manuscripts, as well as Risner, designate it "AEK." This is clearly incompatible with figures 7.5.53a and 7.5.53b, p. 189, where AE and AEK, respectively, would both be lines of incidence, not lines of refraction, when B rather than A is taken as the center of sight.

[119]In other words, what holds for the refraction of B's form to A through the two media holds reciprocally for the refraction of A's form to B through the same

two media. For the Arabic version and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 91-92.

[120]Here we have at least tacit recognition of the critical angle in refraction from a denser to a rarer medium, as is the case represented in figure 7.5.54, p. 190. Given the density differential between the medium behind refracting interface GE and the one in front of it, ZEK is the limiting angle at which the incident light will no longer emerge from the sphere, so EA is tangent to the sphere at point E. Hence, light striking the refractive interface at an angle equal to or greater than ZEK will not be refracted through that interface. Likewise, LEA is the limiting angle of refraction/deviation, which makes AEH = 90° the limiting angle of refraction by modern standards. At any angle of incidence greater than ZEK, then, the light will refract at an angle greater than 90°, so it will reflect back into the sphere from point E. Conversely, if AEH = 90° is the limiting angle of incidence for refraction into the sphere, light traveling along it will not enter the sphere, so ZEK is the limiting angle of refraction, with KET being the limiting angle of refraction/deviation. Moreover, by construction angle KET = half angle LEA, so the first angle of incidence at which refraction into the sphere will occur is infinitesimally smaller than 90°, and the first effective angle of refraction/deviation for that angle of incidence will be infinitesimally smaller than half angle LEA. By the same token, the first angle of incidence at which refraction out of the sphere will occur is infinitesimally smaller than ZEK, and the first angle of refraction/deviation for that angle of incidence is infinitesimally smaller than LEA, which means that angle of refraction AEH will be infinitesimally smaller than 90°. Alhacen's addition of "as far as can be empirically determined" indicates that the values for ZEK and LEA are to be based on the tabulations supposedly made in chapter 3 or given by Ptolemy in book 5 of the *Optics*, so it follows that the construction is based on refraction from water or glass, the only two denser media tested by experiment.

[121]Alhacen's point here is that for refraction between two specific media, the difference between any two angles of incidence can be so slight that the difference between the angles of refraction is imperceptible. Presumably, then, if the light passes through the refractive interface at an angle of incidence at or very near 0°, it will be perceived to pass along the normal even though it may actually be refracted by some imperceptible amount away from the normal.

[122]Angle DZT = angle KET by construction, and angle DZT = alternate angle ZTE, i.e., KTE, so angle KET = angle KTE, leaving triangle KTE isosceles. Exterior angle ZKE = angle KET + angle KTE, so angle ZKE = 2 angle KET = angle LEA. By supposition, however, angle ZEK:2 angle KET = angle ZEK:angle LEA, so angle ZEK:angle LEA = angle ZEK:angle ZKE.

[123]The location of point E in this construction is absolutely specific to the density differential between the two media; the farther E gets toward D from G, the smaller the density differential, until it reaches point X, where the tangent is parallel to axis AD. At that point there is absolutely no distinction between the two media. The simplest case—and the one I suspect Alhacen had in mind when choosing this particular construction—involves refraction from glass to air according to a density differential of 1.5. This is in fact the case represented in figure 7.5.54. Let us start with the reverse passage from air to glass, so that AEH

= 90° is the limiting angle of incidence, ZEK the limiting angle of refraction, and KET the limiting angle of refraction/deviation. A glance at the table for refraction from air to glass in figure 7.3.38, p. 173, will show that the value for the angle of refraction/deviation Alhacen should have found for i = 80° is 39°, which is slightly less than one-half i. Moreover, it is clear from the table that as i increases, the angle of refraction/deviation continually approaches one-half i. This same pattern is evident from the tabulations for refraction from air to glass in book 5 of Ptolemy's *Optics* (Smith, *Ptolemy's Theory*, p. 236), where the values for r are based on a simple algorithm, each successive value increasing by the difference between the previous two values minus half a degree. In order to determine the value of r for i = 80°, for instance, we take the difference between the values of r for i = 60° and i = 70° (38.5° – 34.5° = 4°) and subtract .5° to arrive at a value of 3.5°. Then, adding that to r = 38.5° for i = 80°, we arrive at a value of r = 42° for i = 80°. It follows, therefore, that at the limit of i = 90°, r = 42° + 3° = 45°, which is half i. If the direction of radiation is reversed so that the refraction is from glass to air, the limiting angle of incidence is 45°, which yields a limiting angle of refraction of 90° and thus a limiting angle of refraction/deviation of 45°. Whether in fact Alhacen relied upon Ptolemy's values in this case is by no means certain, but we do know that he drew upon them elsewhere (see note 60, pp. 354-356), so it is not unreasonable to suppose that he did so here as well. Accordingly, if he assumed that refraction from glass to air is the absolute limiting case and that at its very limit the limiting angle of incidence is 45°, then when angle BEZ of incidence in figure 7.5.54 = 45°, angle AEH of refraction = 90°, angle KET = 27.5°, and twice angle KET = 45° = angle of refraction/deviation AEL. Consequently, angle of incidence BEZ at which refraction will actually occur is infinitesimally smaller than 45°, and angle KET is less than half angle BEZ by that same infinitesimal amount. But angle of refraction/deviation AEL = twice KET, so it will be smaller than angle of incidence BEZ by an infinitesimal amount, and as angle of incidence BEZ decreases, angle of refraction/deviation AEL becomes progressively smaller than it. From the very first point of actual refraction, therefore, $i > r/d$, which conforms to the fourth general rule of refraction indicated on p. 259.

Now if we pick some point E′ between E and X, as in figure 7.5.54a, p. 191, and if we carry out the construction as before so that angle B′E′Z is the new limiting angle of incidence and A′E′L the new limiting angle of refraction/deviation, point B′, where E′K′ intersects axis AD below the interface, will fall well below B, and point A′, where EA′ intersects axis AD above the interface, will fall well above A. Furthermore, if we imagine L, where BEL and B′E′L intersect, to be a stationary pivot point and BL and the portion of the axis from G behind the refracting interface to be infinitely long, and if we imagine moving that line up and down along that portion of the axis around pivot-point L, then for every new point B of intersection that this line defines on that portion of the axis, it will also define a new point of intersection E on the refracting interface. With those two points defined, the new angles of incidence and refraction/deviation will also be defined, as will the new point A. Thus, the farther B migrates down the axis, the closer to X point E migrates on the circle and the farther from G point A migrates up the axis.

[124]In other words, if some random point E′ on arc GE in figure 7.5.54b, p. 192, is chosen, if a line is drawn to it from A, and if normal ZH′ is extended through it from Z, then angle of incidence AE′H′ can be subdivided as appropriate for that angle of incidence so that the resulting angle of refraction/deviation is less than half the angle of incidence. Hence, the resulting refracted ray will pass on through the sphere to intersect normal BD. What Alhacen does not tell us, and what the diagram accompanying both the Arabic and Latin text fails to show, is on which side of B the refracted ray intersects the axis, that is, whether it intersects that axis between B and D or beyond B. As we will see later in chapter 7, when we deal with proposition 17, pp. 318-319, this failure to specify where the refracted ray should fall led to serious confusion on the part of either the Latin translator or the draftsman responsible for the accompanying diagram. In fact, the refracted ray will intersect the axis beyond B, and the closer point E′ approaches G, the farther the point of intersection will lie from B. There is, however, a point at which a ray from A strikes some point E so close to G that its refracted ray will never intersect the axis; see note 126 below.

[125]See the reference in note 116, p. 375.

[126]For the Arabic text and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 92-95. Several points are worth noting about this analysis. First, the circle of refraction and the resulting image on it are limited at the outer edge by point E because, according to stipulation, no refraction can occur at or beyond that point on arc EK, which is to say that there is no point between B and D from which the radiation will reach A through arc EK on the refractive interface. Second, the above-mentioned circle of refraction will actually form a ring whose inner circumference is limited by point E′ for some object point B′ on axis ADB, the entire image being of object line BB′. And finally, the longer we make that object line, the smaller the inner circumference of the ring will become. Given that E is the outer limit of refraction and, therefore, that no radiation through arc EK will reach A, the radiation from any point other than B on axis ADB that refracts to A must do so from some point E′ on arc EG. It would seem, therefore, that, since there is an infinitude of possible points E′ between E and G, there must be an infinitude of points on the axis beyond B whose incident rays will refract to A. In short, starting at B, the line forming an image on arc GE must be infinitely long. This is in fact not the case because, as Rashed points out in his analysis of the proposition, there is a limiting point B′ on axis ADB from which a ray striking the refractive interface very near G will not be refracted to A, nor will any point beyond it; see *Géométrie et dioptrique*, pp. xlix-xlx.

[127]For the Arabic text and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 95-96.

[128]Let X be the intersection of lines AH and MG, which form equal vertical angles AXM and GXH in triangles AXM and GXH. Therefore, angle XAM (i.e., HAM) + AMX (i.e., AMG) in triangle AXM = angle HGX (i.e., HGM) + angle XHG (i.e., AHG), from which it follows that AHG + HGM = AMG + HAM. Therefore, AHG – AMG = HAM – HGM.

[129]Angles BHA and BMA lie on the same base AB, so they form triangles BHA and BMA. In triangle BHA, then, angles BHA + HAB + HBA = angles BMA +

MAB + MBA in triangle BMA. Since BMA > BHA, it follows that (HAB + HBA) > (MAB + MBA). Accordingly, BMA – BHA = (HAB + HBA) – (MAB + MBA) = (HAB – MAB) + (HBA – MBA) = HAM + HBM.

[130]For the Arabic text and a French translation of this proposition, see Rashed, *Géométrie et dioptrique*, pp. 96-103.

[131]For the Arabic text and a French translation of this paragraph to this point, see Rashed, *Géométrie et dioptrique*, p. 104.

[132]Although Alhacen seemed to be addressing the doubling of images by double refraction through certain kinds of crystals earlier in this passage, it is clear at this point that he actually has in mind the form of an object point passing through two refractive interfaces. As he mentions at the end of this passage, he will discuss such a case, and indeed he does in chapter 7, proposition 17, pp. 318-319.

[133]Alhacen's point here echoes the point he makes about reflection in book 4, chapter 5, paragraph 5.4: namely, that innumerable cones of reflected light are formed at the surface of any mirror and that, no matter where the center of sight may lie as it faces that surface, it will be at the vertex of one of these cones and will thus see the entire object from which that cone is originally generated (see Smith, *Alhacen on the Principles*, pp. 326-327). See also the analysis in chapter 4, paragraphs 4.1-4.14, pp. 260-265, of this book.

[134]The points made in the preceding two paragraphs are illustrated in figures 7.6.57 and 7.6.57a, pp. 194-195. In all three diagrams of figure 7.6.57, AB is the object, A'B' its image, and DF the interface between the two transparent media within plane of refraction AREB. The "body" imagined to consist of all the normals dropped from every point on visible object AB to the refracting surface is contained within rectangle ADFB, when the refracting interface is plane, and a cone, half of which is represented here, with its vertex at C and its base on DF, when the interface is spherical. Each normal within the imagined body extending from the visible object to the refracting surface constitutes the cathetus of incidence for its object point, so the image of that point will lie at the intersection of the cathetus and the refracted ray within the cone of refraction. Accordingly, the form of A extends to R and is refracted to center of sight E along RE, so the image of A lies at A', and the same analysis holds for any point on the visible object between A and B. Hence, the image A'B' lies where the "body" formed by all the normals from AD to BF is intersected by the cone of refraction with its vertex at E.

In the top-left diagram of figure 7.6.57 the image A'B' is the same size as object AB, which lies in the denser medium, but it will appear larger because the visual angle A'EB under which it is seen is larger than the visual angle AEB under which the object would be seen were the medium between A and E to be of uniform density, and because the object's form is weakened by refraction so that it appears to lie farther away than it should. In the top-right diagram, the object lies inside a transparent sphere of greater density than the medium in which center of sight E lies, whereas in the bottom figure it lies in a transparent sphere of greater rarity than the medium in which E lies. Thus, in the top-right diagram, the image, which lies between the object and spherical surface DF, is actually larger than its object, whereas in the lower diagram it lies beyond the object and is actually smaller.

Finally, if the object lies entirely outside the sphere, as in figure 7.6.57a, p. 195, the intersection of the cone of refraction and the imagined body will occur outside the sphere itself. The case illustrated in this particular figure involves the refraction of object AB's form through a transparent sphere of greater density than the medium in which the object and center of sight E lie. The normals from all the points between A and B on the object will thus form two imagined conical bodies CF′D′ and CDF with a common vertex at centerpoint C of the sphere. The form of A extends along AR to the sphere, is refracted toward the normal at R to pass through the sphere along RR′, and is refracted a second time away from the normal along R′E to E, and the same holds by symmetry for point B. As a result, image A′B′, which is seen along the extension of refracted rays ER′, lies at the intersection of refracted cone EA′B′ and the extension of cone CF′D′ outside the sphere. Image A′B′ is thus actually larger than its object and lies beyond it. Conversely, if the medium within which the object and the center of sight lie is denser than that inside the sphere, the first refractions at R will be away from the normal, and the second refractions at R′ will be toward the normal, so the image will be smaller than the object and will lie between the object and the surface of the sphere. Thus, depending on which of the two media is denser, the image can be smaller than or larger than the object. In no instance, however, can the image actually be the same size as its object, although in the case illustrated in figure 7.6.57a, the object and its image approach each other in size as the object is drawn ever farther away from the sphere.

[135]The points made in this paragraph are illustrated in figure 7.6.57b, p. 196, where center of sight E faces a concave interface between the two media. In the left-hand diagram, center of sight E lies in the denser medium, so ray AR from endpoint A of object AB reaches point R to be refracted toward the normal along RE, and the same holds for all the points between A and B. Thus, image A′B′ is larger than object AB and lies beyond it. In the right-hand diagram, center of sight E lies in the rarer medium, so image A′B′ will be smaller than its object and will lie between the object and the refracting interface.

[136]The analysis that follows in the next five paragraphs is essentially a reprise of Alhacen's account of image fusion and diplopia in book 3, chapter 2, paragraphs 2.2-23, where he casts that account in terms of the two visual axes passing through the centers of the eyes and the common axis dropped from the center of the optic chiasma perpendicular to the line connecting the two centers of the eyes. Perfect image fusion, which yields maximum visual acuity, occurs when all three axes converge on one visible point so that the form of that point impressed on each eye is transmitted through the two optic nerves to the center of the optic chiasma, where each form coincides perfectly with the other. For a sketch of Alhacen's explanation of image fusion, see Smith, *Alhacen's Theory*, pp. lxviii-lxix and pp. lxxiii-lxxvi; for the actual text of that account, see ibid., pp. 563-573.

[137]Chapter 7, paragraphs 6.69-72, esp. 6.70, in Smith, *Alhacen's Theory*, pp. 376-377.

[138]As is evident from the next paragraph, the forms to which Alhacen refers in this paragraph are ones that strike the surface of the cornea at a slant.

[139]In other words, the forms of points that pass through the corneal surface along the orthogonal are also radiated to surrounding points on the cornea along lines that are oblique to its surface.

[140]By "the surface of the eye facing the aperture in the uvea," Alhacen means the portion of the corneal surface in front of the pupil that is demarcated by the cone of radiation, which cuts it along a circle parallel to the circumference of the pupil. Thus, according to Alhacen's analysis, every point on an outlying visible object within the cone of radiation forms a cone with its vertex at that object point and its base on that portion of the corneal surface, which is to say that this particular point radiates its form to every point on that portion of the corneal surface.

[141]In other words, all the points on the surface of the visible object will radiate through any given point between the surface of the visible object and the surface of the eye to form a double cone with this point as a common vertex. One of these cones is based on the visible object's surface, the other on the eye's surface. If the point in question lies on the surface of the eye itself, the form of all the points on the visible object at the base of the cone will radiate to that one point on the surface of the eye. Hence, according to the analysis based on this cone, any single point on the base at the surface of the visible object will radiate its form to every point on the facing surface of the eye, and all but one of those forms will strike that surface obliquely. According to the analysis based on the cone of radiation, on the other hand, all the points on the base at the visible object's surface will radiate their forms to any single point on the surface of the cornea, and all but one of those forms will strike that point obliquely.

[142]To this point, Alhacen's account is based on that provided in book 1, chapter 7, paragraphs 6.1-44 (in Smith, *Alhacen's Theory*, pp. 355-369). As laid out there, Alhacen's theory of image selection is predicated on supposing that the visual faculty perceives everything from the perspective of the center of sight, which is the center of the eyeball as a whole, or at least the part of it that functions in the selection of visible forms. As illustrated in figure 7.6.58, p. 197, this part consists of the cornea at the very front of the eye, the albugineous humor (i.e., the aqueous fluid) just behind the cornea, and the glacial humor just behind the albugineous humor, this last component forming the crystalline lens. The anterior surfaces of these three components are all centered on C, which is the center of the eyeball and thus constitutes the center of sight. Unlike the other two components, the glacial humor is not just refractive; it is also sensitive to the impingement of light and color radiated to its anterior surface through the cornea and albugineous humor.

Now when the eye faces a luminous or illuminated object, the forms of all points on that object's surface are radiated to every point on the outer surface of the cornea and thence to portion AGD of the anterior surface of the glacial humor that is exposed by the pupil, i.e., what Alhacen refers to as the aperture in the uvea, which lies between the two black lines nesting against the posterior surface of the cornea. Thus, as illustrated in figure 7.6.58, if O, O_1, O_2, and O_3 are object points within the field facing the eye, each of them will radiate its form to every point on portion AGD of the glacial humor's anterior surface. That being the case, surface AGD of the glacial humor will be bombarded at every point by the forms reaching it from every point in the visual field. In order to explain how the glacial humor makes sense of this incoherent bombardment, Alhacen appeals to both its optical and its sensitive nature. Optically, the glacial humor differs in transparency, and thus density, from the albugineous humor, which in turn differs

in transparency or density from the cornea. Although Alhacen never specifies the order of transparency for these three media, we can safely assume that the cornea is most transparent and the glacial humor least transparent. Accordingly, any form reaching the cornea obliquely—along O3R1, for instance—will be refracted toward the normal passing through R1 at its anterior surface to follow path R1R2 until it reaches the anterior surface of the albugineous humor, where it will again be refracted toward the normal passing through R2 along path R2R3. On reaching R3 at the surface of the glacial humor, it will be refracted yet again toward the normal along line R3F. And the same holds for any other form, such as the one from O1, that reaches the cornea obliquely; that form will undergo a series of two refractions before reaching the surface of the glacial humor at point A, and, arriving there at a slant, it will be refracted along some line AE. In all cases, therefore, the forms reaching the cornea's anterior surface obliquely will eventually reach the anterior surface of the glacial humor at a slant and will thus be refracted toward the normal dropped from centerpoint C but will never be refracted along it—and this will obtain no matter what the order of density among the three media may be. Consequently, the form will always be shunted away from the center of sight at C. Meantime, the forms passing along the orthogonals, such as OC, O1C, and O2C, will pass straight through all three tunics unrefracted and in the aggregate will form what Alhacen calls the "cone of radiation" (*piramis radialis*), whose vertex lies at center of sight C and whose amplitude is constrained by the circumference of the pupil. This relatively narrow cone, which is represented as OACDO2 in figure 7.6.58, extends to the edge of the cosmos, and anything that lies within it is seen along the orthogonals, which are dubbed *linee radiales* ("radial lines") by Alhacen.

The optical selectivity of the glacial humor is complemented by its innate selectivity as a sensitive organ, according to which it feels the impingement of light and color as a low-level "pain" (*dolor*). How intensely it feels that impingement is a function of the intensity of the impression the incoming light and color make upon it. Here Alhacen has recourse to the physical analogy he used in accounting for the deviation of light by refraction: the strongest, most effective impingement is along the perpendicular, and so the more slanted or glancing it is, the less effective it becomes. Thus, the anterior surface of the glacial humor is disposed to be affected only by those forms that reach it orthogonally. The rest it simply ignores as too weak and ineffective to be taken into account. Given its optical and sensitive nature, then, the glacial humor is peculiarly disposed to accept only those forms that reach it orthogonally and thus to select a point-by-point representation of any outlying visible object from the chaos of forms reaching it from every point on that object. As Alhacen makes clear at the end of paragraph 6.24, p. 304, he intends to modify this account of image selection by allowing for the perception of forms that reach the anterior surface of the glacial humor obliquely. One pressing reason for him to make this modification is to explain the peripheral vision of point objects, such as O3 in figure 7.6.58, which lie outside the cone of radiation and should therefore be invisible.

[143]According to Alhacen's understanding of ocular anatomy, the uvea, or "grapelike tunic," is an opaque sphere that encloses the albugineous humor, the glacial humor, and the vitreous humor behind it. Perforated at the front by the

small hole that forms the pupil, this sphere is black on the inner surface but can be various hues from grey or green to black on its outer surface, a portion of which forms the iris. As a rule, however, the iris is black, according to Alhacen. The black portion of the eye (*nigredo oculi*) to which Alhacen refers here is therefore most likely the iris, although it could be the inner surface of the uvea. Alhacen's description of ocular anatomy is to be found in chapters 6 and 8 of book 1, in Smith, *Alhacen's Theory*, pp. 348-355 and pp. 387-394; specific references to the uvea are to be found in 6, 5.6-11 and 8, 7.4, in ibid., pp. 348-349 and pp. 387-388.

[144]Here Alhacen appears to be defining the field of vision according to whether a line can be drawn from a given object point at either side of the eye to the middle of the eye's surface without intersecting the eye before reaching that point. It is at that point, of course, that the visual axis intersects the eye's surface. Theoretically, then, the field of vision for either eye is defined by a plane tangent to the middle of the eye's surface, the visual axis being perpendicular to that plane at the point of tangency. Any point above that plane will be visible if a line can be extended from it to the middle of the eye without interference. Thus, as illustrated in figure 7.6.59, p. 197, XM is a line within the aforementioned plane tangent to M at the middle of the eye and perpendicular to visual axis CM. Since line O3M can be drawn to M, and thus to the axis, without interruption, point O3 will be visible. The field defined by such a plane is obviously restricted by the nose, eyelids, and brow, which occlude everything beyond them but not inside the area ringed by them, which is why the needle is visible when placed by the tear duct or just below the upper or lower eyelids. The lateral area to the outer side of the eye, on the other hand, is entirely open to view. Thus, when the two eyes come into play, each with its respective lateral area open to full view, the horizontal field of vision spans 180°, which defines the limit of peripheral vision.

[145]Alhacen's explanation for why we see objects that lie to the side of the cone of radiation rests on the idea that we somehow see them along virtual radial lines—i.e., radial lines "by analogy" (*transumptive*). For instance, in figure 7.6.59, p. 197, object point O3 radiates its form to the cornea along lines O3R1, O3R2, and an infinite number of lines between those, so as to cast its form on the entire portion of the cornea's surface between R1 and R2. All of these forms will reach the anterior surface of the glacial humor at a slant, so all of them will be refracted away from center of sight C along such lines as R1'E and R2'F. According to the cathetus rule, the resulting image I3 should be seen where the extensions FI3 and EI3 of the refracted lines meet the extension of normal CO3, but instead, it is seen directly along normal CO3, which is a virtual rather than a real radial line. Thus, although the glacial humor senses all the forms of O3 that reach it at all the points on its surface between R1' and R2', the visual faculty is constrained to see those forms coalesced along the virtual radial line CO3, as if the form of O3 had actually radiated to it along that line through the opaque portion of the eye to the side of the exposed portion of the glacial humor's surface.

[146]In book 2, chapter 3, paragraph 3.171, Alhacen does claim that, when viewed too close, objects look larger than they should, but he gives no explanation at that point, deferring it to book 3, chapter 7, paragraphs 7.24-25; see Smith, *Alhacen's Theory*, pp. 493-494 and pp. 605-606. Later, in paragraph 7.46 of that same chapter, he

adverts to the apparent transparency of a needle held close to the eye; see ibid., p. 608.

[147]Let AB in figure 7.6.60, p. 198, be a portion of a white wall facing surface DK of the cornea, let the black circle N represent a cross-section of the needle, let CD and CK be the edges of the cone of radiation, and let OO4 be a portion of AB blocked from center of sight C by the needle. Points A and B on the wall will be seen from center of sight C directly along orthogonals AEC and BHC, and all the points between A and O and B and O4 will also be seen directly along the orthogonals. Being blocked by N, the points between O and O4 cannot be seen directly, although the facing surface of the needle can. However, all the points, such as O1, O2, and O3, between O and O4 radiate their form to all portions of surface DK on the cornea to the sides of the needle. That, of course, is what Alhacen defines as the surface facing the aperture in the uvea. All three points radiate their forms to segments DE and HK of that surface, and in addition points O and O4 radiate their forms to segments EF and HG, leaving FG untouched. Those three points will thus be seen along virtual radial lines CO1, CO2, and CO3, and the two outer points O and O4 will be seen along radial lines CO, and CO4, which are both real and virtual. Accordingly, the portion of the wall behind the needle will be seen conjointly with the needle itself, so the needle will appear to be somewhat transparent. When the needle is withdrawn, not only will all the points between O and O4 be open to direct view, but they will also be able to radiate their forms to the surface between F and G that was previously occluded. Thus, the visual impressions made directly along the orthogonals from those points will be reinforced by the added oblique impressions to yield a clearer visual impression than before. From this, Alhacen will conclude that visual acuity is greatest within the cone of radiation because the visual impressions made along the orthogonals within that cone are reinforced by the refracted impressions made within it, whereas the visual impressions made by objects outside that cone, being entirely due to refraction, are weaker and less clear.

In modifying his theory of visual selection to accommodate obliquely incident forms, Alhacen has placed an extraordinarily heavy burden on the glacial humor. Problematic enough is that it be required to distinguish orthogonal impingements from all others, even those that follow neighboring paths at the slightest of slants, but now it must be able to differentiate the full spectrum of impingements in order not to confuse the overlapping forms reaching each point on its exposed surface. Worse yet, it must be able to distinguish between two forms reaching any point on its surface from different directions but at the same slant, since both forms will strike with precisely the same force, as is the case for the forms of O and O2 in figure 7.6.58, p. 197, both of them arriving at point G on the glacial humor at precisely the same inclination. Thus, aside from recognizing the precise force with which these forms strike its surface, the glacial humor must also recognize the precise direction of inclination. Without knowing the direction of impingement, moreover, the visual faculty would have no way of ascertaining that a given set of refracted forms distributed throughout the glacial humor's exposed surface belongs to a given object point, so it would be unable to determine the virtual radial line along which the form of that point *would* be radiated to the center of

sight, if it *could* pass to that point through the opaque portion of the eye. The very determination of this line, of course, takes the process of image selection out of the domain of physics and into that of pure psychology. For further discussion of Alhacen's modified theory of image selection and its implications, see pp. lxvii-lxx of the Introduction.

[148]The empirical test of visual acuity within the cone of radiation to which Alhacen refers here is part of his experimental analysis of diplopia in book 3, chapter 2, the specific tests being described in paragraphs 2.55-79, in Smith, *Alhacen's Theory*, pp. 580-586.

[149]The Latin word rendered here as "misperception" is *fallacia*, whereas the Latin word rendered as "misperception" in the title just above is *deceptio* (see p. 114, lines 208 and 209). As pointed out in Smith, *Alhacen on Image-Formation*, p. 250, note 92, the term *fallacia* connotes not simply a misperception or illusion but also a mistake in deduction or judgment.

[150]In book 6, chapter 2, paragraph 2.3 (in Smith, *Alhacen on Image-Formation*, p. 162), Alhacen makes the same point about reflected forms, whose color is affected by the color of the reflecting surface, the color after reflection being a blend of the two. That blending can add to the natural weakening of the form due to the reflection itself.

[151]For full details, see book 3, chapter 3, in Smith, *Alhacen's Theory*, pp. 588-593.

[152]Alhacen's overall account of size perception is to be found in book 2, chapter 3, paragraphs 3.135-145, in Smith, *Alhacen's Theory*, pp. 475-479. In that account Alhacen is explicit about the need to correlate the size of the visual angle subtended by the object to the distance of the object from the center of sight in order to determine its size. This determination is not simply mathematical; it depends heavily on factors grounded in experience. Not only must we know what kind of thing the object is, but we must also know how large such things generally tend to be. Constant perception of horses, for instance, teaches us that they are uniquely determined by certain properties that include a characteristic size. We must also be able to reckon the distance of the object and, on that basis, correlate it to its characteristic size. Accordingly, a horse seen from a distance of 1000 meters will be judged to be the same size as one seen from a distance of 10 meters, if the angles they subtend at those distances are commensurate to the appropriate size of such an animal. Such a correlation will allow us to determine that a large statue of a horse seen at a certain distance is not actually a horse. Distance perception also permits us to determine the slant of an object. Thus, if an object facing the eye directly and an object slanting away from the eye subtend the same visual angle, and if both converge at one endpoint, the visual faculty will realize that the segments of the two objects at the other endpoint subtending the same visual angle lie at different distances and, therefore, that the segments lying farther away on the end of the slanted object are larger than the ones on the end of the directly facing object. From this the visual faculty will conclude that, as a whole, the slanted object is larger than the one facing the eye directly.

To this mix of correlations Alhacen adds the weakening of the form caused by refraction. Such weakening makes the form appear dimmer and less clear, and, as he observes in book 2, chapter 3, paragraph 3.159 (ibid., p. 486), experience

teaches us that an object seen up close is perceived more clearly than it is when seen from a greater distance. The weakening or dimming of the refracted image thus causes us to judge it to lie farther away than it would if it were not so dimmed and seen under the same visual angle. In correlating the visual angle subtended by the object to its apparent enlarged distance, then, the visual faculty judges it to be magnified. Otherwise, since the image and the object are the same size, they would be perceived to be the same size according to the size-distance invariance hypothesis, to which Alhacen clearly subscribes (see Smith, *Alhacen's Theory*, pp. lxv-lxvi).

[153]By "the proximity of the angle to the center of sight" (*propinquitas anguli ex visu*) Alhacen apparently means not the proximity of the angle itself but of its base LK. Thus, since LK is nearer the center of sight than BG, the angle it subtends is larger than the one subtended by BG.

[154]The argument here is based on assuming that AM is far greater than MO, much more so than represented in figure 7.7.61a, p. 199. That being the case, the difference in distance between A and IK and A and BG may be small enough to be negligible. Although BG and KI share the same orientation in that both are slanted upward from respective endpoints G and K, they will never be parallel, although the less slanted GB is, the closer to parallel the two will become. On the other hand, if GB is slanted such that B coincides with D at the refracting surface, then KI will be far more gently slanted than GB because I will also coincide with D. Nonetheless, if A lies far enough away from BG and KI that they appear to lie the same distance from it, the difference in slant will be imperceptible to the visual faculty from the perspective of A, so they will appear to share essentially the same orientation.

[155]Since AZ lies beyond LO, it never intersects it, which means that, strictly speaking, it cannot be perpendicular to it. The ulterior point here is to establish that AZ lies within a plane, i.e., AZH, that is perpendicular to LO.

[156]Again, these two claims are based on assuming that A lies far enough above the refracting surface that the difference in distance between it and both CO and BG is so small as to be imperceptible.

[157]The qualifications here are meant to account for LC's being the image of a slanted line, in which case point C will lie above KL if the line is slanted upward. However, since the image and the object line will both be slanted and thus equivalently oriented, according to Alhacen's analysis, image LC will appear to be essentially the same size on the slanted line as it would be on line LK.

[158]This sentence is an extremely loose translation of the Latin text, which reads *causa ergo que facit formam BZ videri maiorem facit ut BN habeat maiorem proportionem ad ipsum quam illam quam habet BZ ad BN*. More or less literally, this reads "thus, what causes the form of BZ to appear magnified causes BN to have a greater proportion to it than BZ has to BN." This seems to be a statement of proportionality, but the elements of that proportionality are far from clear. First, "ipsum," which occurs in all seven manuscripts, has no obvious antecedent because it is either masculine or neuter, whereas the proximate antecedent, "the form of line segment ZN" (*forma partis linee ZN*) is feminine. Even if we assume that this *is* the intended referent, the resulting proportionality would be "BN:form

of ZN > BZ:BN," which makes no sense. I take the intended point of this passage to be that the apparent size of image of BN in relation to the apparent size of the image of BZ is as BN is to BZ. Accordingly, CL in figure 7.7.63, p. 201, is the image of NB as seen by refraction under angle CAL; NAB is the angle under which NB would be seen without refraction; OL is the image of ZB as seen by refraction under angle LAO; and BAZ is the angle under which ZB would be seen without refraction. Accordingly, CL:PS = OL:OS, which is to say that the apparent size of the images in relation to the actual size of their objects is constant and, therefore, that the images always appear commensurately larger than their objects. It is crucial to note that this relationship between apparent and real size is based on assuming that the difference in those sizes is proportional to the difference in the angles under which the image and object are seen. To assume this is to assume that the apparent enlargement of distance that causes the apparent magnification of the object correlates perfectly with the enlargement of the angle under which the image is seen. Although Alhacen never makes this assumption explicit, his later analysis of atmospheric refraction and its effect on our perception of the size of heavenly bodies is evidently based on it.

[159]In this case, image LK is actually larger than object BG, so even without the weakening of the form it ought to appear magnified despite its lying closer to the center of sight. Presumably, then, the weakening of the form and the consequent enlargement of apparent distance simply adds to the apparent magnification.

[160]What "in front of line DM" means is unclear because in fact AO will intersect KL on line DM itself. As illustrated in figure 7.7.65, p. 202, AD lies in a plane oblique to DM, which lies in vertical plane DNME. GB lies above D within that vertical plane, and its image LK lies above GB in that same plane. Thus, LK will be seen under angle KAL, whereas GB would be seen under angle GAB if it were seen unrefracted, so LK will actually be larger than its object GB. Point Z on line BZ is seen at point O on line LK, so the image of BZ is KO, and it is seen under a larger angle KAO than the angle BAZ under which its object BZ would be seen if it were unrefracted.

[161]As Alhacen observed earlier in chapter 3, paragraph 3.4, p. 248, that center of curvature is the Earth's center, which lies well beyond anything seen under water. Alhacen's point here is that, since the surface of any body of water is convex, the image of anything seen under water will actually be larger than its object. Since, however, the surface of any body of water on Earth is virtually flat, the difference in size between object and image will be far too small to be perceived.

[162]For the Arabic version and a French translation of this paragraph, as well as proposition 17 following it, the experimental confirmation of proposition 17 that follows it, and proposition 18 following that—i.e. paragraphs 7.34-7.42, pp. 317-320—see Rashed, *Géométrie et dioptrique*, pp. 105-110.

[163]Although the majority of manuscripts designate this line HAB, it is clear from the diagram that it should be HBA, which in fact is how Risner relabeled it.

[164]Here, as well as in every other subsequent reference to the circle in figure 7.7.66, p. 204, the manuscripts all agree in designating it BGDZ, the letters thus being out of proper consecutive order when read in a counterclockwise direction.

Accordingly, I have followed Risner in reordering the letters to conform to that direction.

[165]Although all the manuscripts have "B" here, it is clear that "D" is actually intended. As we have seen several times before, the designation of propositions by number is problematic throughout the text of the *De aspectibus*. In this case, for instance, all the manuscripts agree in having "ninth" (*nona*) as the proper reading in the text, although three of them (*P1*, *L3*, and *C1*) have either changed it to "eighth" or have added "or the eighth" (*vel octava*). In this edition, in fact, proposition 9 is the ninth theorem of the entire book but the eighth theorem in chapter 5. It is worth noting that the Arabic version of the text cites the "ninth" proposition.

[166]As Alhacen shows in proposition 6, pp. 287-289 above, when the image location is the center of sight itself, as in this case, where the image of KO lies on normal OA dropped from all the object points on KO, the object will appear to lie on the refracting surface. Unlike the Latin version, the Arabic version of the text includes a diagram showing the projection of the ring on the surface of the sphere facing center of sight A; see Rashed, *Géométrie et dioptrique*, p. 109.

[167]Figure 7.7.66, p. 204, upon which this analysis is based reflects the diagrams in all the manuscripts of the *De aspectibus* that have it. It is also the diagram used by Witelo in his version of proposition 17—i.e., book 10, proposition 43 of the *Perspectiva*—and Friedrich Risner used it in his 1572 tandem edition of Alhacen's *De aspectibus* and Witelo's *Perspectiva*. In all those diagrams the original line-pairs TL and GH and the refracted line-pairs ZO and MK fail to intersect before they reach line AH. This in fact is a telling misrepresentation of the analysis. Both pairs of lines should have been shown to converge before reaching AH, the result being that the order of intersections on AH should be H, L, K, O, as represented in figure 7.7.66a, p. 205, rather than L, H, O, K, as represented in figure 7.7.66.

In order to understand the implications of this misordering of points, let us start with a brief look at figure 7.5.54, p. 190, which accompanies proposition 9, pp. 291-292. According to the analysis in that proposition, the entire medium from G through the extension of the normal from D beyond is homogeneous and denser than that between G and center of sight A. E is the point of refraction at which angle of incidence BEZ and angle of refraction/deviation AEL form the limiting pair for the medium filling the circle and the space behind it. That being the case, no point on the arc at or below E toward K can serve as a point of refraction for light passing to the refracting interface from B, whereas any point on arc GE can serve as a point at which the light passing from some point on normal BD will be refracted to A, so virtually the entire arc GE will be the locus of refraction for an indefinite number of points on normal BD.

Now let us adapt this analysis to the current case according to figure 7.7.66a, p. 205, which represents the analysis correctly, and let us follow the same lettering as in figure 7.7.66, p. 204, while continuing normals ET and EG to points Y and X, respectively, and continuing lines LT and HG to points S and P, respectively. Let LTE and STA be a pair of angles of incidence and refraction/deviation according to Alhacen's stipulation for figure 7.5.54. In order to avoid confusion during the rest of this analysis, I will designate the angle of refraction according to modern usage, i.e., with respect to the normal, rather than according to Alhacen's usage, in which

the angle of refraction is actually the angle of deviation. Thus, if the medium occupied by L is of the same density as that contained in the sphere, the light from L will radiate straight through Z to T and will strike the surface of the sphere at angle of incidence LTE. It will then refract to A away from normal ETY at angle ATY. Under the same conditions, the light from H will radiate straight through M to G at angle of incidence HGE and will refract to A away from normal EGX at angle AGX. During this radiation, rays LT and HG will intersect to the left of normal AH and outside circle BZD. If we switch the direction of radiation so that A constitutes the source, the light passing along AT will strike the sphere at angle of incidence ATY, refract toward normal ET at angle ETL, and continue through point Z to L, whereas the light passing along AG will strike the sphere at angle of incidence AGX, refract toward normal EG at angle EGH, and continue through point M to H. Now let L and H lie in the same medium as A so that DMZ is a refracting interface. In that case the light passing from A along AT will strike the sphere at angle of incidence ATY and refract toward normal ET along TZ at angle ETZ, as before, and the light along AG will strike the sphere at angle of incidence AGX and refract toward normal EG along GM at angle EGH, as before. When it reaches Z, however, the light radiating along TZ will strike the sphere's surface at angle of incidence EZT and will refract away from normal EZF along ZO at angle OZF. Since angle ETZ of refraction at point T is equal to angle of incidence EZT at point Z, then according to the principle of reciprocity angle of refraction OZF formed by the light originating at A and exiting the sphere from Z is equal to angle of refraction ATY formed by the light originating at L and exiting the sphere at T. Applied to the light passing along AG, the same line of reasoning dictates that angle of refraction KMC for the light originating at A and exiting the sphere at M be equal to angle of refraction AGX for the light originating at H and exiting the sphere at G. Hence, as far as the relevant angles are concerned, the passage of light from A to L is perfectly symmetrical with the passage of light from L to A, and the same holds for the passage of light from A to K and from K to A.

From this analysis it is evident that, if T is the limiting point of refraction for the medium in the sphere, no point between O and D can radiate its form to A because Z will be the limiting point on the other side, leaving angle OZF as the limiting angle of incidence. Hence, O is the limit at which the *inner* end of the object line can be seen from A, the rest of the points necessarily extending beyond it in the direction of H. In figure 7.7.66, on the other hand, O represents the *outer* limit of radiation through the sphere and thus the outer limit of that object line because, according to the model represented in that figure, any point beyond L must radiate its form to some point beyond Z on arc ZT; otherwise the line of radiation will cross OZ. Hence, the angle of incidence at that point will be greater than angle of incidence OZF, so no refraction can occur. Moreover, according to figure 7.7.66, virtually all the points between O and D can radiate through the sphere to A, whereas none of them can do so according to figure 7.7.66a. We have also seen in the discussion of proposition 9 that there is an outer limit of radiation from axis AD beyond the sphere, at which point the light will not refract to A if the entire medium behind BT is homogeneous (see note 126, p. 378). It therefore follows that there is an outer limit along that axis when the sphere is suspended

in the rarer medium. In other words, there is some point K' beyond K at which the light striking very close to D will not reach A after being refracted into and out of the sphere. One last point is evident from figure 7.7.66a and the analysis based on it: because rays OZ and KM cross one another, the image of KO will be inverted from A's point of view. Oddly enough, Alhacen ignores this point in the proposition.

Thanks to Roshdi Rashed, we know that, unlike its counterpart in the Latin version of proposition 17, the figure that accompanies the Arabic version represents the analysis correctly, with the equivalents of line-pairs ZL and MH and ZO and MK intersecting as they should (see Rashed, *Géométrie et dioptrique*, pp. 105-106). Rashed goes on to discuss various technical implications of the analysis in proposition 17 with regard to focal properties and spherical aberration, but these implications are drawn in the context not only of the correct version of the diagram but also of Ibn al-Haytham's analysis of focusing in the "Treatise on Burning Spheres," neither of which was available in the Latin West. For further discussion of these implications and their possible bearing on the development of lens theory in the Latin West, see pp. lxx-lxxv of the Introduction.

[168]In doing the experiment with a wax sphere, as Alhacen suggests, one sees not only the annular image but also an image of the sphere in the center of that ring along the normal. Moreover, one cannot help but notice that, when the wax sphere is brought fairly close to the back surface of the transparent sphere, the space between the central image and the annular one around it is quite large, whereas the farther the object is drawn away, the smaller that space becomes until the ring and the image at its center begin to coalesce. This experiment would be more effective if the object approximated a line in order to reflect the case represented in proposition 17. To that end, one might use a short piece of wire or a thin strip of metal, placing it lengthwise along the axial line through the center of the eye and the center of the transparent sphere. That way the central image would be minimized while the two outer images would be clearly visible.

[169]Although all the manuscripts have KZ instead of KB, I have followed Risner in choosing the latter because it makes more sense. Since KB is perpendicular to the lines of longitude passing through points B and D, and therefore perpendicular to the refracting interfaces on both sides of the cylinder, then within the plane of refraction defined by those lines of longitude it is impossible for the form of any point on KO to be doubly refracted to the center of sight at A. Thus, as illustrated in figure 7.7.66b, p. 206, within the plane of refraction bounded by lines of longitude DL and BM, the form of K will necessarily strike line of longitude DL at a slant, except when it passes perpendicularly along axis AK. When it strikes point L, it will then refract toward the normal along LM, and finally refract away from the normal along MN. The same holds for any point on KO; all rays from it, except the one passing along the axis, will strike the line of longitude on the cylinder's surface at a slant, so after double refraction the resulting ray will incline away from center of sight A. By extension, moreover, every ray from K or any other point on KO that reaches the cylinder's surface above or below the plane of circle DB will strike the cylinder's surface at a slant and will therefore incline away from

A after double refraction through the cylinder. Altogether, then, it is impossible for any point on KO to refract to A within any plane other than that of circle BD. So endpoint O of line KO will be the only point on KO visible to A, and it will be seen along the axis because it is unrefracted.

[170]The Latin phrase *quarum altera reflectitur ad alteram* is at best cryptic. Presumably *altera* refers to either of the two forms, and if *alteram* is presumed on that basis also to refer to either of the two forms, then the phrase implies that one form is refracted onto another. This makes no sense whatsoever, so I take the intent of this passage to be that each of the two forms is refracted respectively on each of the two corresponding arcs on the surface of the cylinder facing the eye.

[171]The atmospheric model Alhacen proposes here emphasizes that atmospheric refraction occurs at the interface between the heavens, which consists of aither, and the sphere of fire below it, not at the interface between fire and air or anywhere else in the atmosphere itself. But if, as Alhacen claims, the atmosphere becomes increasingly dense as it approaches the Earth's surface, it stands to reason that, instead of passing straight through after refraction at the aither-fire interface, the incoming light from celestial objects will be refracted along a curved path continually verging toward the normal in its approach to the Earth. Interestingly enough, although Alhacen ignored this implication of his atmospheric model, the fourteenth-century scholastic thinker, Nicole Oresme, took it seriously and elaborated on it in a brief treatise entitled *On Seeing the Stars*, which deals with the effect of parallax and atmospheric refraction on celestial observation. For details, see Dan Burton, ed. and trans., *Nicole Oresme's* De visione stellarum (On Seeing the Stars) (Leiden/Boston: Brill, 2007), pp. 149-171. Oresme in fact quotes Alhacen to justify his atmospheric model: *Et hoc est quod dicit Alhacen in septimo capitulo septimo, quod scilicet "aer, quanto magis appropinquat celo, tanto magis purificatur donec fiat ignis. Ergo eius subtilitas sit ordinate secundum successionem, et non in differentia determinata"* (ibid., 150, lines 9-12) Aside from two minor changes in word order from *quanto magis appropinquat aer celo* to *aer quanto magis appropinquat celo* and from *subtilitas ergo eius* to *ergo eius subtilitas*, along with the addition of *et* before *non in differentia*, this quotation mirrors verbatim the passage in paragraph 7.43, lines 42-44, on p. 130 of the critical Latin text. It is worth noting that none of the manuscripts compiled for that text, nor Risner's edition, has these three variants found in Oresme's version of the passage.

[172]This "not" is missing in all the manuscripts, as well as Risner's edition, but the passage makes no sense without it because the analysis just before it claims explicitly that the light from heavenly bodies is refracted at the aither-fire interface. I have therefore added "not" here and in the Latin text to reflect what I think the passage was intended to say.

[173]See book 3, chapter 7, paragraph 7.23, in Smith, *Alhacen's Theory*, pp. 604-605.

[174]In other words, as shown in proposition 14, pp. 313-314, above, the farther off to the side of the center of sight an object lies in a denser medium, the more pronounced the refraction of its form and, therefore, the more pronounced the displacement of its image. By the principle of reciprocity, the same applies when the object lies in a rarer medium, such as the heavens, and the eye lies in a denser

medium, such as the atmosphere. In addition, the more pronounced the refraction, the more the object's refracted form is weakened, so the object's apparent distance is magnified, which magnifies its apparent size.

[175]See chapter 4, paragraphs 4.28-4.30, pp. 270-272.

[176]Suffice it to say that because of the scale of the figure the refraction of rays KM and KZ at points M and Z is represented in a grossly exaggerated way, as if the atmospheric shell surrounding the Earth were far denser than the aither above it.

[177]See book 2, chapter 3, paragraphs 3.76-3.93, in Smith, *Alhacen's Theory*, pp. 451-457.

[178]The specification "below the zenith and the horizon" (*infra verticem capitis et orizonta*) is peculiar; indeed peculiar enough that Risner decided to omit it. I take it to distinguish the special case in which the star or interval actually lies below the horizon but is seen above it on account of atmospheric refraction. Whatever the particular conditions specified, however, the key criterion for this proposition is eminently clear: i.e., that the star or interval lie below the zenith.

[179]That is, since they lie closer to vertex B of spherical triangle BDE than do E and D, F and N lie on a narrower portion of that triangle.

[180]The claim that A is virtually the center of circles BD and BE is based on comparing AM, the radius of the Earth, to the radius of the celestial sphere, which is measured by lines MT, MK, MF, and MN. According to the Ptolemaic model, the Earth is as a point in comparison to the celestial sphere, so its radius is as nothing in comparison to the radius of the celestial sphere. For all practical purposes, then, we can take point A on the Earth's surface as the centerpoint of the universe.

[181]In note 92, pp. 368-370, we saw that Alhacen's analysis of stellar and lunar refraction in chapter 4 leaves no clear idea of how high above the Earth he thought the shell of air and fire extends. Here we have a rough clue: it extends high enough that angle AZM in figure 7.7.69, p. 209, is tiny (*valde parvus*). Assume that it extends all the way or almost all the way to the moon. Ptolemy calculated the Earth-moon distance at perigee to be 33 Earth radii (see Albert van Helden, *Measuring the Universe* [Chicago: University of Chicago, 1985], p. 27), so let us assume that the atmosphere extends that high. Accordingly, the Earth's radius AM in figure 7.7.69 is to the closest possible Earth-moon distance ZM as 1:33, which means that angle AZM is around 1.7°. Furthermore, as Alhacen measures it, the angle of refraction/deviation is always less than half the angle of incidence. Hence, angle AZM + angle of refraction/refraction AZY can be at most 2.55° and in fact considerably less, given the slight density differential between aither and pure fire at the interface between the heavens and the atmosphere.

[182]Alhacen's argument here hinges on the virtual equality of AD and AE. Accordingly, let them be combined into line AD/AE in figure 7.7.69a, p. 210, and let angles AZE and AHD be posed upon that line to form triangles AHD and AZE. Let the circle centered on C′ circumscribe triangle AHD, and let the circle centered on C circumscribe triangle AZE. Combined line AD/AE will thus form a chord on each circle, and since the circle centered on C is smaller than the circle centered on C′, angle ACE subtended by arc AZE is greater than angle AC′D subtended by arc AHD, which is to say that arc AZE > arc AHD. Now suppose that HD = ZE. H will thus lie below Z and to the left of AZ, so clearly angle HAD subtended by that

line, which is a chord on circle AHD, will be smaller than angle ZAE subtended by ZE, which is a chord on circle AZE, and since HD < ZE, angle HAD will be even smaller relative to angle ZAE.

[183]This claim is based on the assumption that the diminution in the apparent size of the celestial body caused by refraction is constant throughout, an assumption that is perhaps reinforced by the demonstration in proposition 19, where it is concluded in paragraph 7.50, pp. 322-323, that, if the actual diameter KL of the star or interval in figure 7.7.67, p. 207, is rotated about axis TB, it will form a circle, as (by implication) will its apparent diameter QR. While this is true if axis TB, which bisects the two diameters, passes through the zenith, it is not true if it reaches any point between the zenith and horizon because in that case the vertical diameter will be shorter than the horizontal one so that the image will be somewhat flattened. The reason is that the light from the endpoint closer to the horizon will be refracted more severely than the light from the endpoint closer to the zenith. This is in fact implied in proposition 21, paragraph 7.59, pp. 324-325, where it is demonstrated that angle AHD in figure 7.7.69, p. 209, is greater than angle AZE, which means that angle of refraction/deviation AHX (according to Alhacen's usage) is less than angle of refraction/deviation AZY. Thus, angle of refraction AHM (according to modern usage) is less than angle of refraction AZM. Kepler in fact makes this argument in *Paralipomena*, chapter 4, section 7, to show that heavenly bodies do not appear perfectly round at horizon; see Donahue, *Kepler*, p. 144.

[184]For a brief account of Alhacen's theory of distance and size perception, along with the pertinent textual loci, see the introduction to Smith, *Alhacen's Theory*, pp. lxiv-lxvi. The gist of that theory is that we learn to gauge distance according to bodily measures, such as an arm span or pace, and work our way outward from our immediate surroundings by increments, thus gaining a determinate impression of ever-longer distances. For fairly long distances we need clues along the way, such as a succession of trees, but for the most part the succession of bodies according to which we gauge distances consists of portions of the ground. Accordingly, our ability to determine distances is entirely earthbound and extends only as far as the horizon, at which point our determination of distance becomes increasingly vague, yet still dependent on landmarks, such as mountains. Without any such clues, we are left to guess at the distance, which we can only do according to an estimate of what size the object seen ought to be according to our knowledge of the normal size of such things and a correlation of that estimated size to the visual angle it subtends.

[185]The Latin term I have translated as "bodily mass" is *corporeitas*. According to Alhacen, we perceive *corporeitas* in three dimensions, and since we grasp the celestial sphere according to length and breadth only, we are constrained to perceive it as a plane rather than as a body; see Smith, *Alhacen's Theory*, pp. 469-471, for Alhacen's actual account.

[186]The color term *glaucus*, which I have translated as "bluish," has a rather wide range of meanings that includes bright and shiny, grey, blue-green, and even green. In choosing this color over black (*niger*), Alhacen apparently meant to designate the hue of the sky during daylight rather than at night, although in that case he (or his translator) might have used the less equivocal term *ceruleus*.

[187]For Alhacen's full account of how we perceive convexity and concavity, see book 2, chapter 3, paragraphs 3.129-3.132, in Smith, *Alhacen's Theory*, pp. 472-473.

[188]In other words, although the color of the object is apprehended immediately at the level of pure sensation, its shape—if it is apprehended at all—is apprehended inferentially and therefore mediately. In this case, of course, the shape of the sky cannot be apprehended because of its vast distance from the viewer on Earth.

[189]As pointed out earlier, Alhacen supposes that for the most part we learn to gauge distances according to incremental portions of the ground ahead of us, and from that we learn to gauge them according to the angle formed by the normal through our standing bodies and the line of sight to some point on the ground ahead of us. The closer that point lies to us, then, the more acute the angle, and conversely the farther that point lies from us, the less acute the angle. Consequently, if we see two objects under the same visual angle, and if the angle under which the distance of one is seen is less acute than the angle under which the distance of the other is seen, the one seen under the larger angle of distance will appear larger.

[190]Technically, the angle under which a given celestial body or interval between celestial bodies is seen will be somewhat different at the zenith than at the horizon because the viewpoint of an observer standing on the Earth is not at the center of the universe but eccentric to it by the amount of the Earth's radius. Hence, the observer's line of sight to the zenith will be shorter than his line of sight to the horizon by that amount, which means that the celestial body or interval will be somewhat closer at the zenith than at the horizon and will thus appear commensurately larger. As we have already seen, however, the Earth is taken by Alhacen to be so small in relation to the heavenly sphere that the eccentricity of viewpoint caused by the Earth's radius is nugatory, even if we are looking at the moon or sun, which lie far closer to us than the stars. A second, more significant factor is that a given vertical distance in the heavenly vault will appear smaller than an equal horizontal one because, as pointed out in note 183, p. 393 above, the refraction of the light at the endpoint nearer the horizon will be more severe than that of the endpoint nearer the zenith. Thus, the closer to the horizon the celestial body is, the more oblate its image will be and, therefore, the smaller it will appear in overall area. If the atmosphere were completely homogeneous and of the rarity of clear air, this distortion would be imperceptible no matter how close the body was to the horizon. In reality, however, the sun and moon can appear quite oblate at the horizon because of pressure and temperature gradients in the atmosphere.

[191]See chapter 4, paragraphs 4.28-29, pp. 270-271. According to the account given there, an armillary sphere is used to measure the angular distance of a given star from the north pole when the star is at the horizon and at the zenith. As it turns out, there is a disparity between the two measurements, but, being around 13′, it is minimal.

[192]See book 3, chapter 7, paragraphs 7.13-21, in Smith, *Alhacen's Theory*, pp. 603-604.

[193]Alhacen supposes that we perceive the heavenly vault as a sort of ceiling that appears to meet the Earth's horizon, and since we perceive that ceiling as

parallel to the plane of our horizon on the Earth's surface, we gauge distances on it in precisely the same way we gauge distances on the Earth's surface. Accordingly, we take the zenith point to lie closest to us and therefore assume that the farther a star lies away from that point in arcal distance, the farther away it lies in horizontal linear distance until, finally, its linear distance is maximum at the horizon. By correlating visual angle, which remains virtually constant, to apparent distance, which varies significantly, we are led to perceive the star as much larger at the horizon than at the zenith. Thus, as illustrated in figure 7.7.70, p. 210, when the moon is seen at position 1, it is perceived under the same angle as it is throughout positions 2-5, but at each position it is perceived to lie on a straight line tangent to the hemispherical vault of the sky at position 1. Consequently, because of the continuing increase in its apparent distance from the viewpoint along that line, the moon will appear ever larger as projected upon the straight line between positions 2 and 5 . One obvious problem with this model, if taken quantitatively rather than qualitatively, is that at horizon the moon should be perceived as filling almost the entire sky, since the line passing over the top of the moon to form the visual angle at that position will meet the straight line along which the moon appears to travel at an inordinate distance from the viewpoint. By the eighteenth century it was commonly assumed that the sky is perceived as a flattened dome and, therefore, that the variation in perceived size is not nearly as great as Alhacen's model implies. Thus, as illustrated in figure 7.7.70, if the moon is perceived to move on the elliptical line, it appears perhaps twice as large at the horizon, not immeasurably larger as it should according to Alhacen's model. In the third book of his *Optics*, Ptolemy acknowledges that the apparent enlargement of heavenly bodies at the horizon is psychological, but he offers no real explanation; see Smith, *Ptolemy's Theory*, p. 151. What sets Alhacen apart from Ptolemy, then, is his effort to provide a fully coherent explanation of the phenomenon based on a systematic theory of perceptual psychology. As far as we know, Alhacen's account is original, but interestingly enough he makes no claim to originality in this case, whereas he did not hesitate to make that claim in at least two others; see book 6, chapter 4, paragraph 4.71, in Smith, *Alhacen on Image-Formation*, p. 175, and book 7, chapter 6, paragraph 6.33, p. 307 above. The Moon Illusion has not yet been satisfactorily explained, although both Alhacen's model and the flattened dome alternative have been discredited in fairly recent times. For a relatively current account of the Moon Illusion in all its aspects, see Helen E. Ross and Cornelis Plug, *The Mystery of the Moon Illusion* (Oxford: Oxford University Press, 2002).

[194]The analysis in this paragraph was inspired by Ptolemy's claim in *Almagest*, I, 3, that the stars appear larger at the horizon because of vapor rising from the Earth and intervening between the stars and the observer on the Earth's surface. Alhacen's account is thus a vindication, or at least a partial vindication of that claim. The model upon which Alhacen seems to be basing his account is illustrated in figure 7.7.71, p. 211. Rather than envelop the entire Earth, thick vapors rising into the atmosphere from the Earth's surface coalesce in a band that encircles the Earth and forms a spherical segment between the Earth and the juncture between the heavens and the topmost surface of the atmosphere. How far above the Earth's surface this band of vapor lies and how thick it is are left unspecified, as is

its upward extent toward the north pole. It is clear, however, that it must extend far enough toward the north pole to reach somewhat, but not too far above the horizon of a viewer in the northern latitudes—at Cairo, for instance—so as to affect observations at or near the horizon without affecting observations in mid-sky.

Alhacen's explanation of how such vapor causes heavenly bodies to appear magnified near the horizon is illustrated in figure 7.7.72, p. 211, where E represents the observer's viewpoint on the Earth's surface, C the Earth's center, EA′ the viewer's horizon, Z his zenith, AE and BE rays from the endpoints of the vertical diameter on a heavenly body's form after atmospheric refraction, and the thick grey arc the band of vapor encircling the Earth between E and the concave interface between the aither filling the heavens and the atmospheric shell surrounding the Earth. If there were no vapor, diameter AB of the star's image after refraction at that interface would be seen directly under angle AEB. With the vapor present, however, ray AE will strike the outer surface of the vapor at R to be refracted toward normal NC, after which it will be refracted away from the normal at R1 to reach the center of sight along R1E. Accordingly, point A will be seen along horizontal line ER1A′. Likewise, ray BE will be refracted toward normal N′C at R′, after which it will be refracted away from the normal at R1′ to reach E along R1′E, so point B will be seen along ER1′B′. Diameter AB will thus be seen under angle A′EB′. That angle A′EB′ > angle AEB follows from the fact that angle of incidence ARN > angle of incidence BR′N′ because AR strikes the outer surface of the vapor at a more oblique angle than does BR′. Consequently, AR is is more intensely refracted than BR′, from which it follows that angle BEB′ < angle AEA′, so angle A′EB′ > angle AEB. Because the arcs subtended by all the heavenly bodies are small, the largest being slightly over 30′ (the arc subtended by the moon and sun), the disparity in angles will be negligible, so magnification by vapor contributes almost nothing to the apparent enlargement of heavenly bodies at the horizon. Alhacen 's claim that the inner surface of the wall of vapor facing the eye must be plane is somewhat puzzling, unless he means by it that the form of any heavenly body on it is so small that the surface on which it lies is virtually plane and can therefore be treated as actually so. See A. I. Sabra, ed. and trans., "On Seeing the Stars II," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 10 (1995/96): 1-59, for a more elaborate and illuminating treatment of vapor as a cause of magnification, which Alhacen included in a set of "doubts" concerning Ptolemy's *Almagest* and which he composed after the *De aspectibus*; see esp. ibid., p. 32 and pp. 41-43. See also A. I. Sabra, "Psychology versus mathematics: Ptolemy and Alhazen on the Moon Illusion," in Edward Grant and John Murdoch, eds., *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages* (Cambridge: Cambridge University Press, 1987), pp. 217-247, esp. pp. 227-236. For Kepler's argument against the vapor-based account of stellar magnification, see *Paralipomena*, chapter 4, section 7, in Donahue, *Kepler*, pp. 144-146.

[195]What Alhacen means here is that at some locations on the Earth thick vapor is always present above the horizon, whereas at others it is sometimes present and sometimes not, which implies that the band of vapor is variable in its northward extent. Moreover, the observer may be at a latitude far enough north that the band of vapor never reaches his horizon, in which case the magnification will never occur; see Sabra, "On Seeing the Stars," pp. 44-46. Unlike the magnification

of heavenly bodies by vapor, their apparent enlargment at horizon on account of our misperceiving the celestial vault as a ceiling is constant (*perpetua*) insofar as it depends on no extraneous factors and thus occurs anywhere we stand on the Earth's surface. For an English translation from the Arabic text of the passage beginning with "Furthermore, we have said that every star lying at the zenith," in paragraph 7.62, p. 325, and ending with the sentence concluded by this note in paragraph 7.74, see Sabra, "Psychology versus mathematics," pp. 237-243.

FIGURES FOR INTRODUCTION AND LATIN TEXT

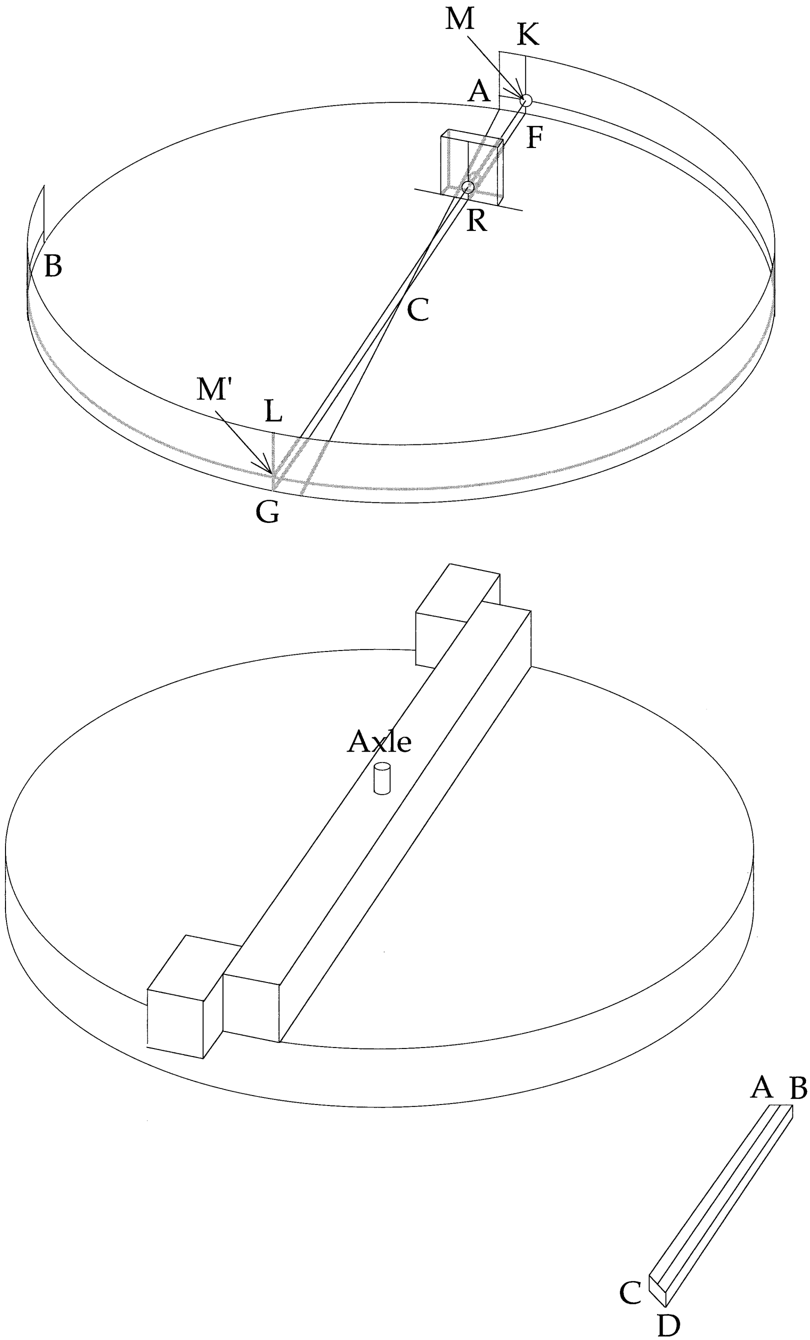


figure 1

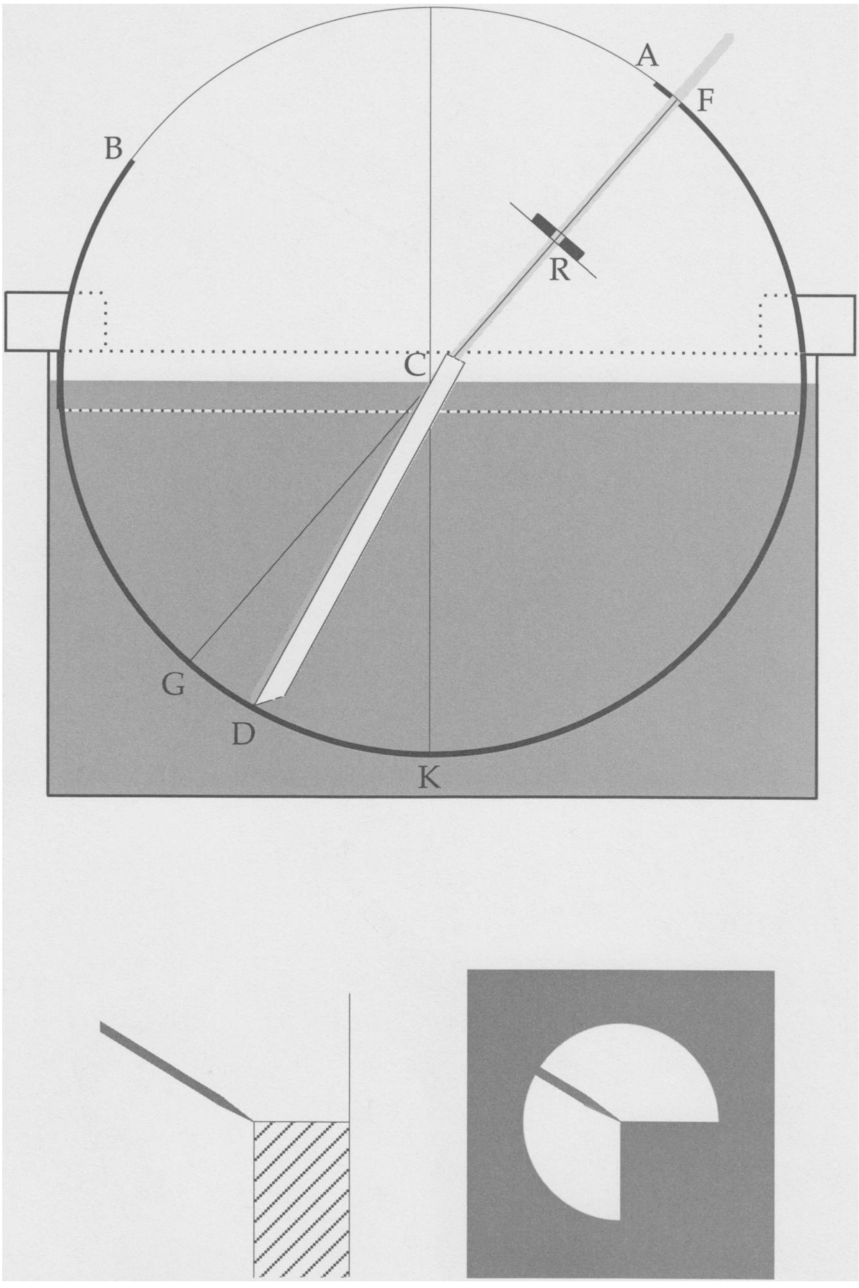


figure 2

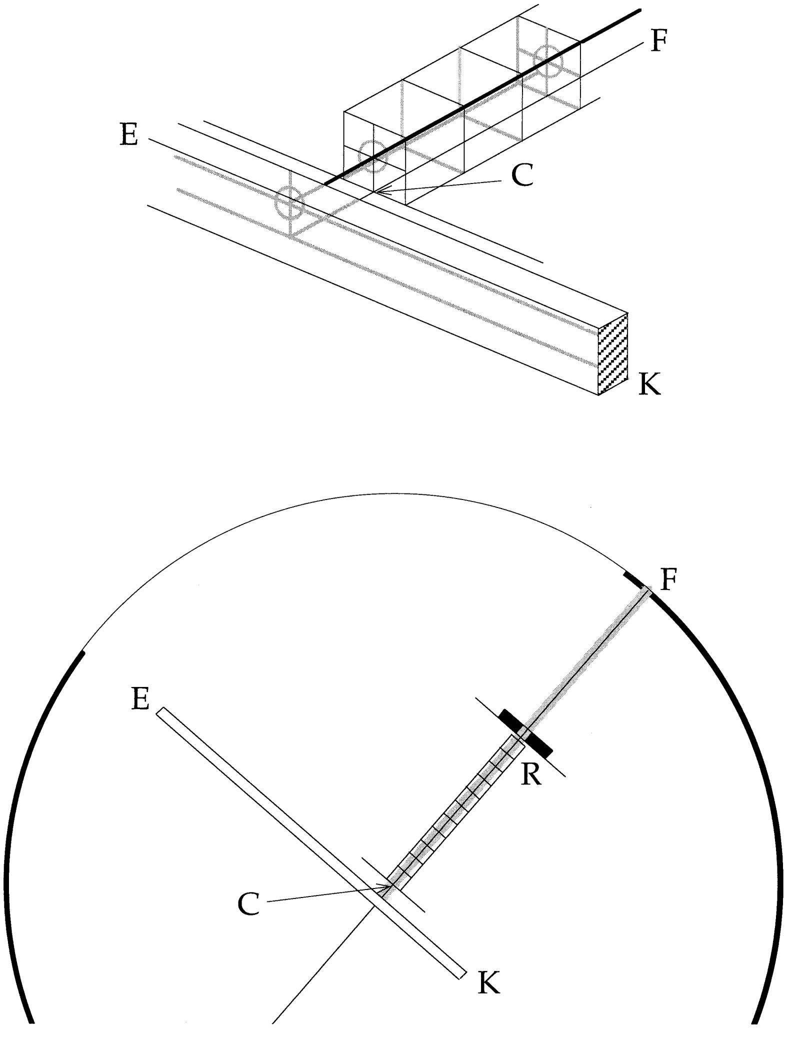


figure 3

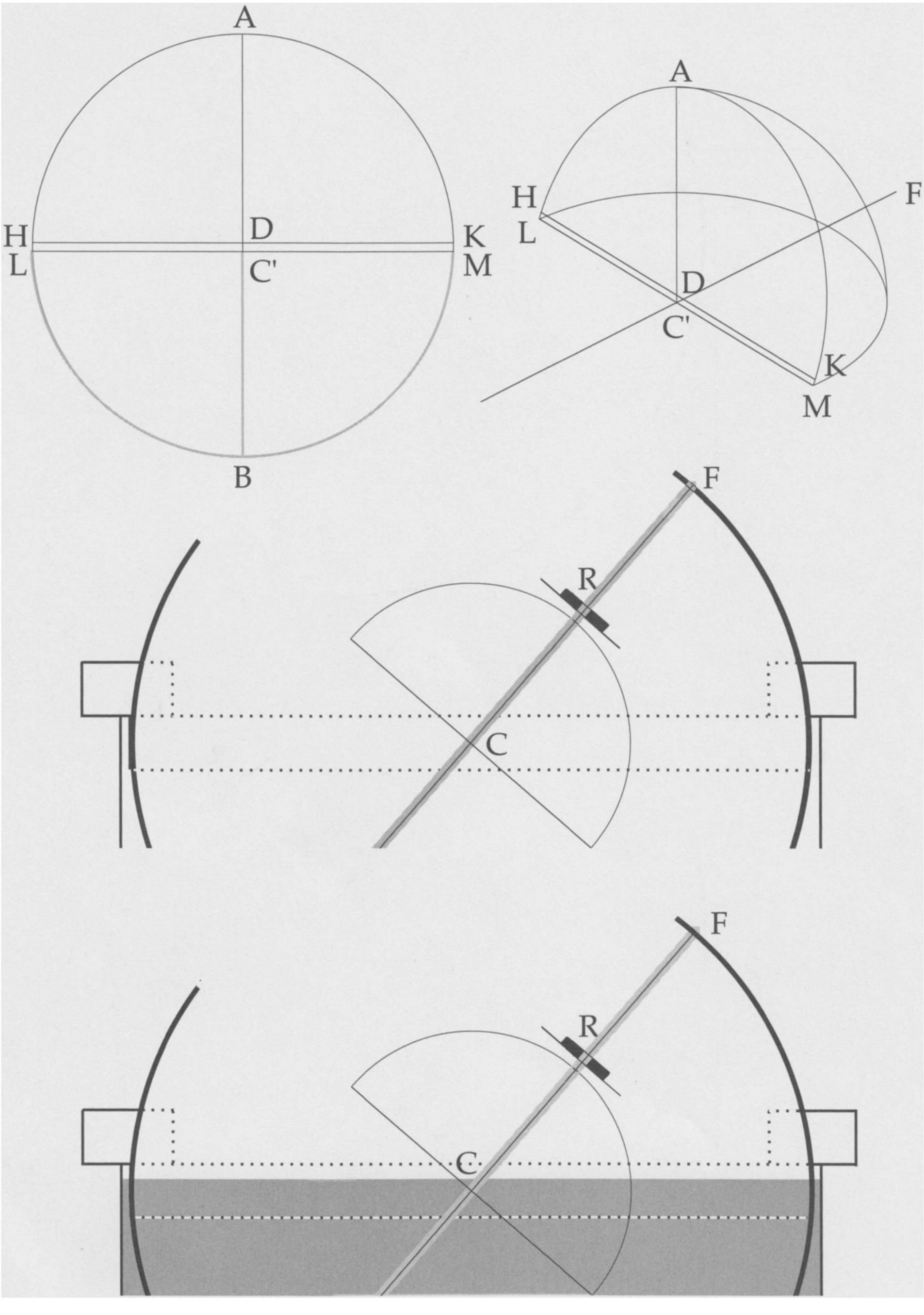


figure 4

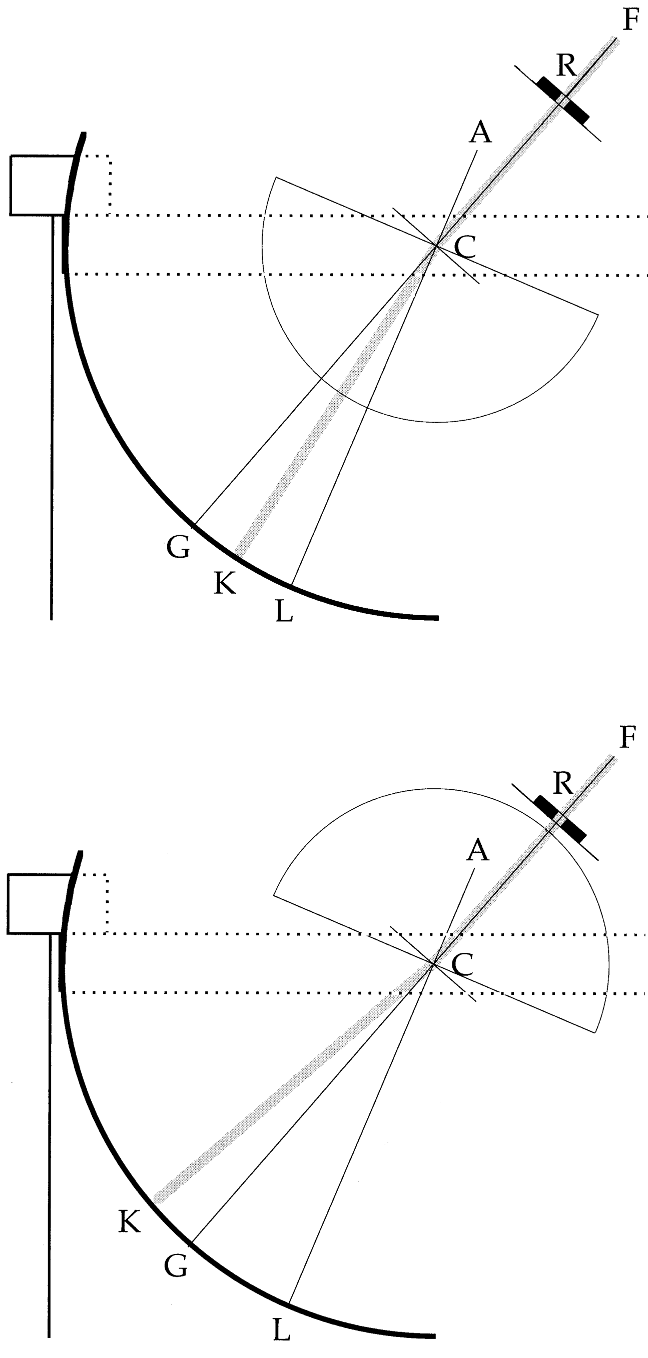


figure 5

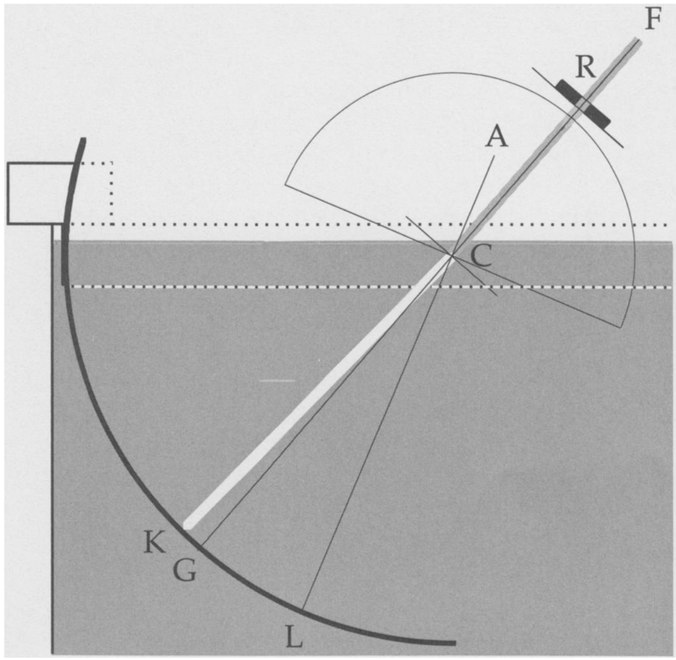


figure 6

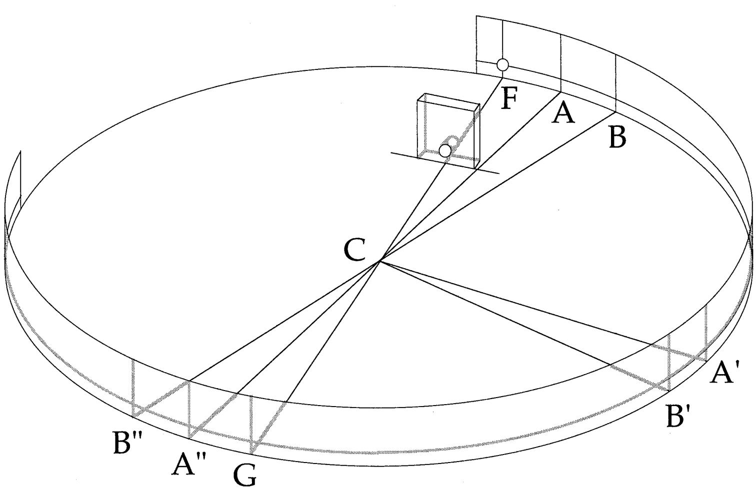


figure 7

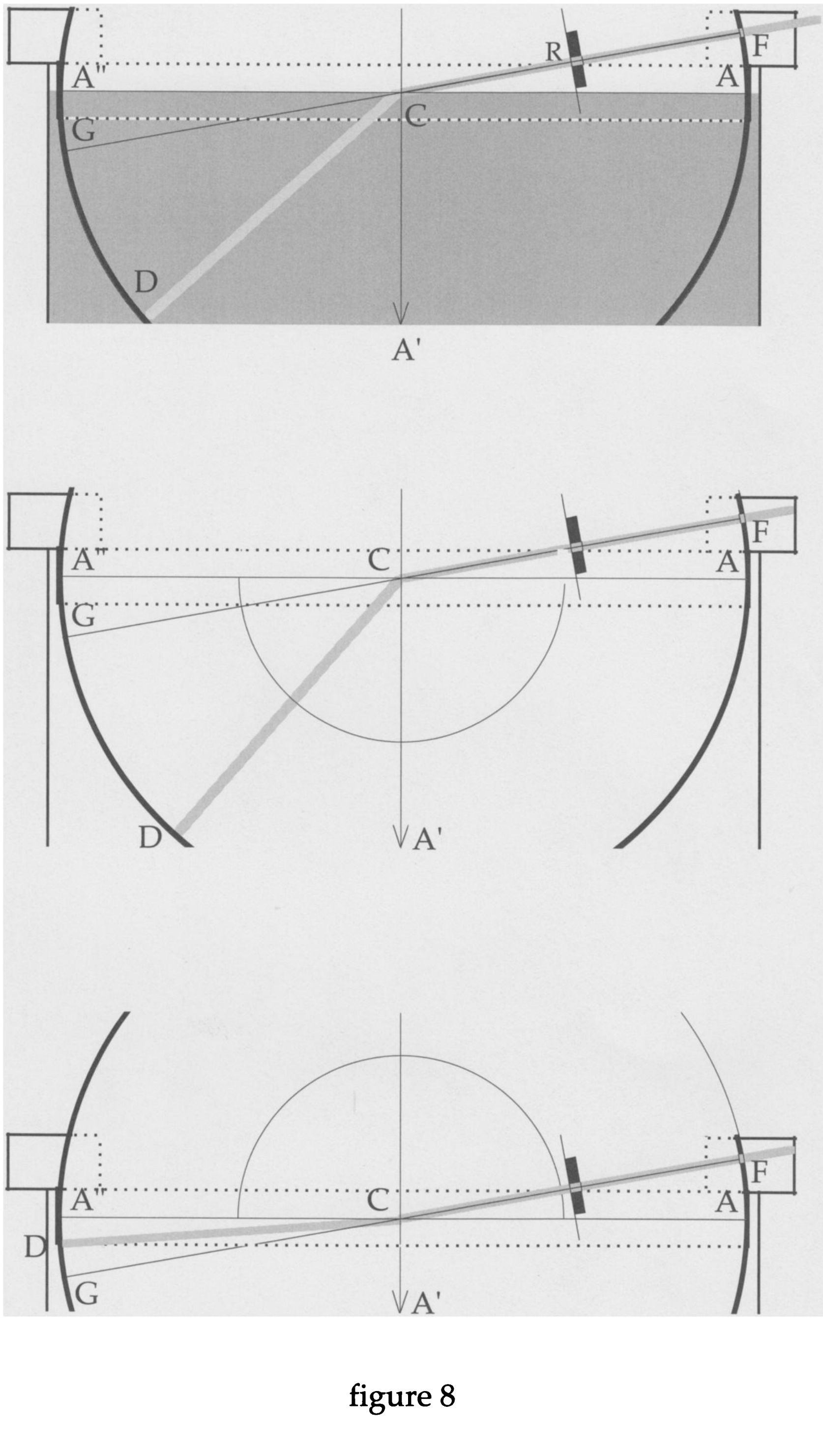


figure 8

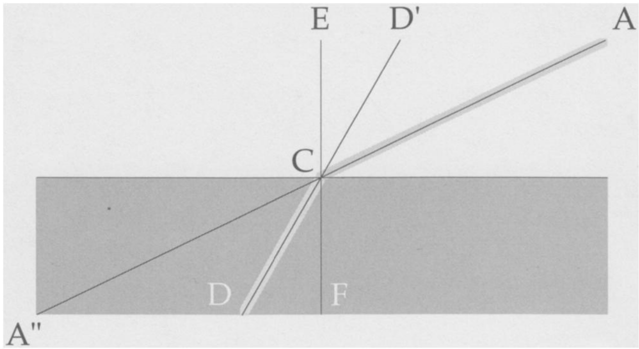


figure 9

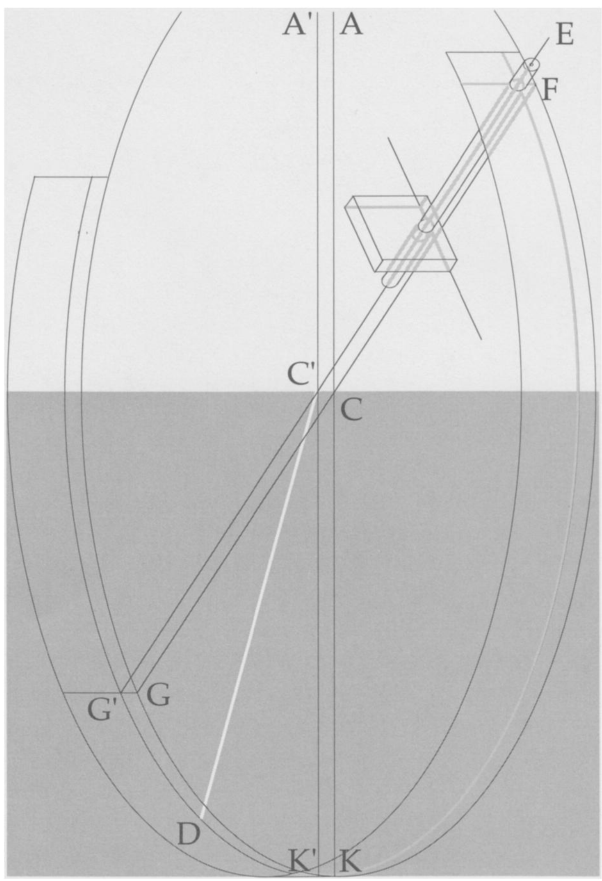


figure 10

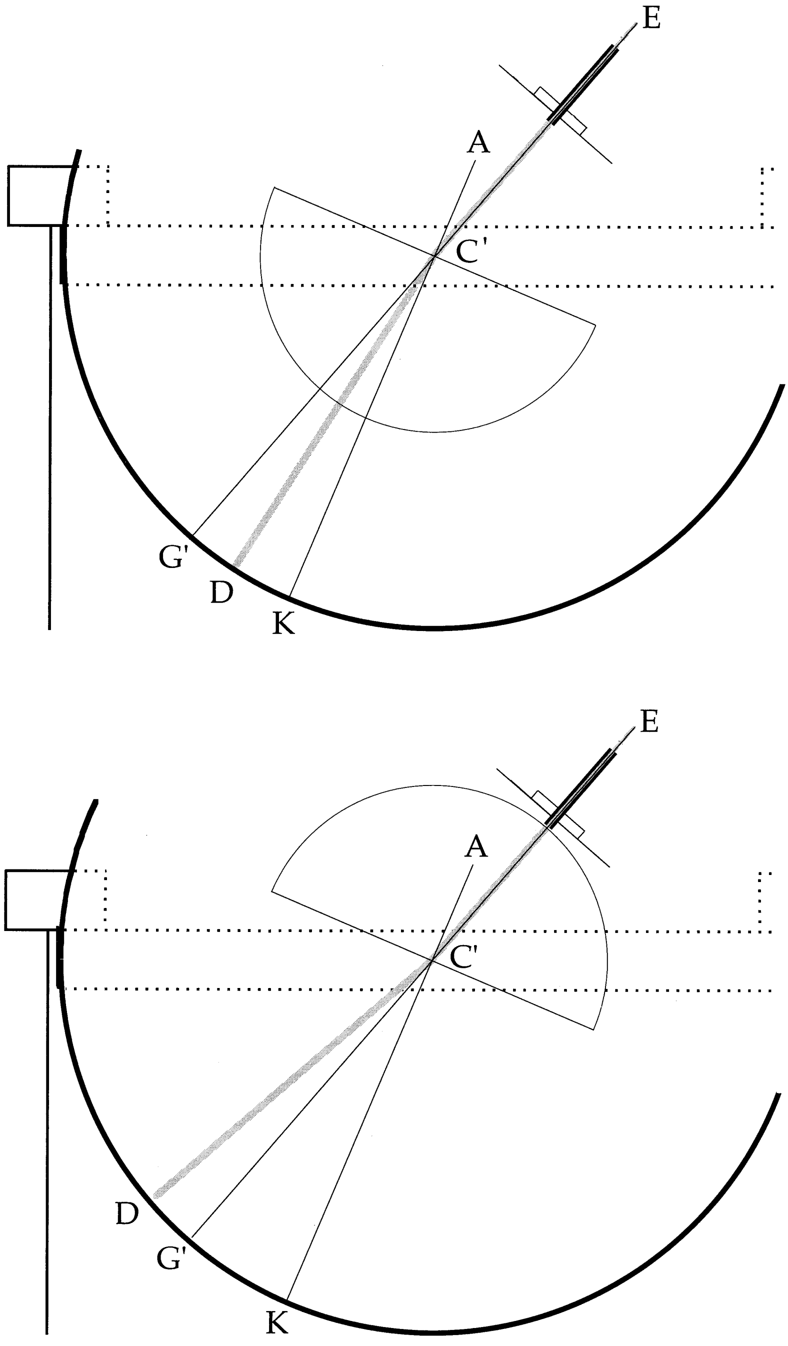


figure 11

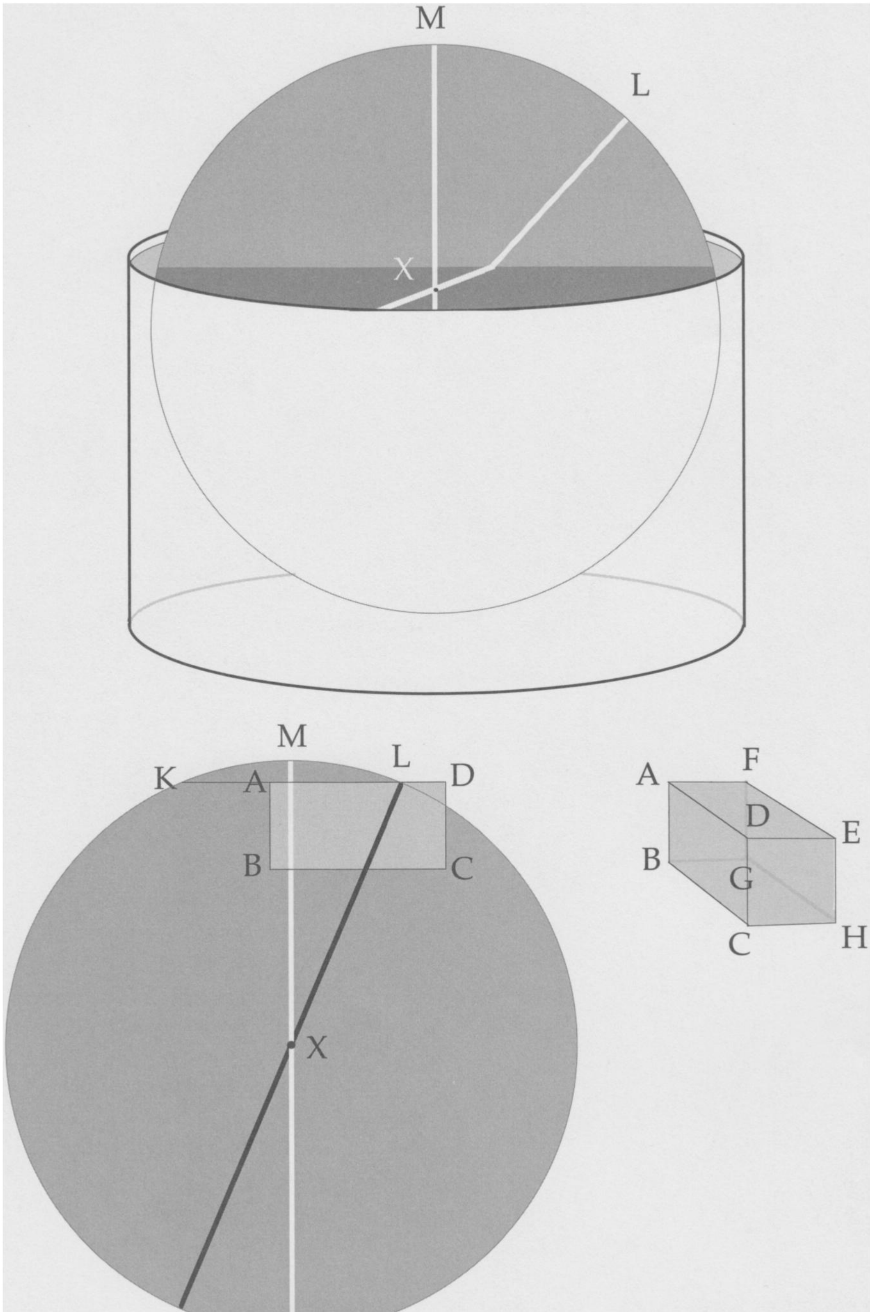


figure 12

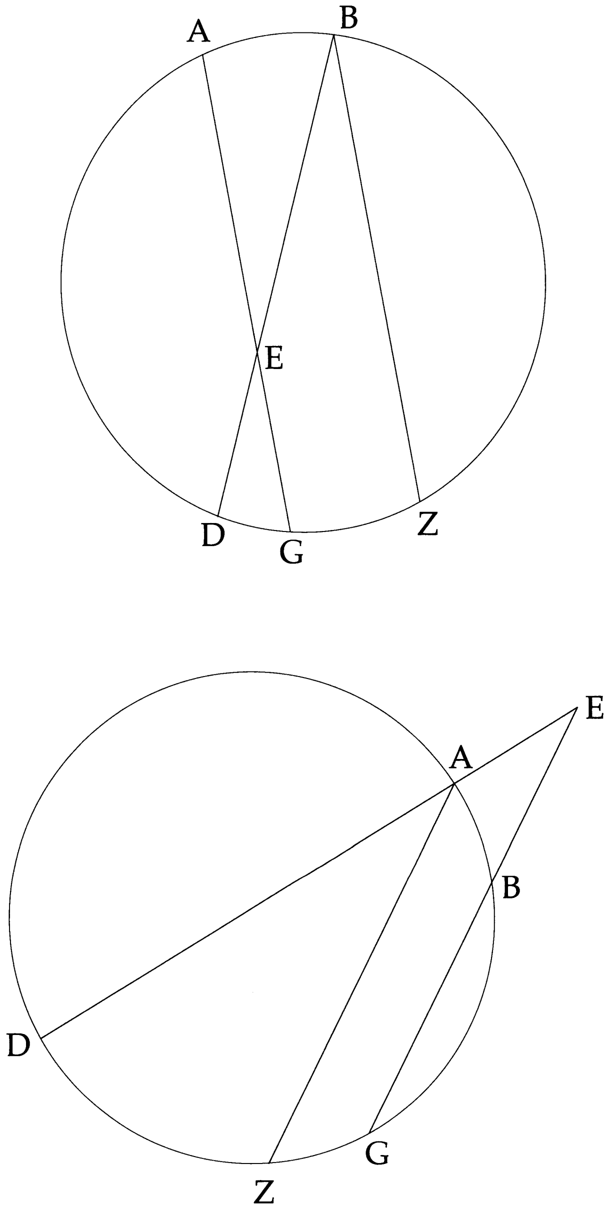


figure 13

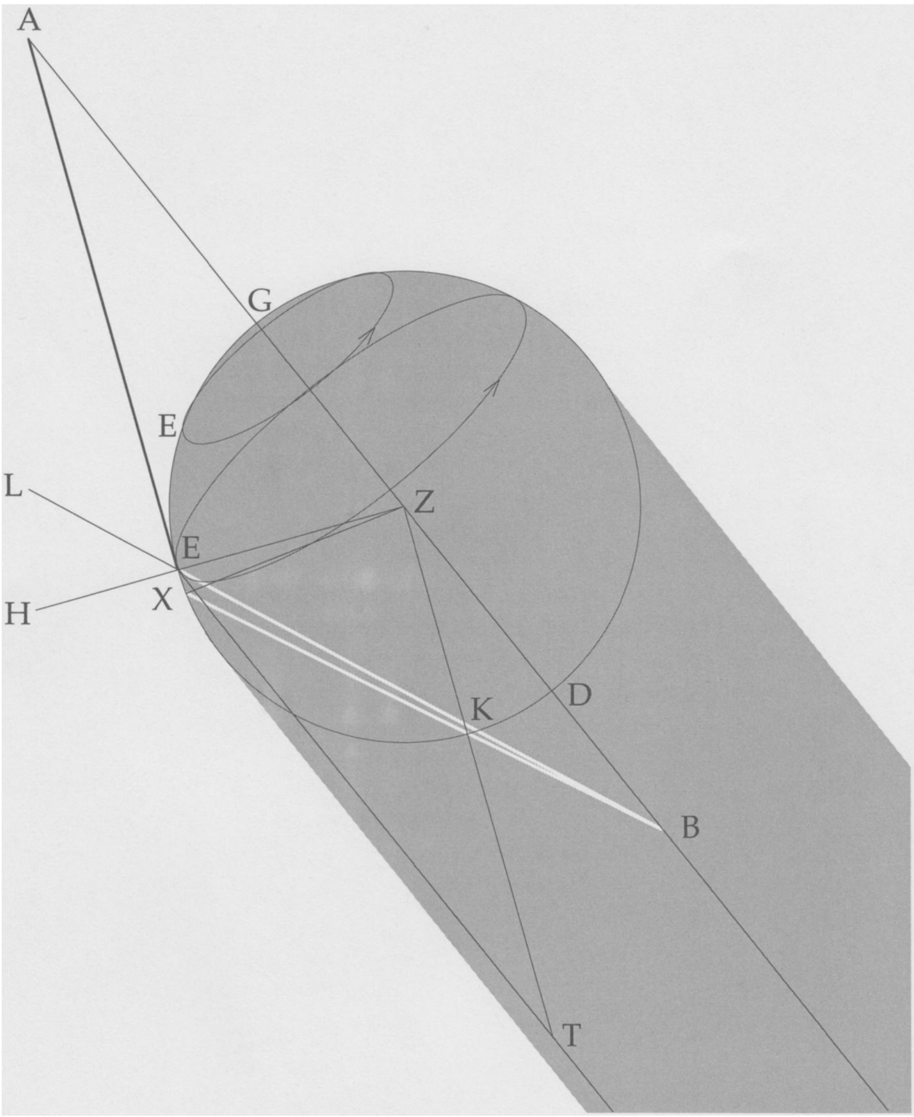


figure 14

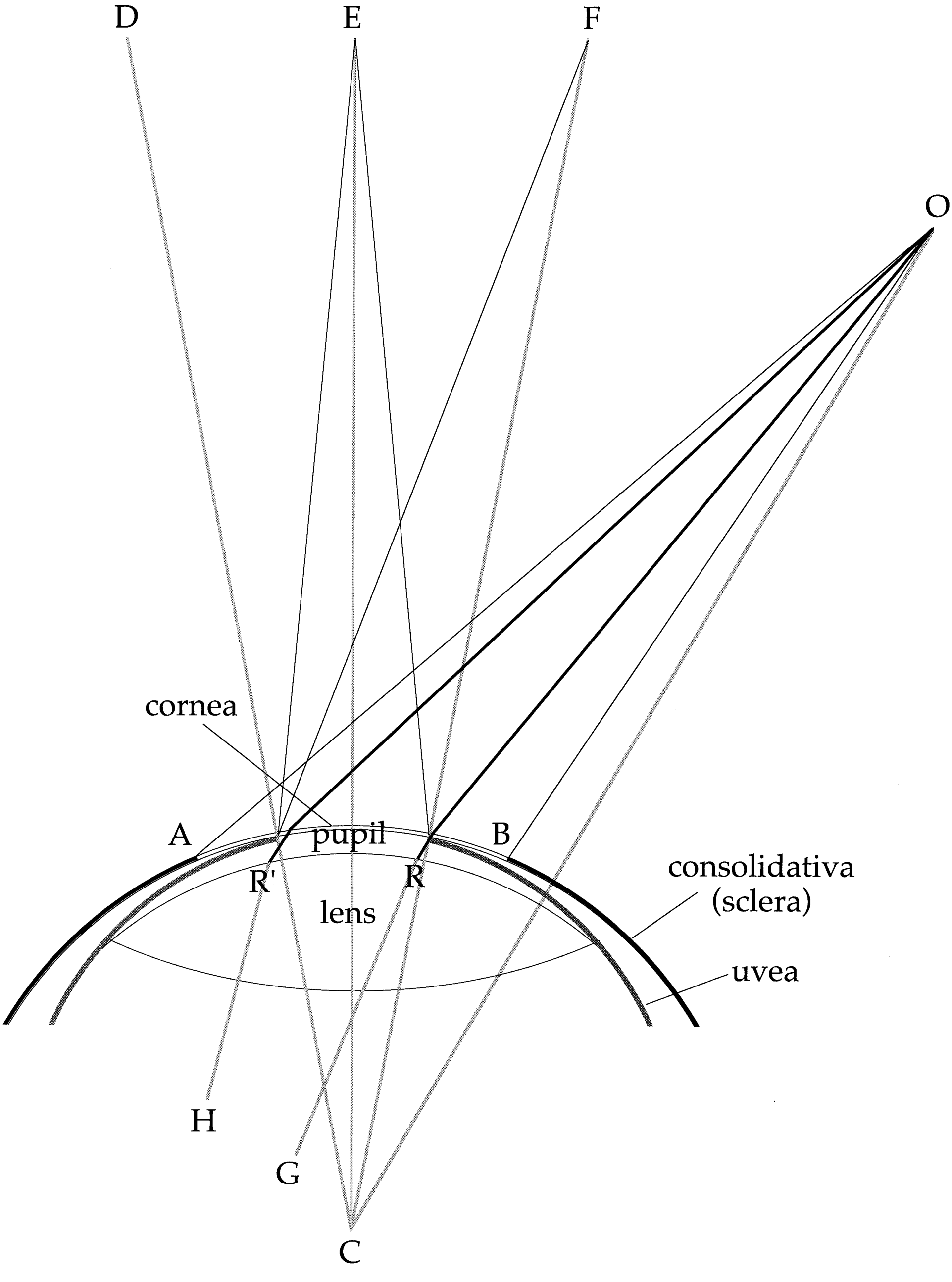


figure 15

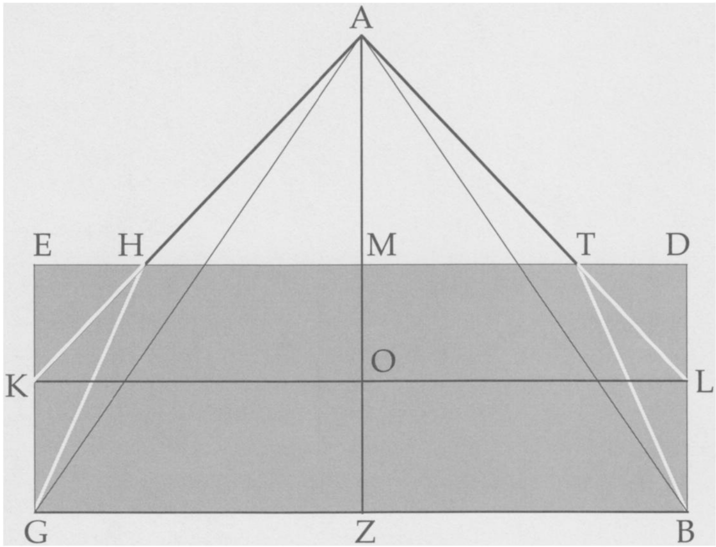


figure 16

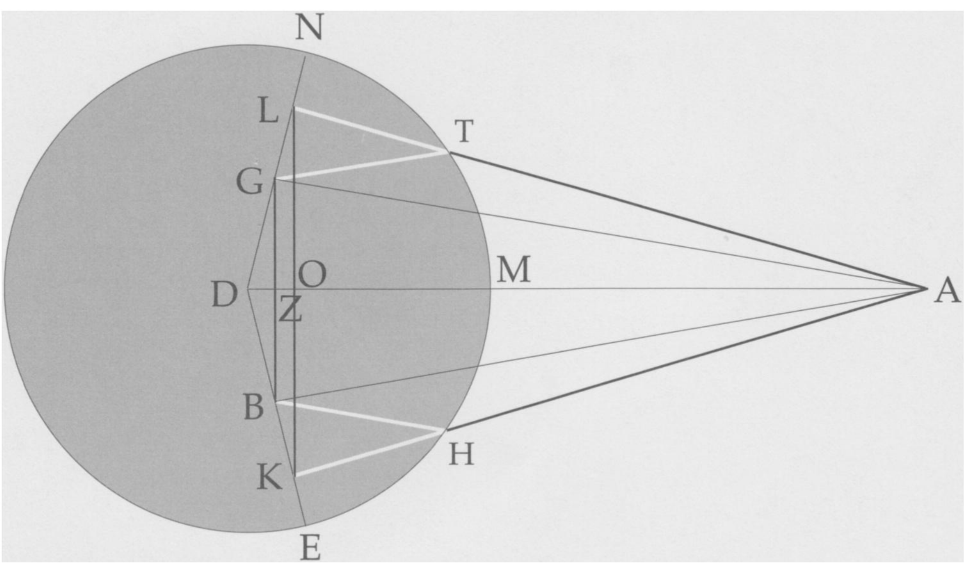


figure 17

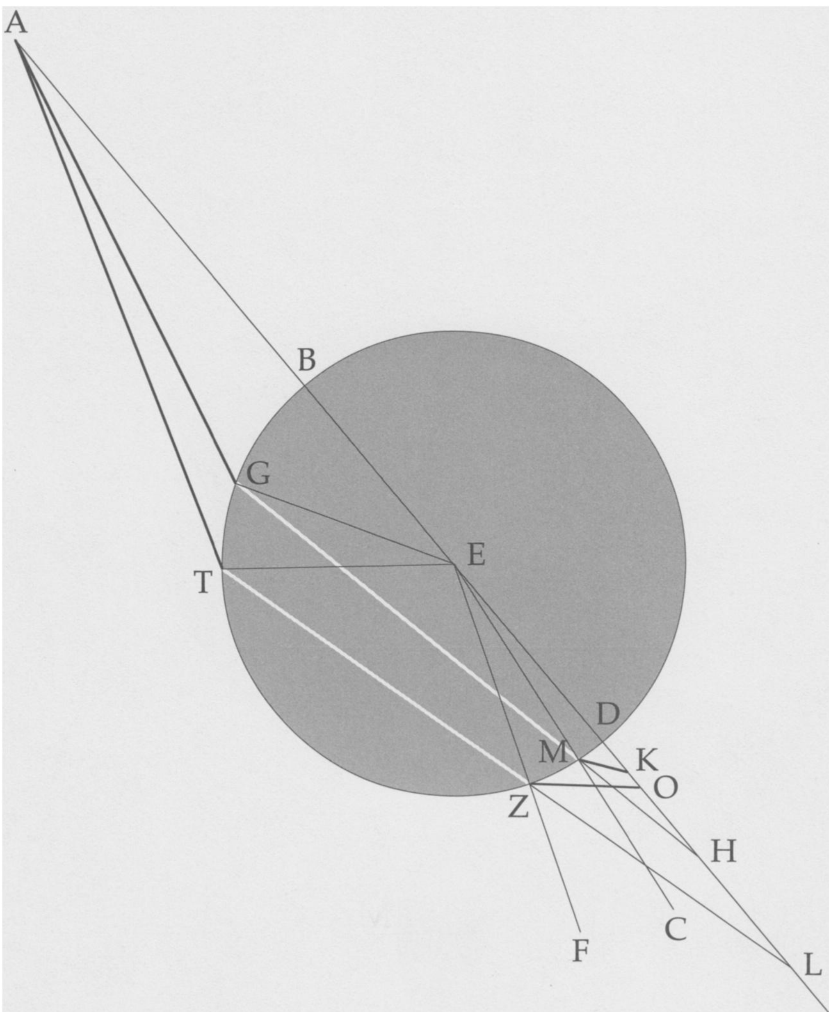


figure 18

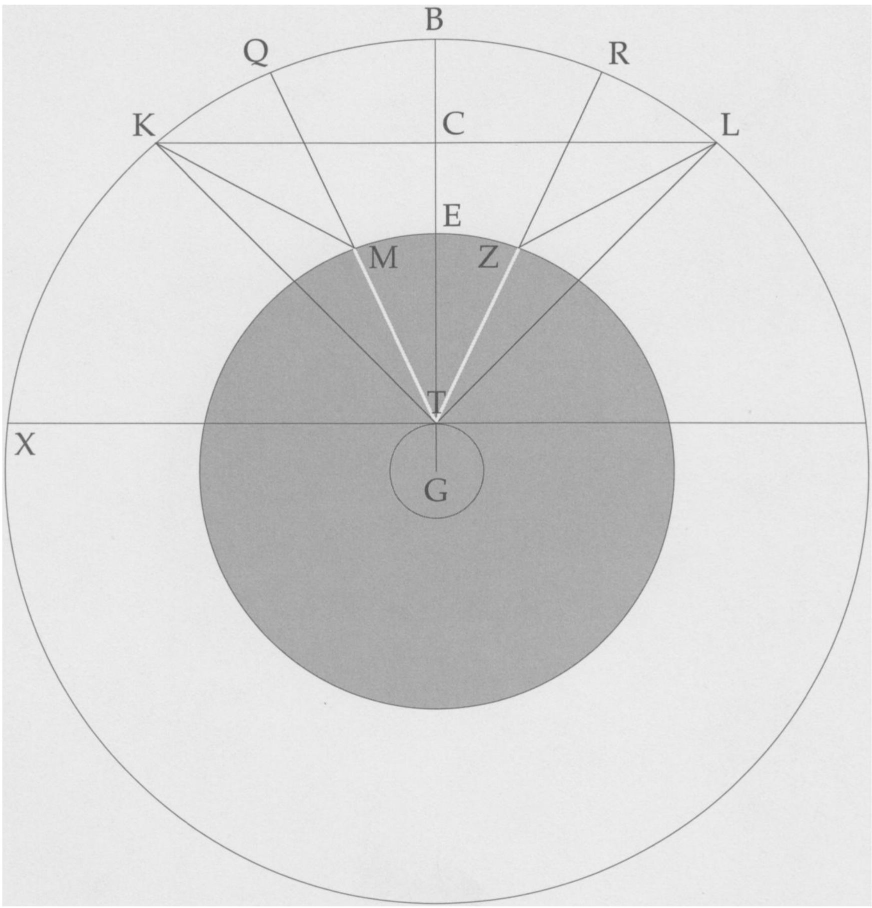


figure 19

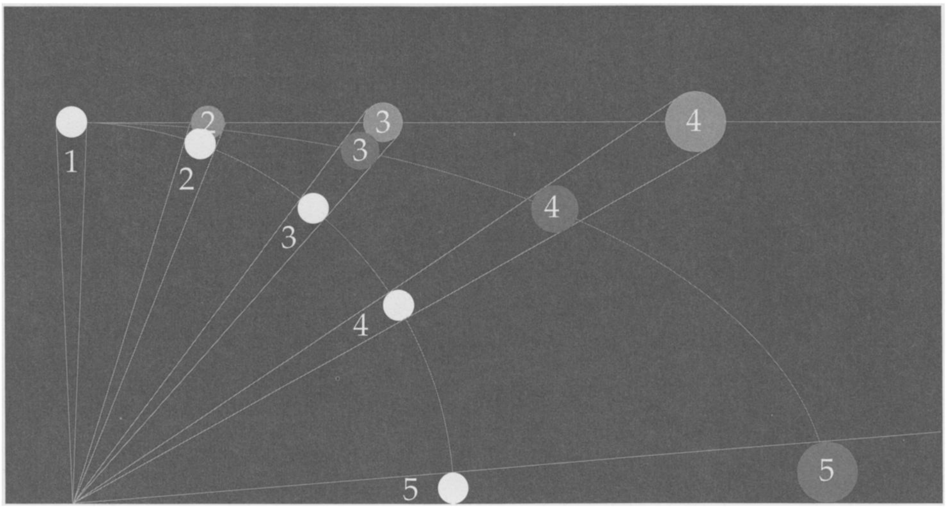


figure 20

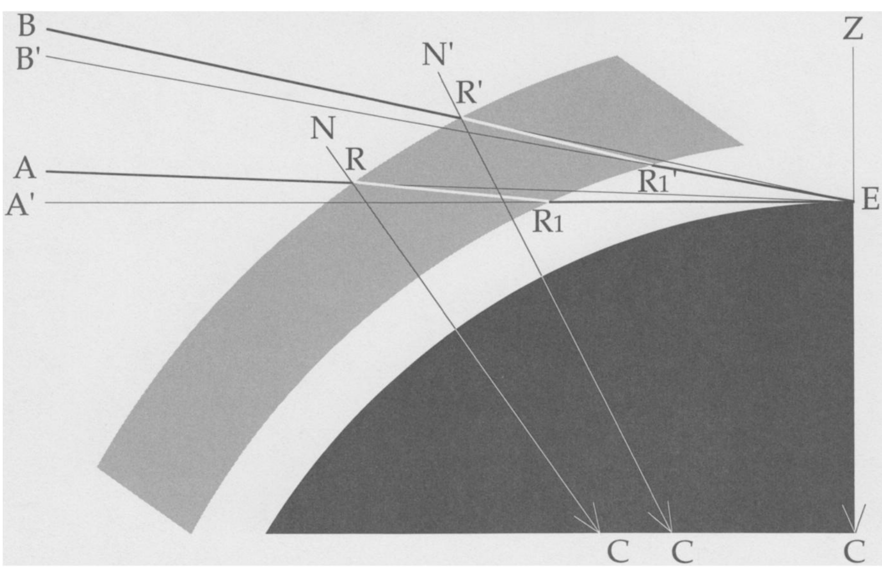


figure 21

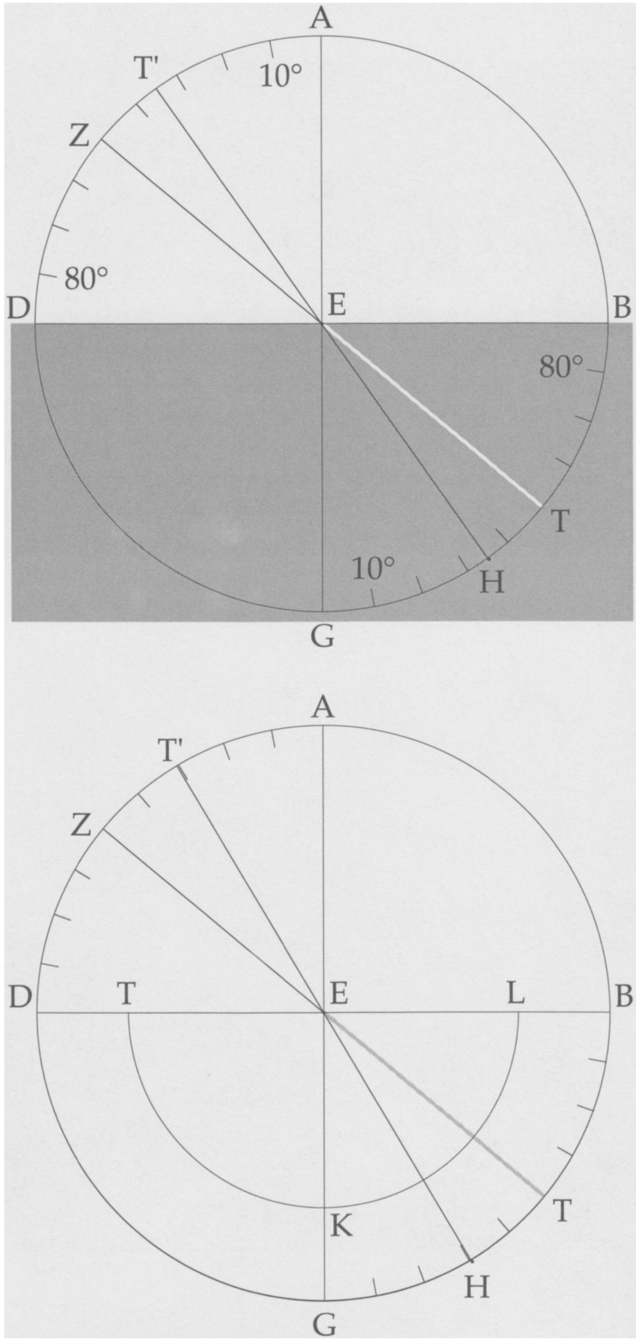


figure 22

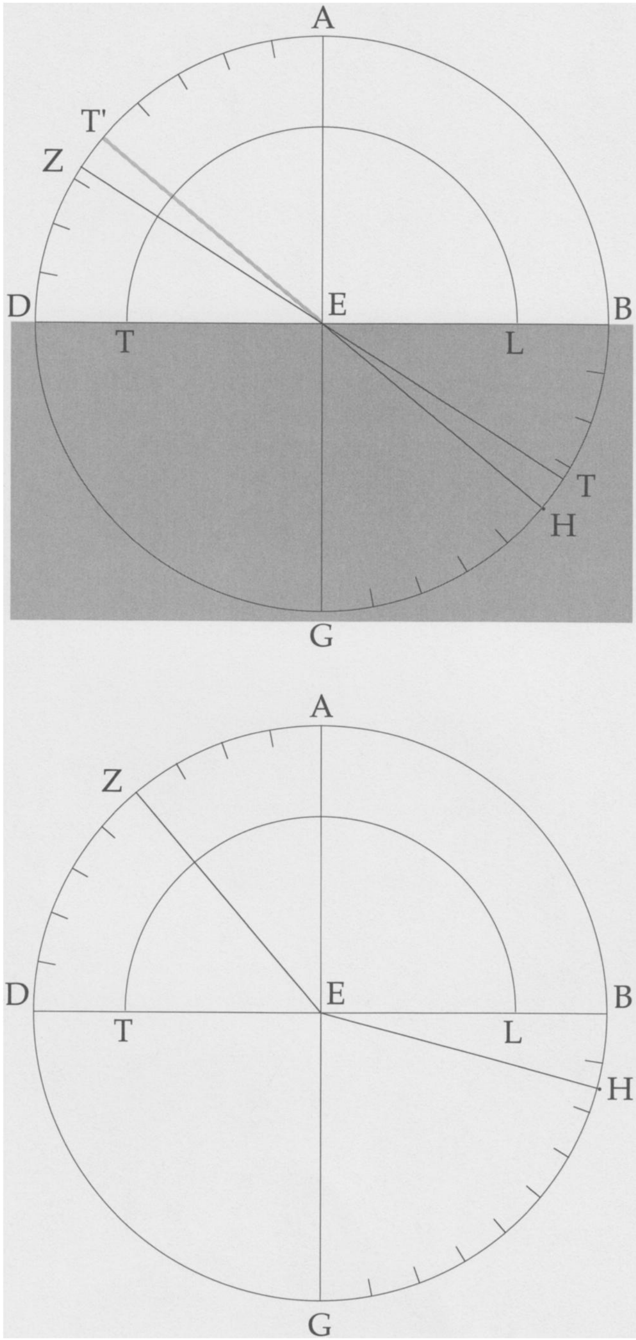


figure 23

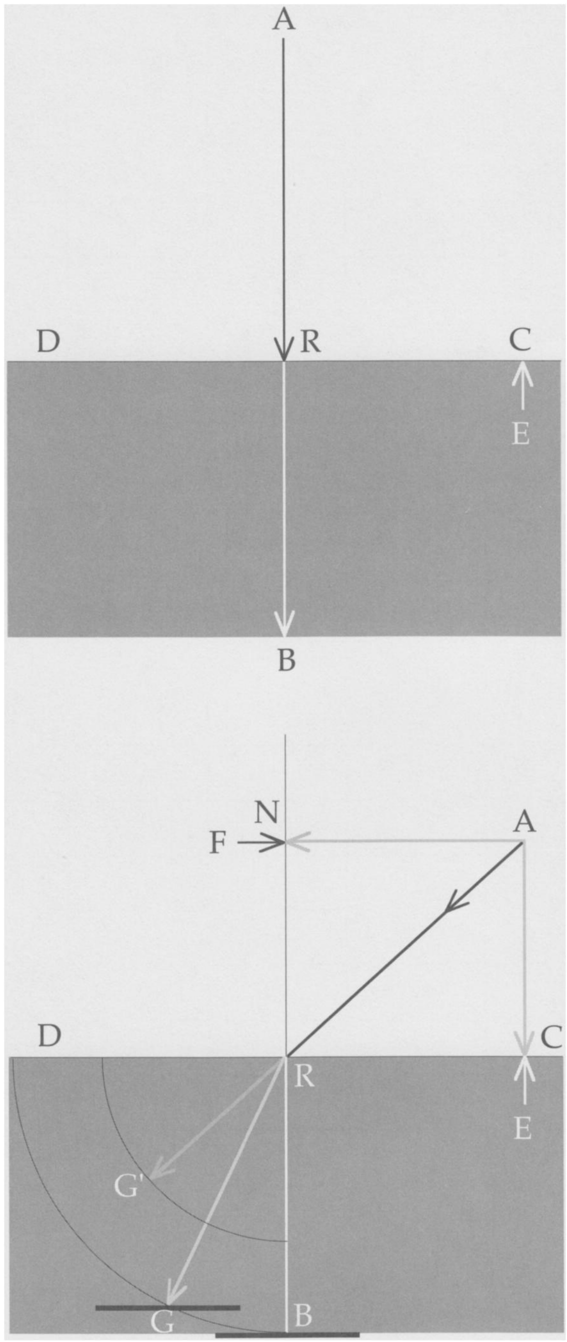


figure 24

PTOLEMY'S VALUES

i	$i-i'$	r	r/d	$r/d\text{-}r/d'$
42.0°		80.0°	38.0°	
	3.5°			6.5°
38.5°		70.0°	31.5°	
	4.0°			6.0°
34.5°		60.0°	25.5°	
	4.5°			5.5°
30.0°		50.0°	20.0°	
	5.0°			5.0°
25.0°		40.0°	15.0°	

MODERN VALUES

i	$i-i'$	r	r/d	$r/d\text{-}r/d'$
41.0°		80.0°	39.0°	
	2.2°			7.7°
38.8°		70.0°	31.3°	
	3.5°			6.6°
35.3°		60.0°	24.7°	
	4.6°			5.4°
30.7°		50.0°	19.3°	
	5.3°			4.4°
25.4°		40.0°	14.9°	

figure 25

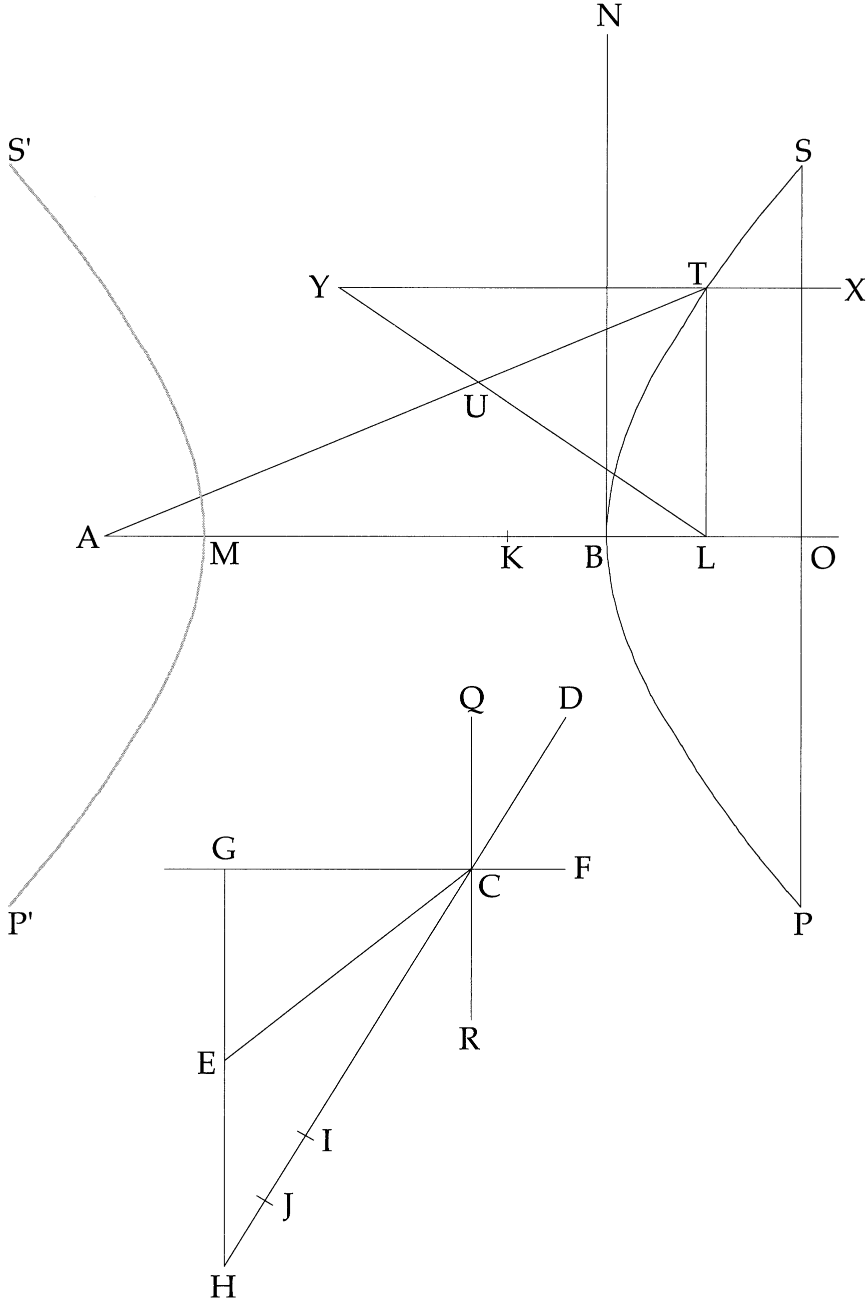


figure 26

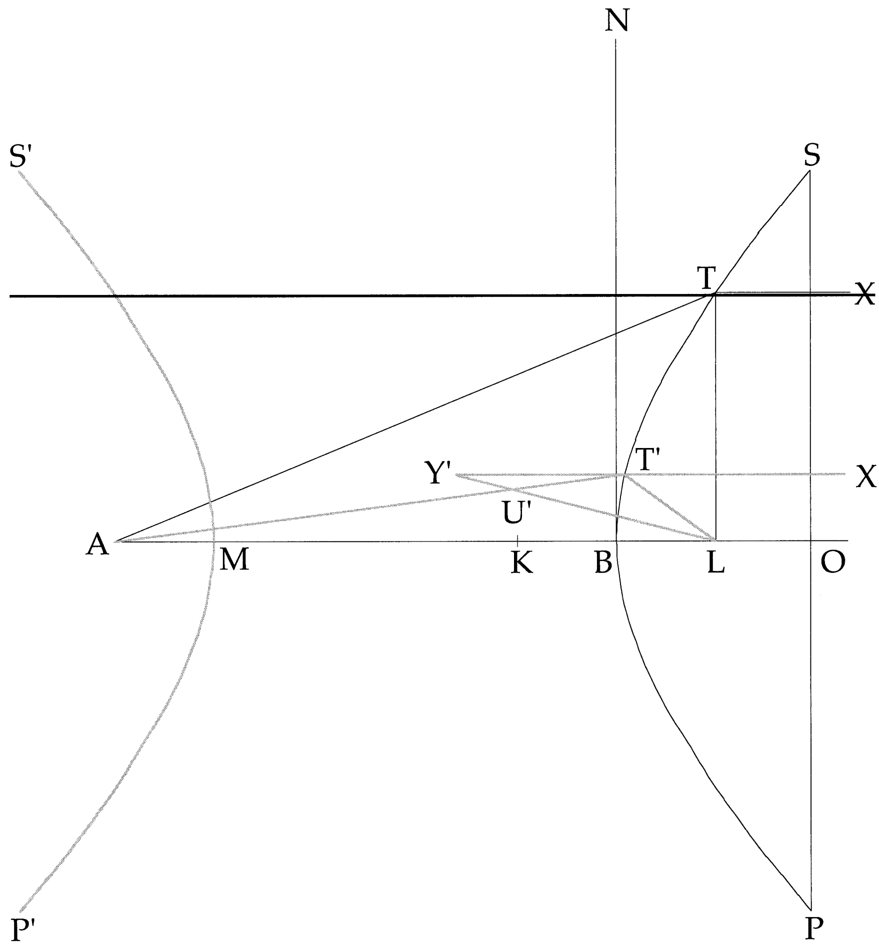


figure 26a

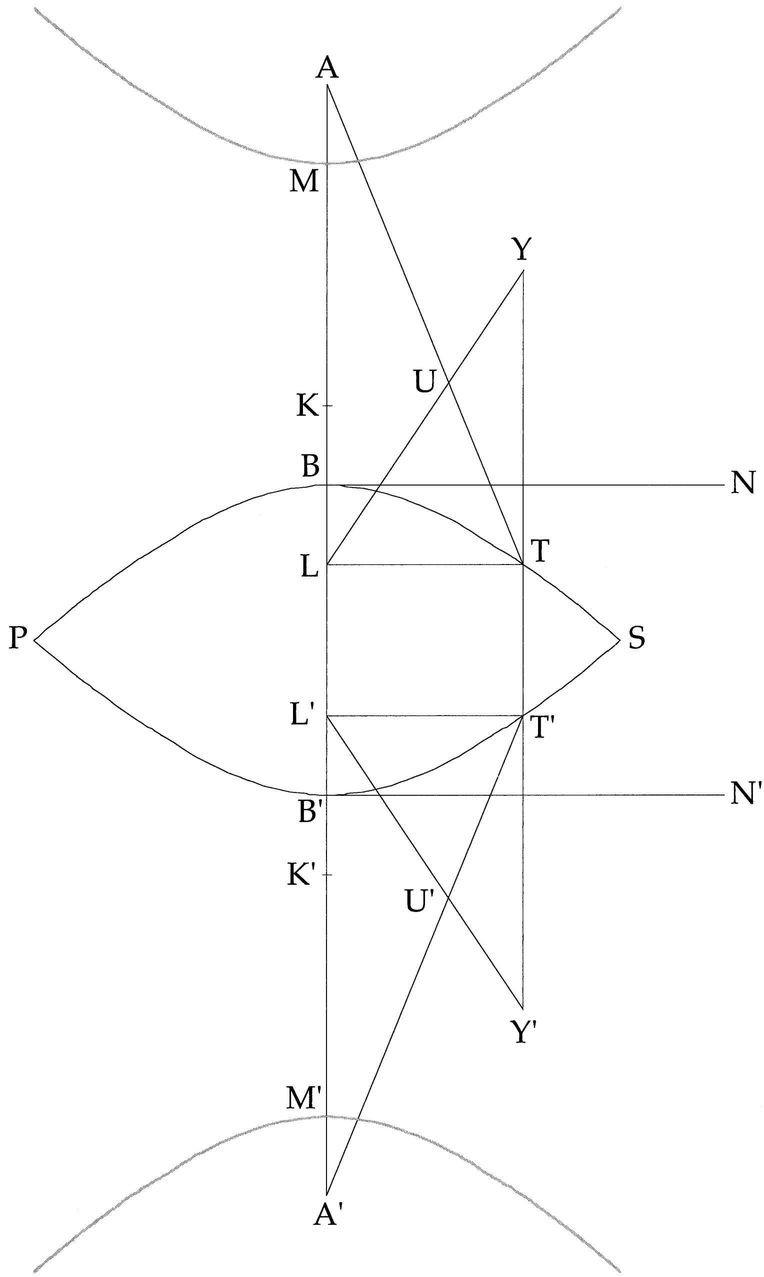


figure 26b

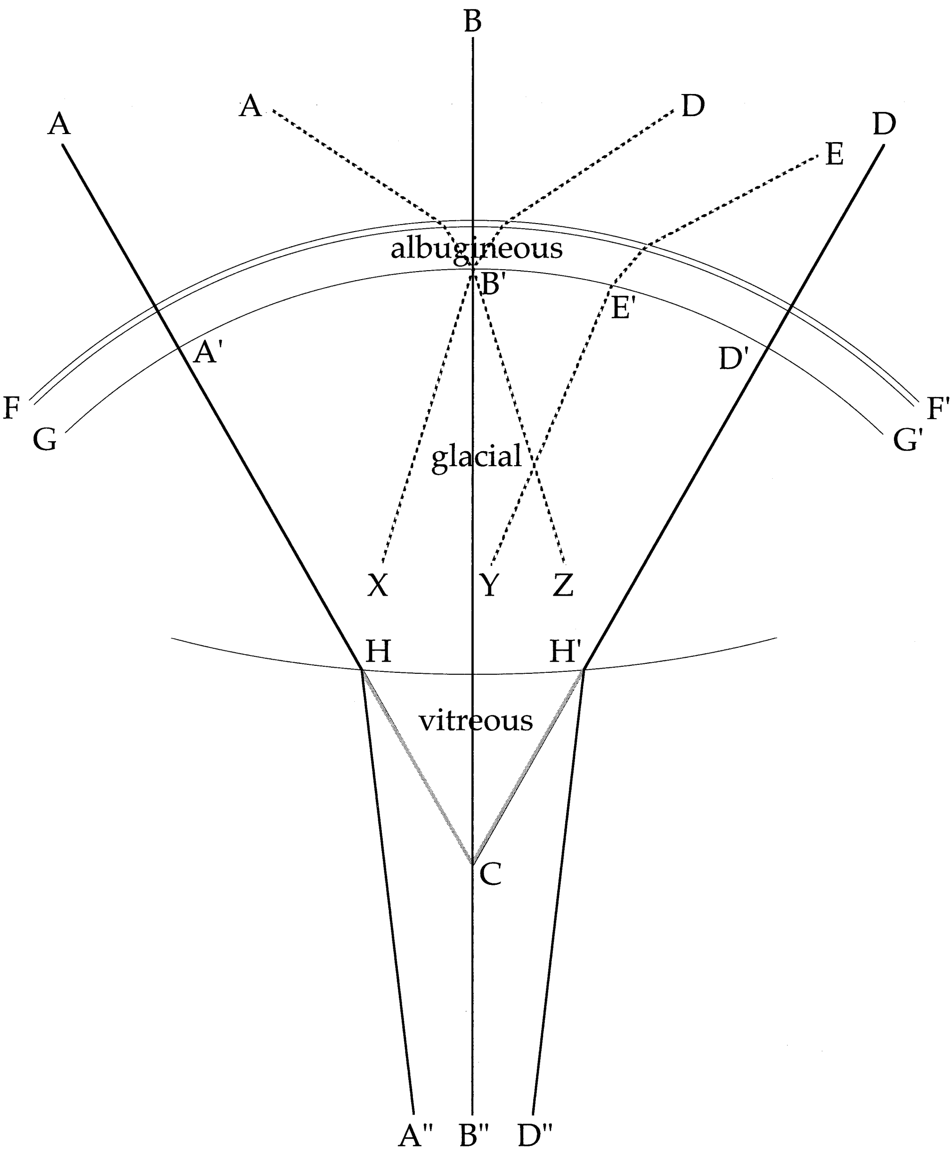


figure 27

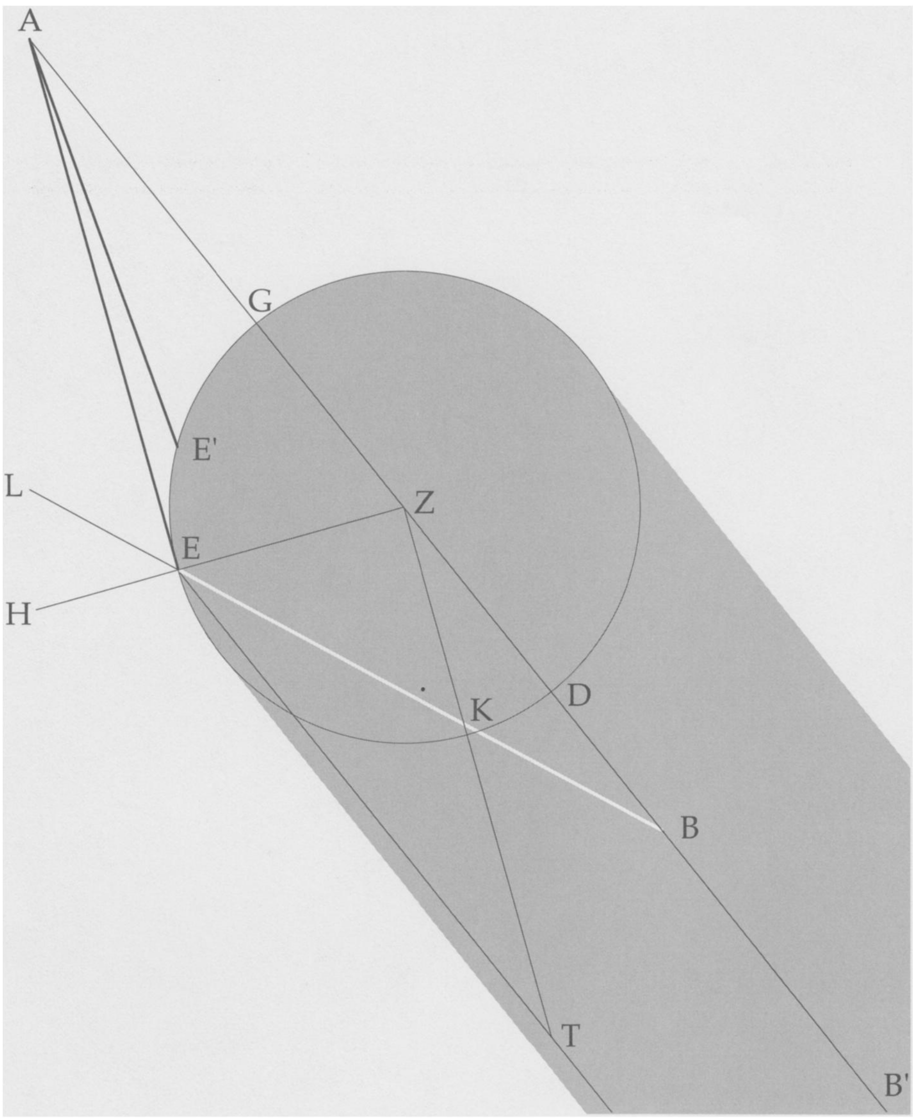


figure 28

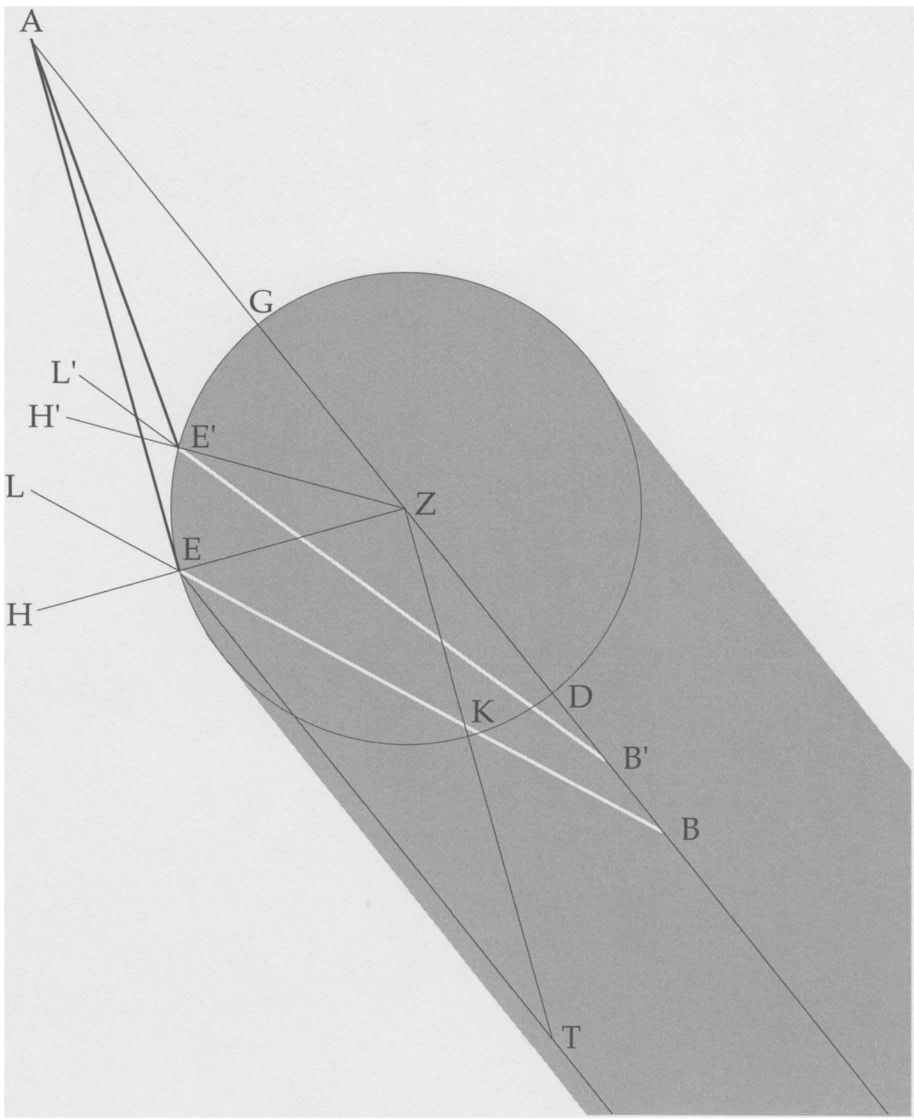


figure 28a

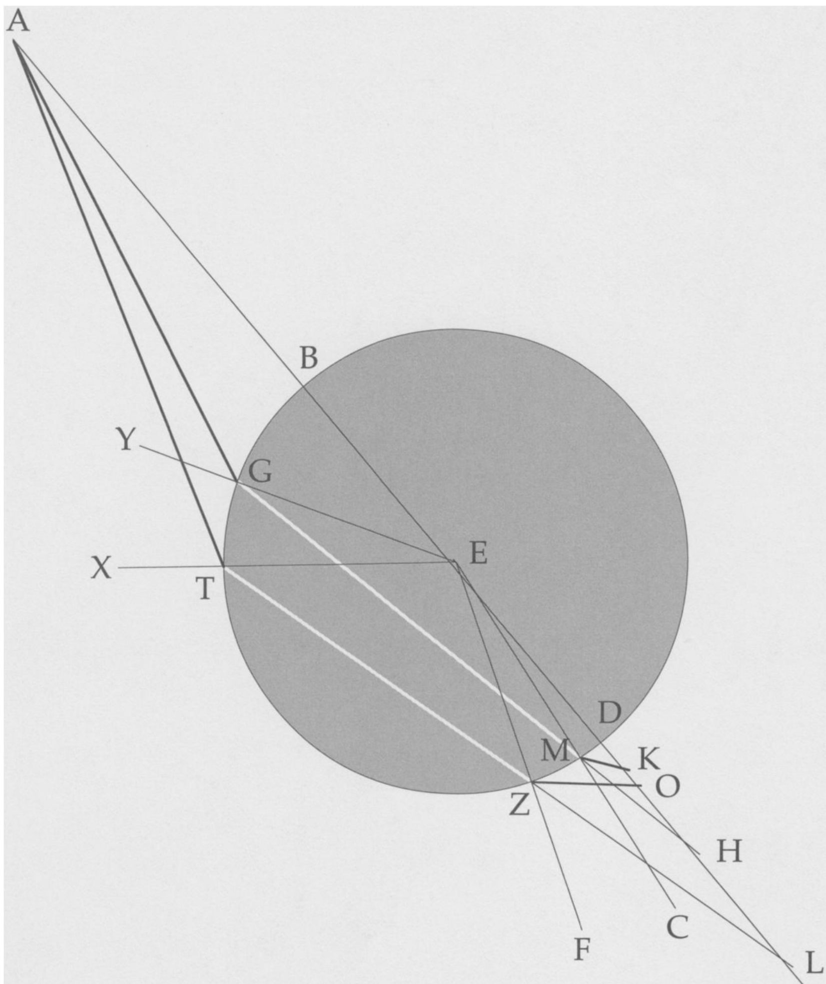


figure 29

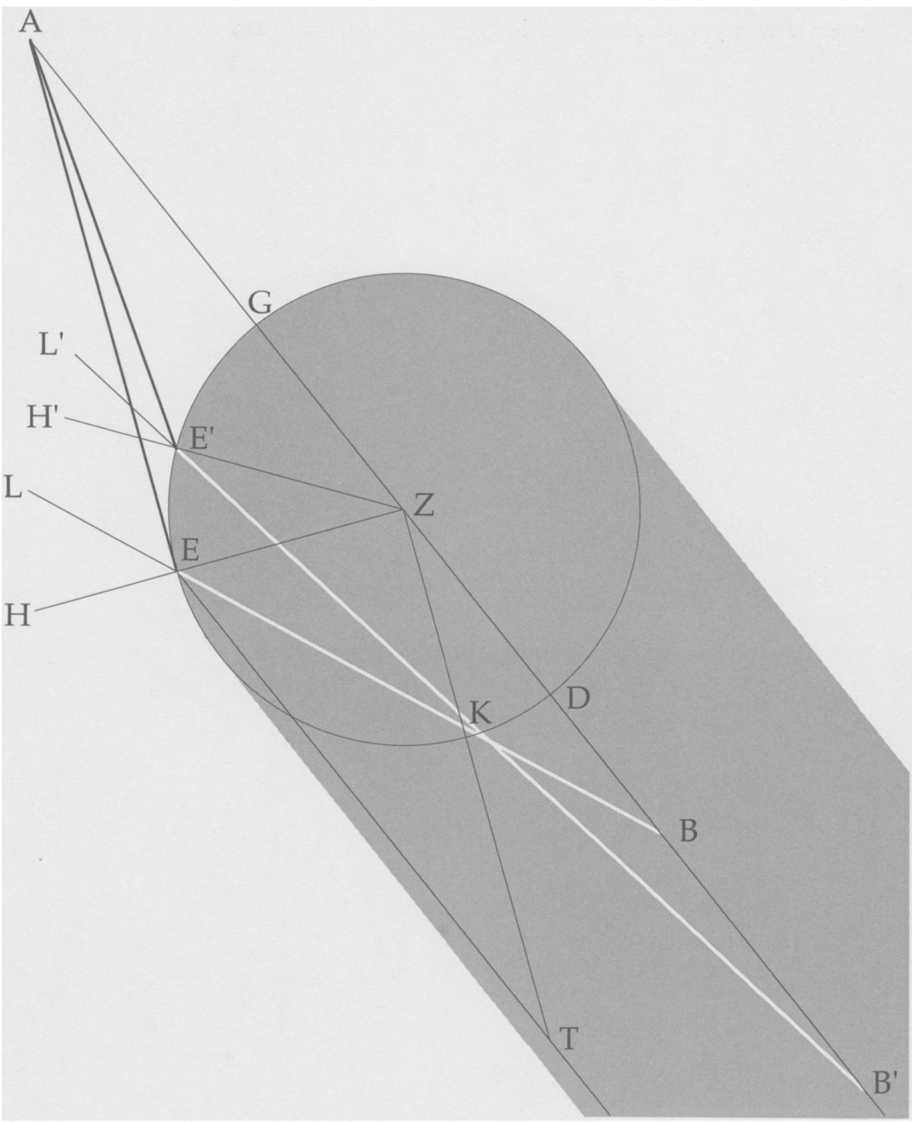


figure 29a

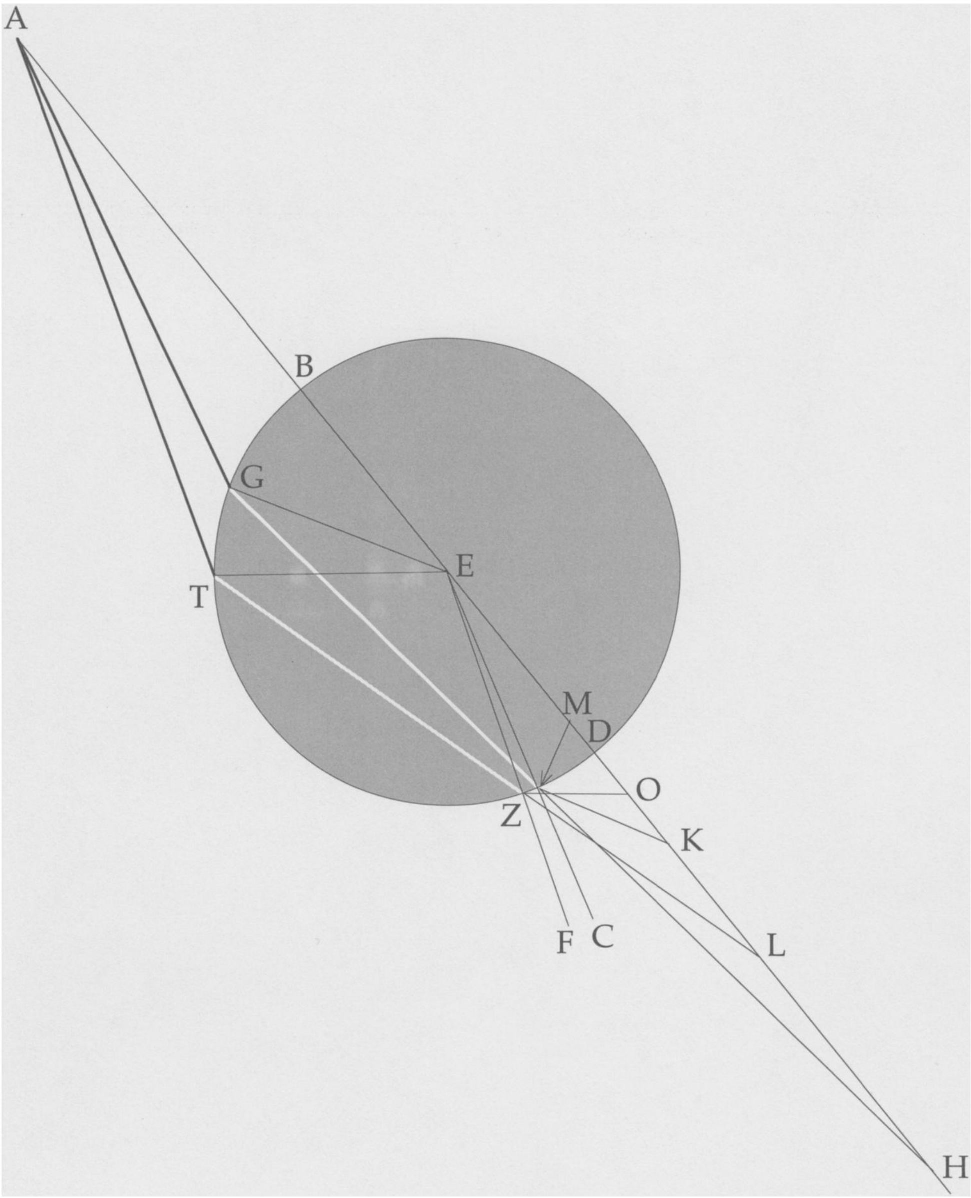


figure 29b

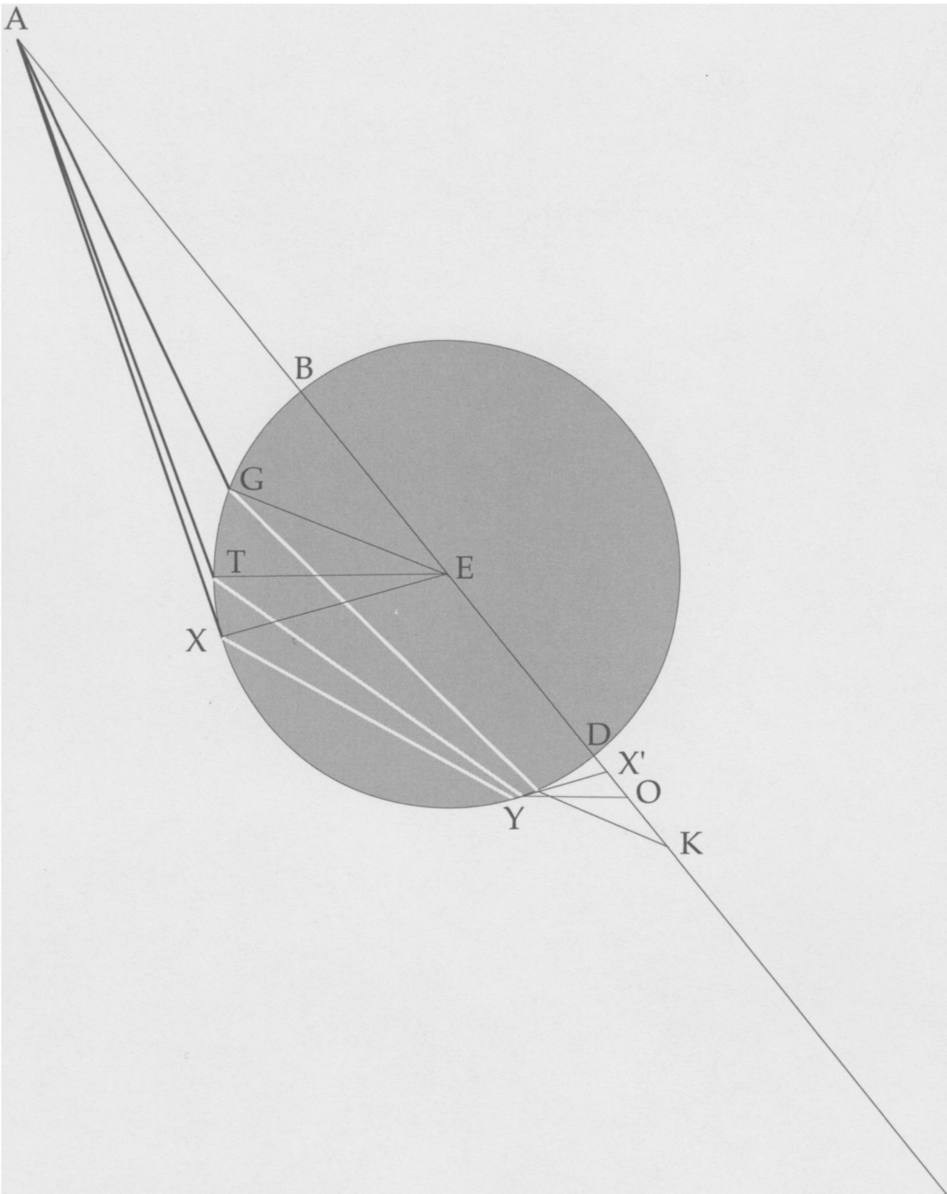


figure 29c

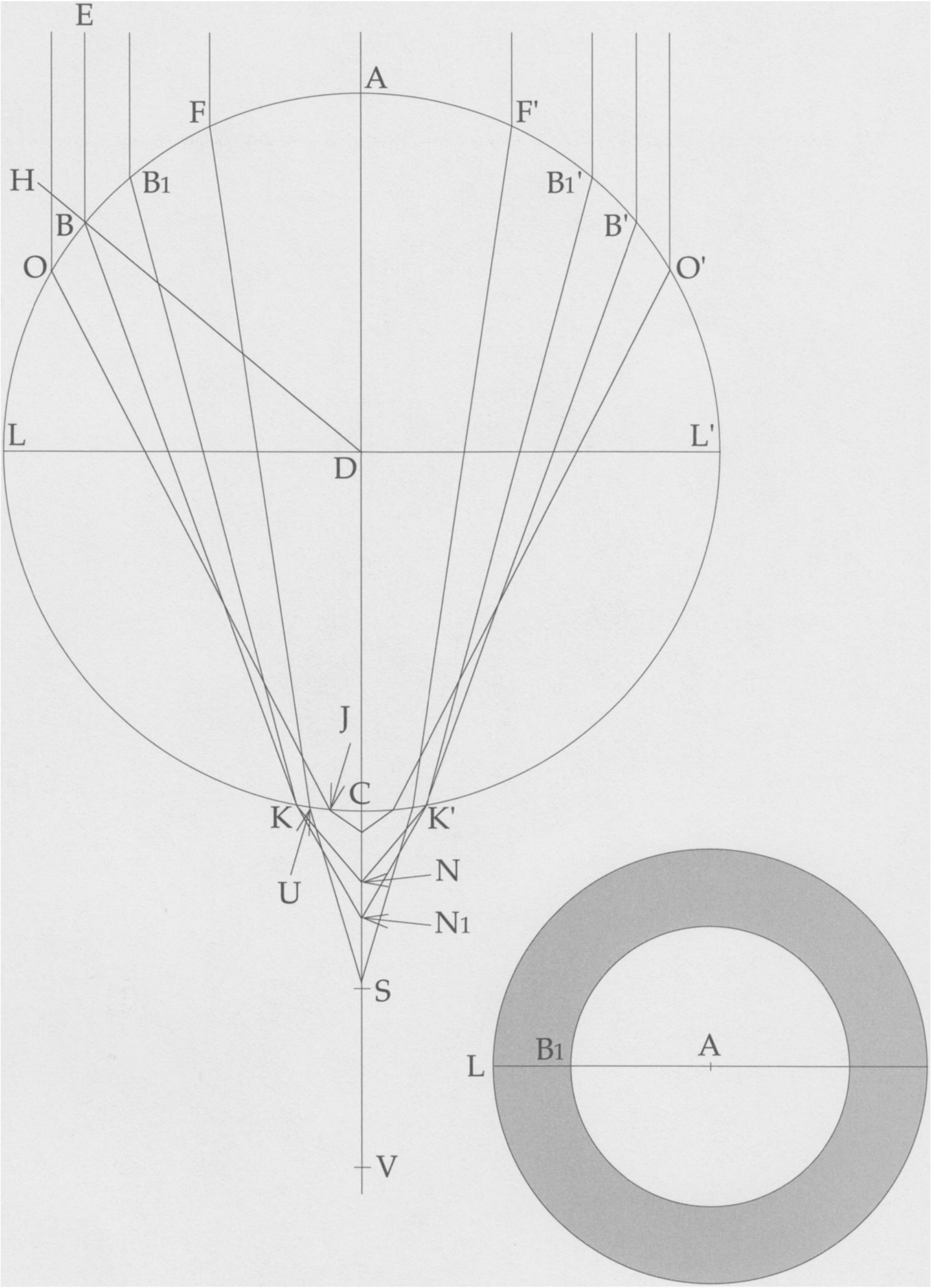


figure 30

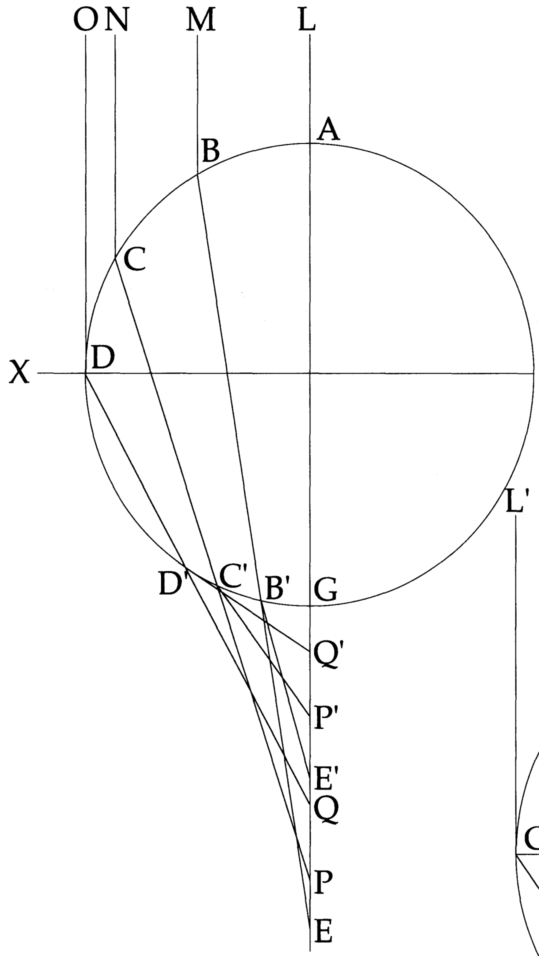


figure 31

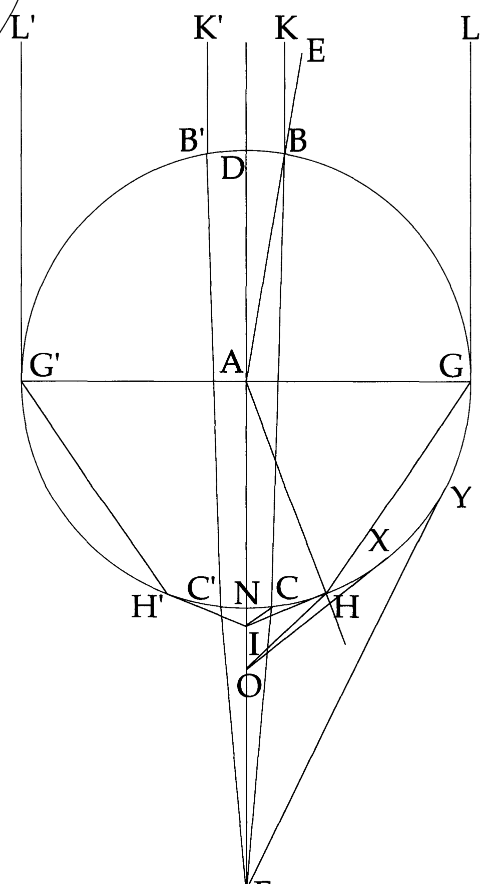


figure 32

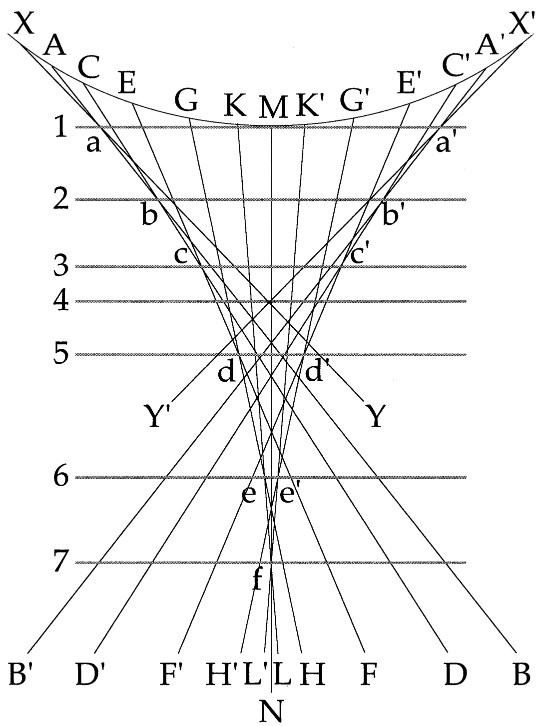


figure 33

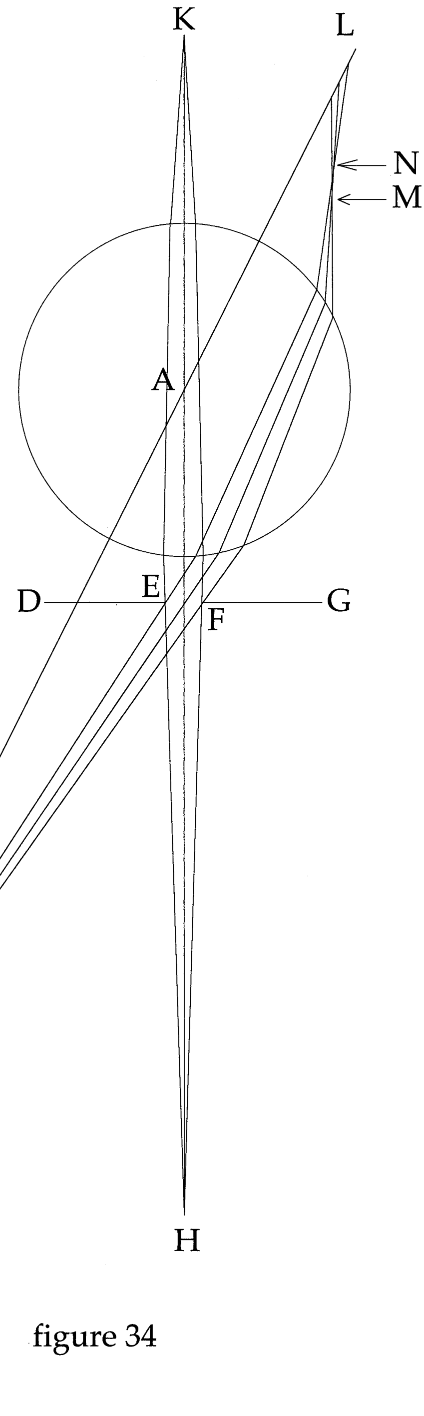


figure 34

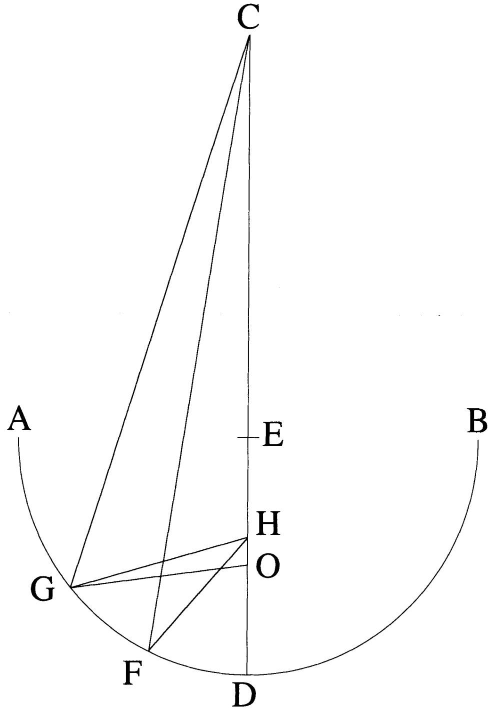


figure 35

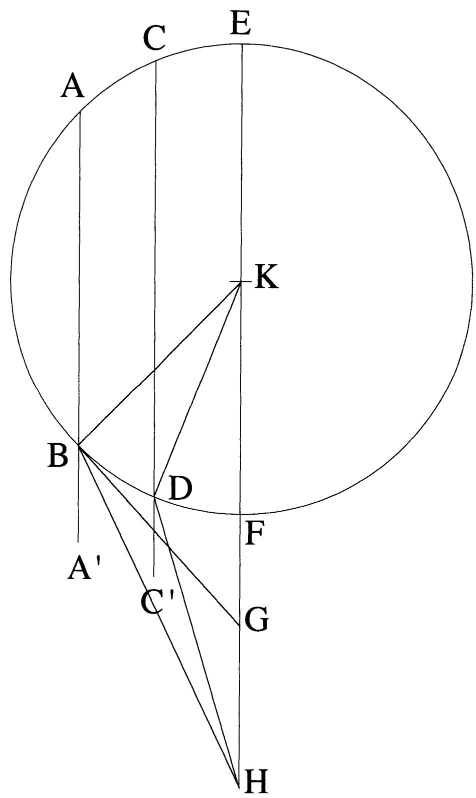


figure 36

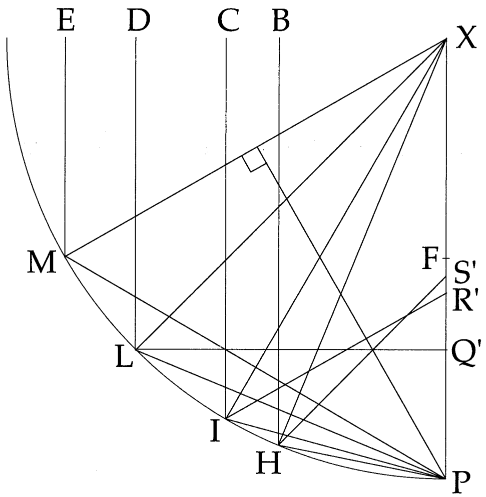


figure 37

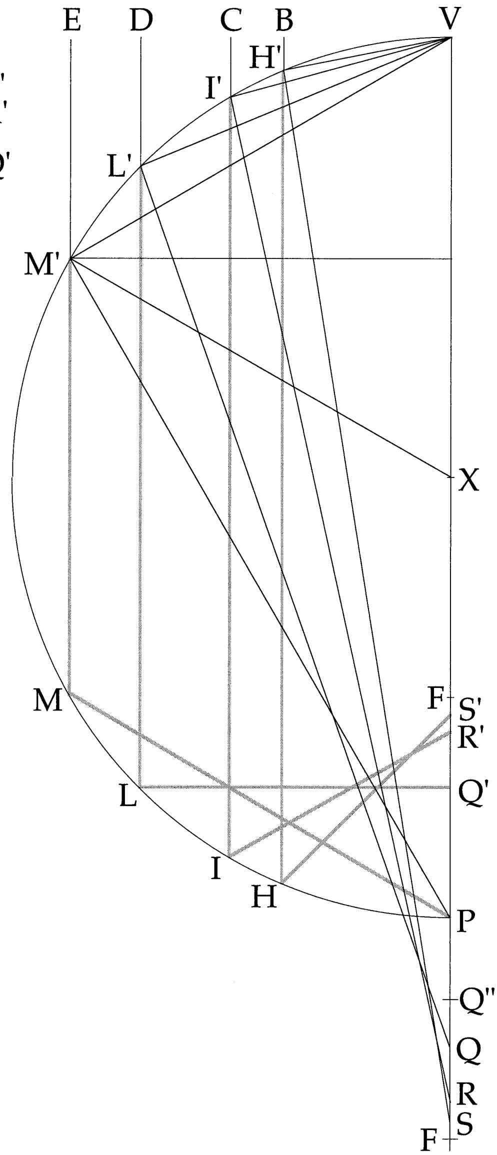


figure 38

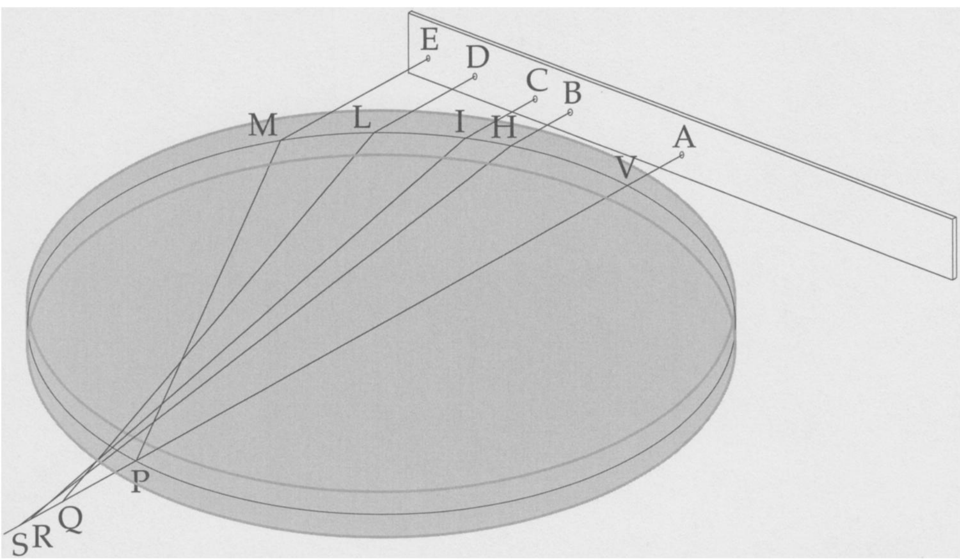


figure 39

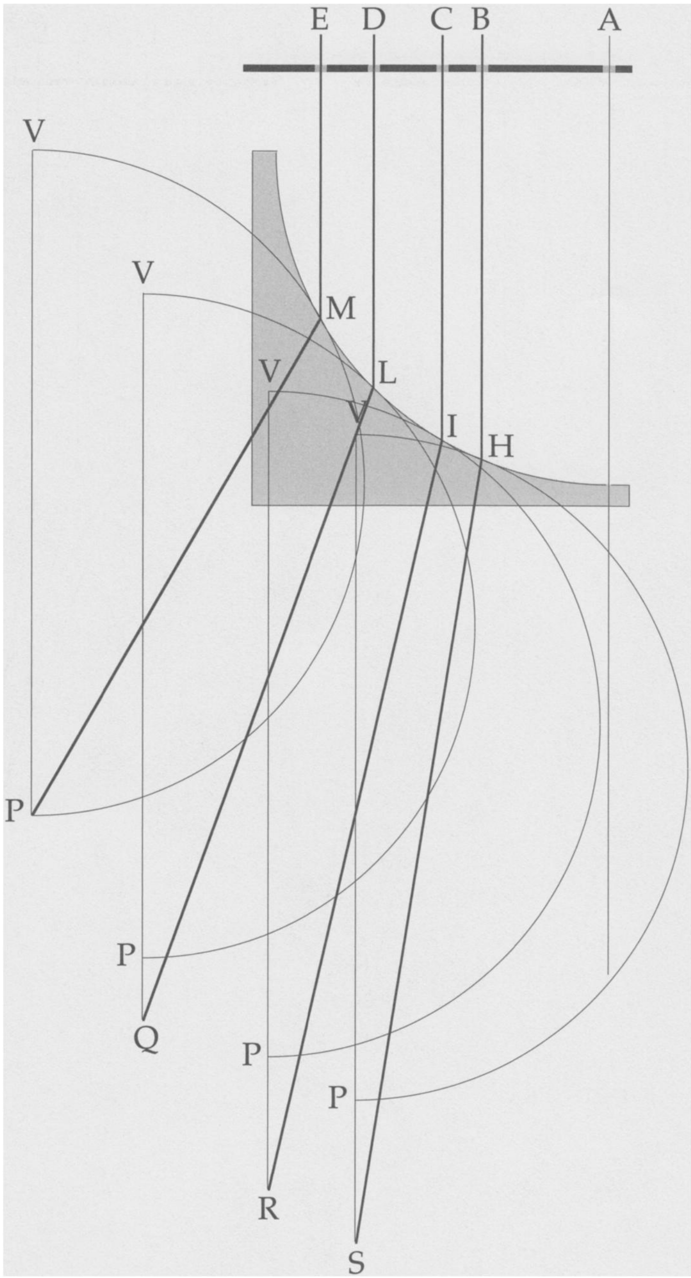


figure 40

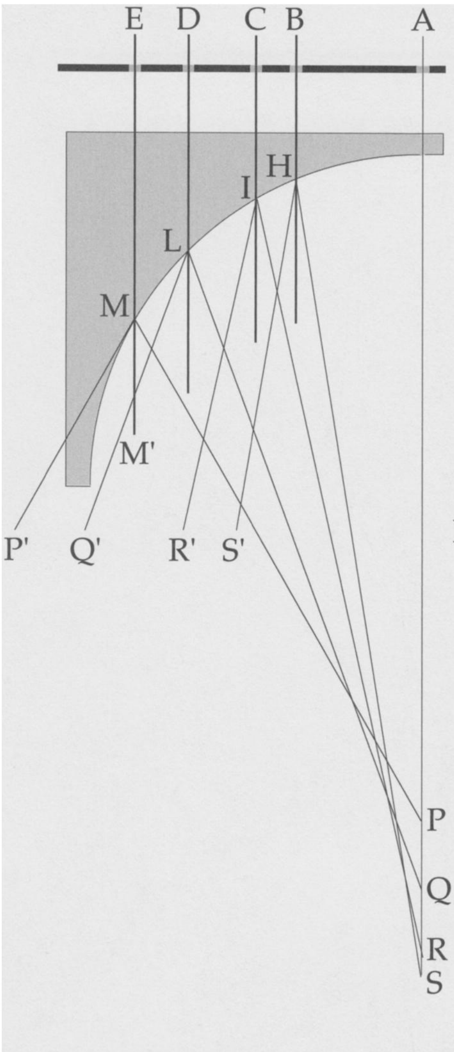


figure 41

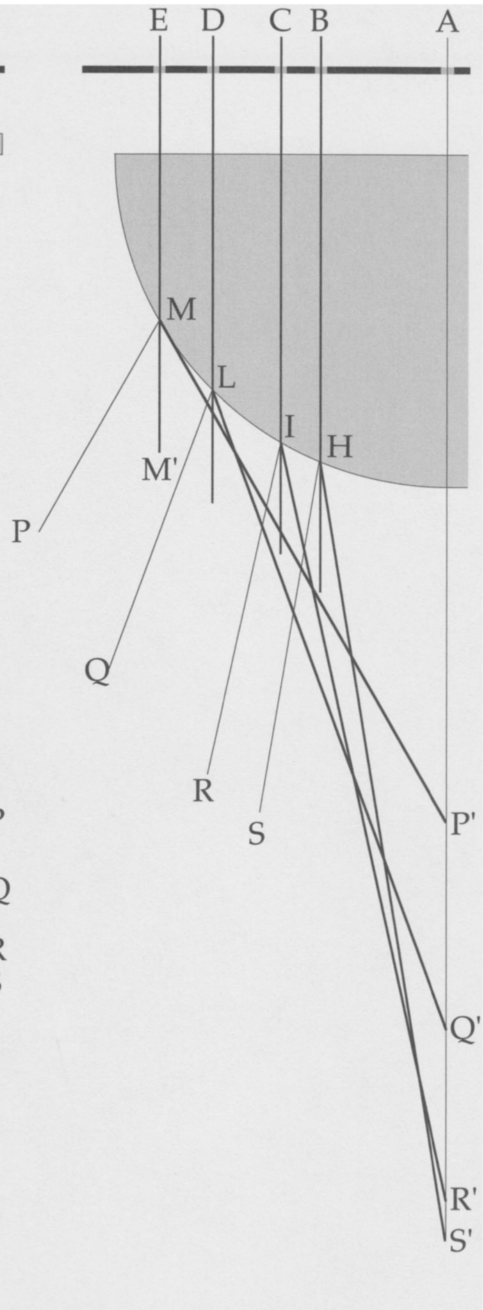


figure 42

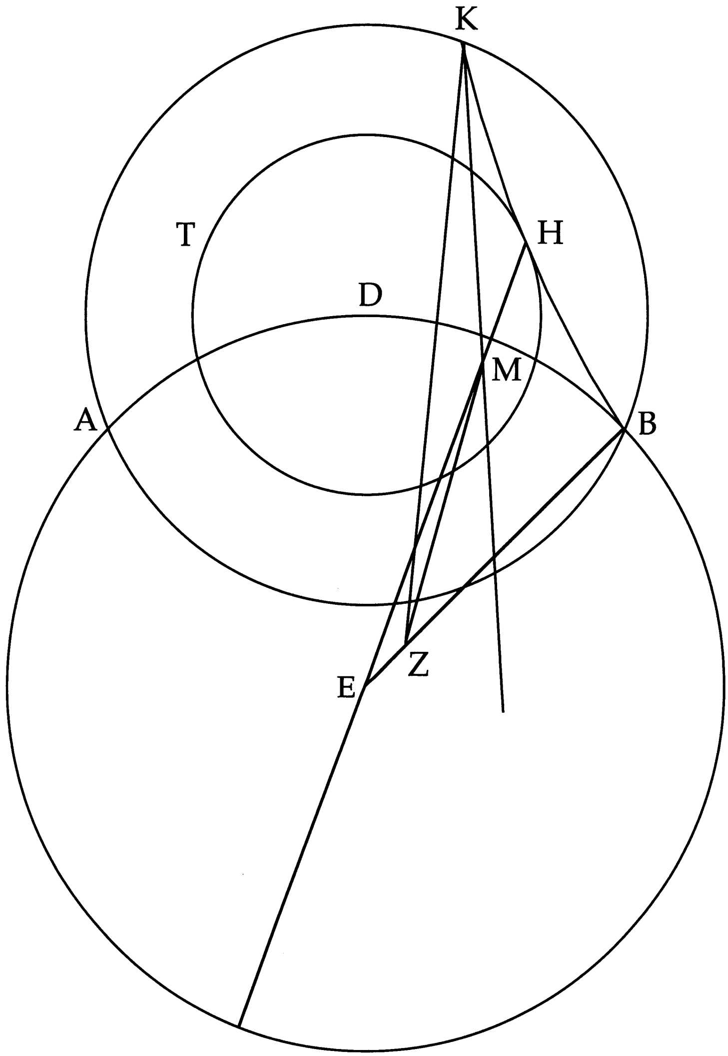


FIGURE 7.4.1

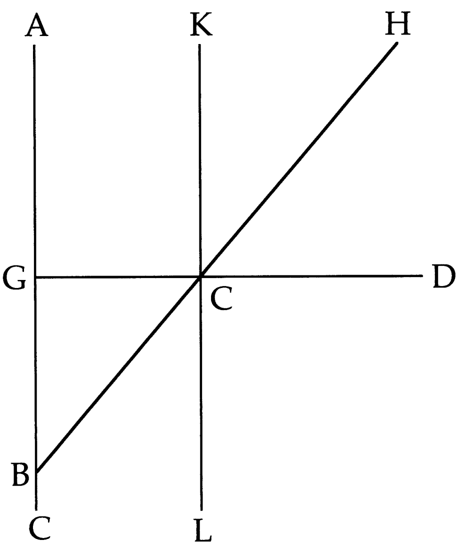


FIGURE 7.5.2

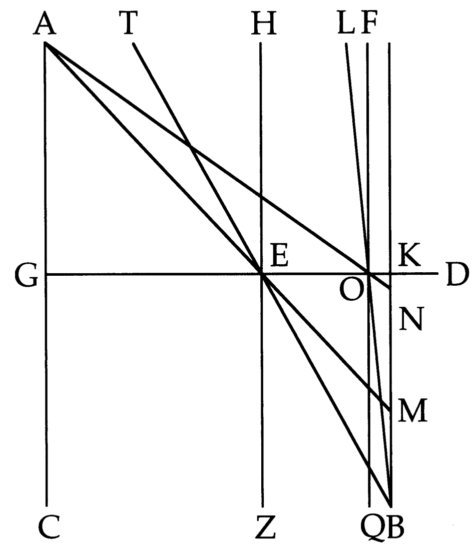


FIGURE 7.5.3

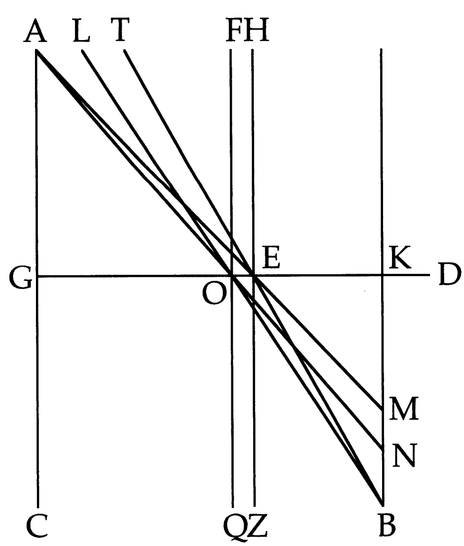


FIGURE 7.5.3a

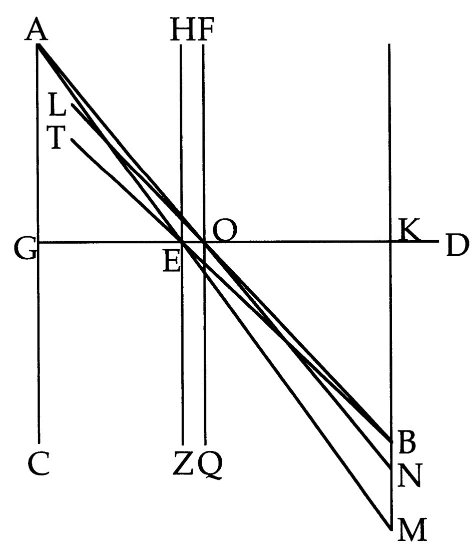


FIGURE 7.5.4

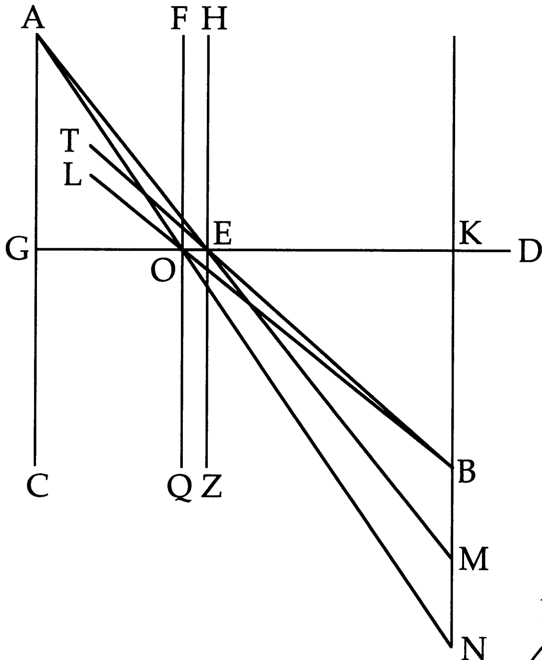


FIGURE 7.5.4a

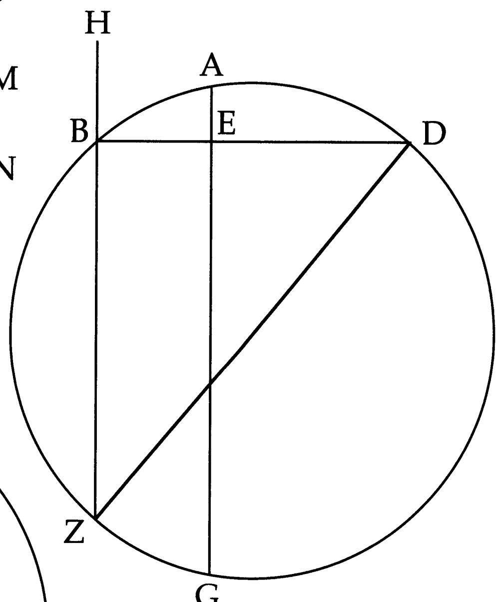


FIGURE 7.5.5

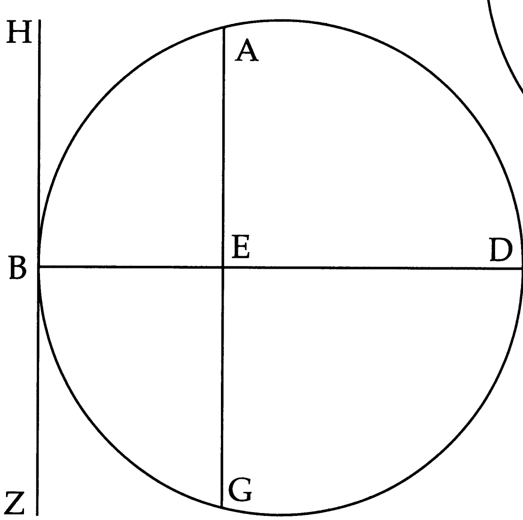


FIGURE 7.5.5a

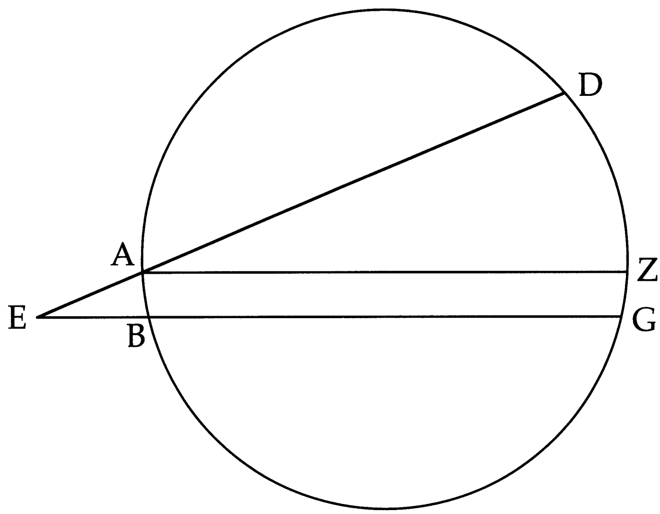


FIGURE 7.5.5b

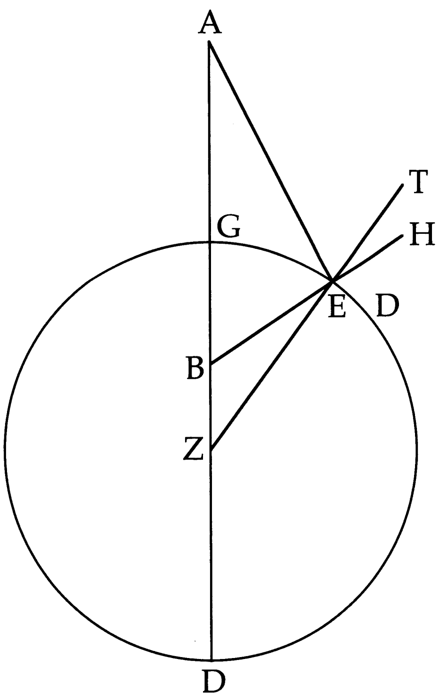


FIGURE 7.5.6

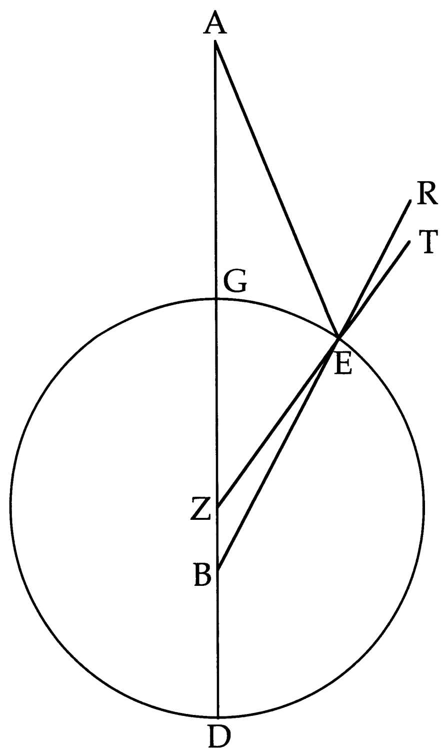


FIGURE 7.5.6a

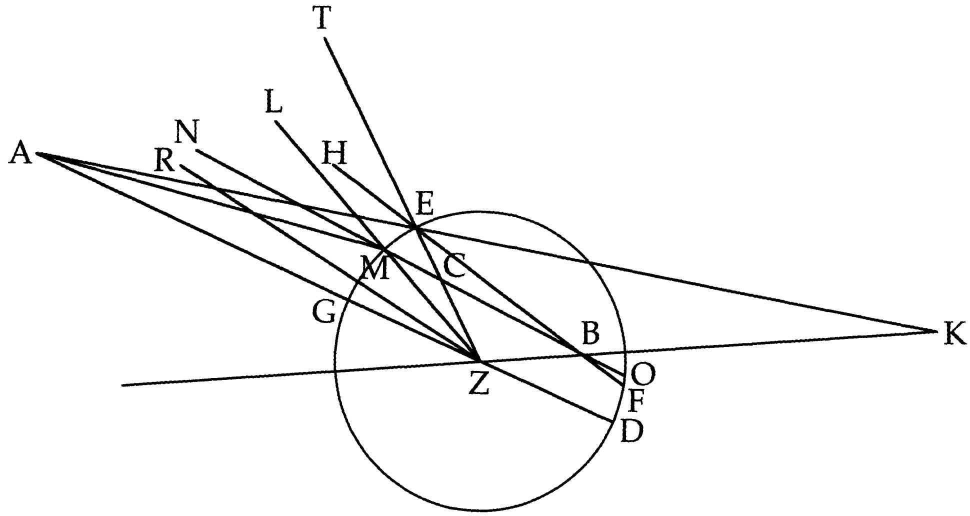


FIGURE 7.5.6b

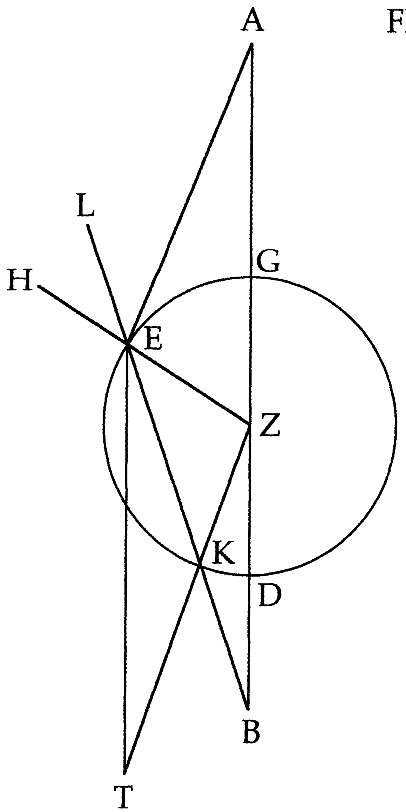


FIGURE 7.5.7

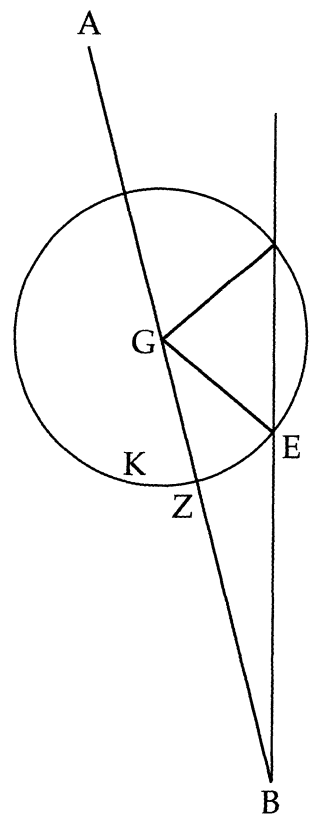


FIGURE 7.5.8

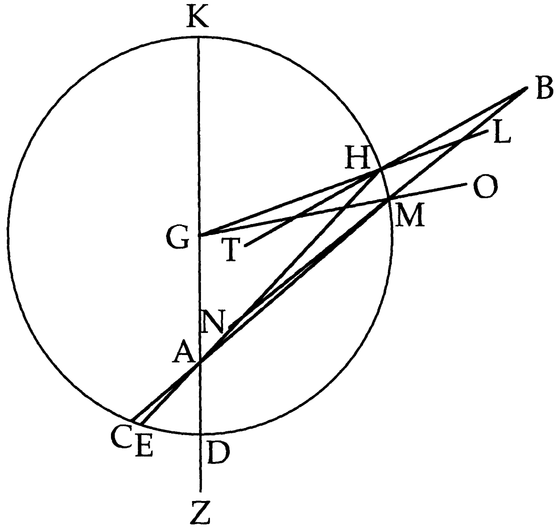


FIGURE 7.5.9

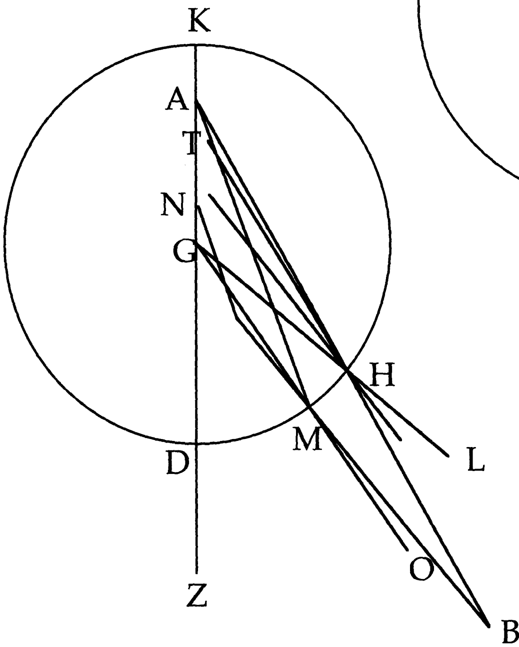


FIGURE 7.5.9a

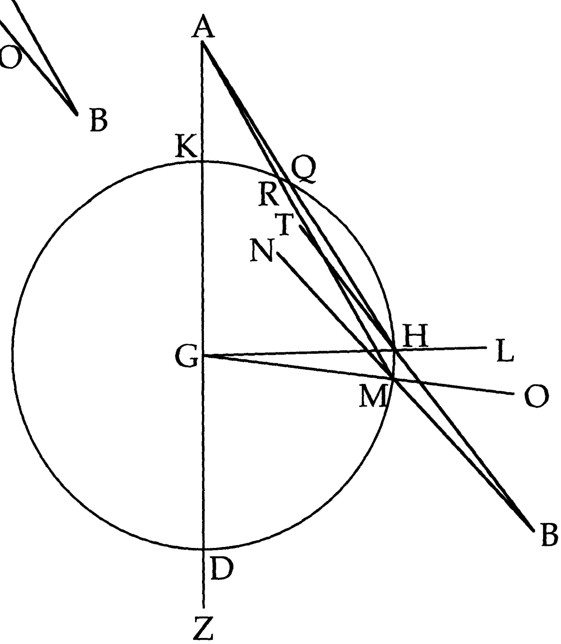


FIGURE 7.5.9b

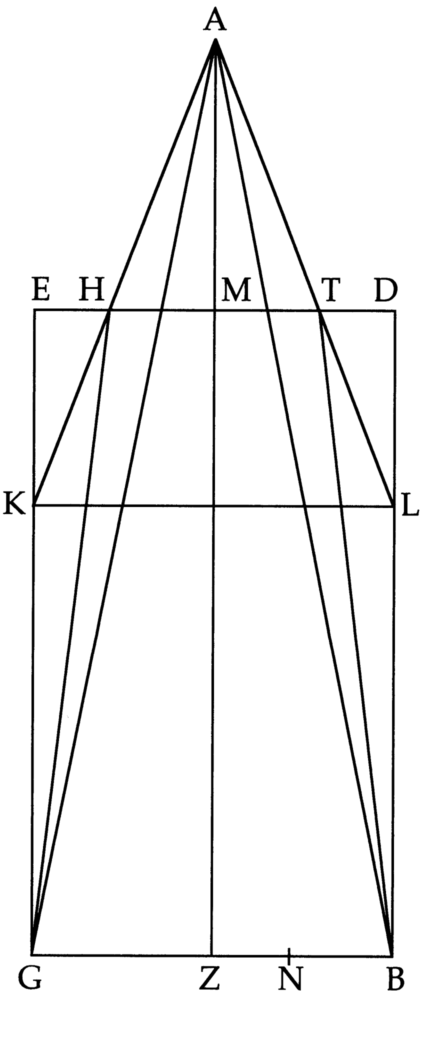


FIGURE 7.7.10

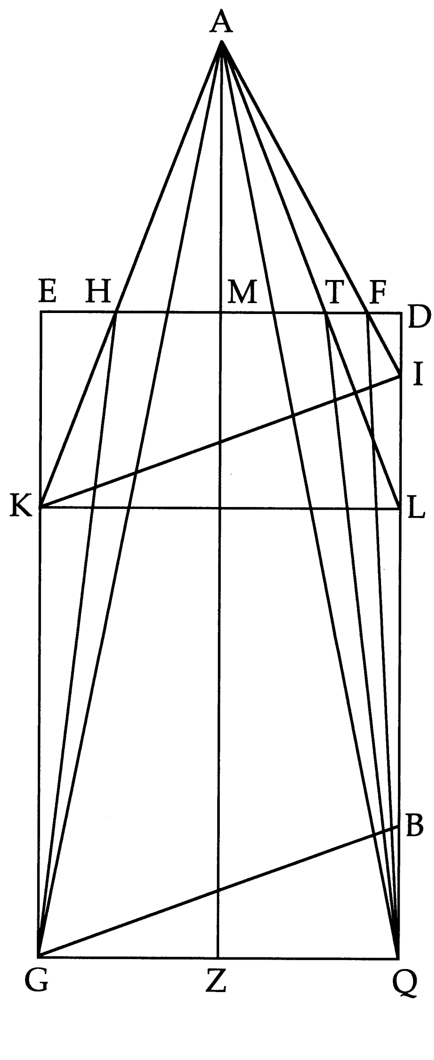


FIGURE 7.7.10a

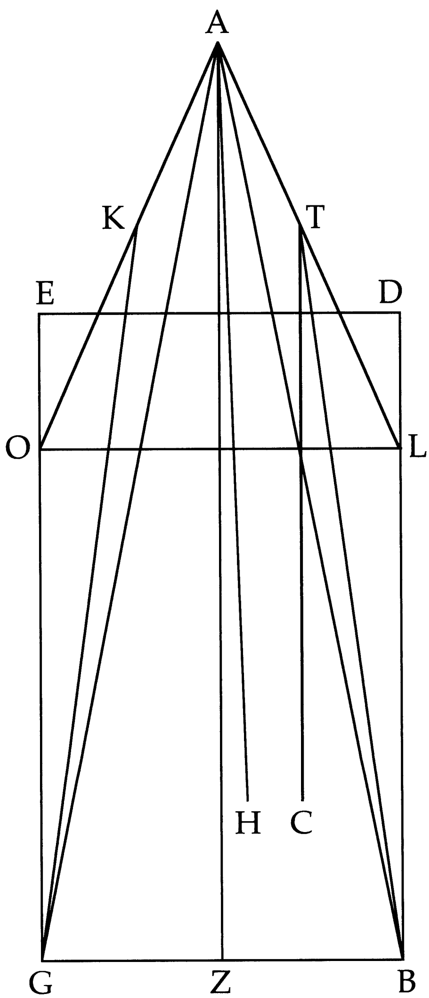


FIGURE 7.7.11

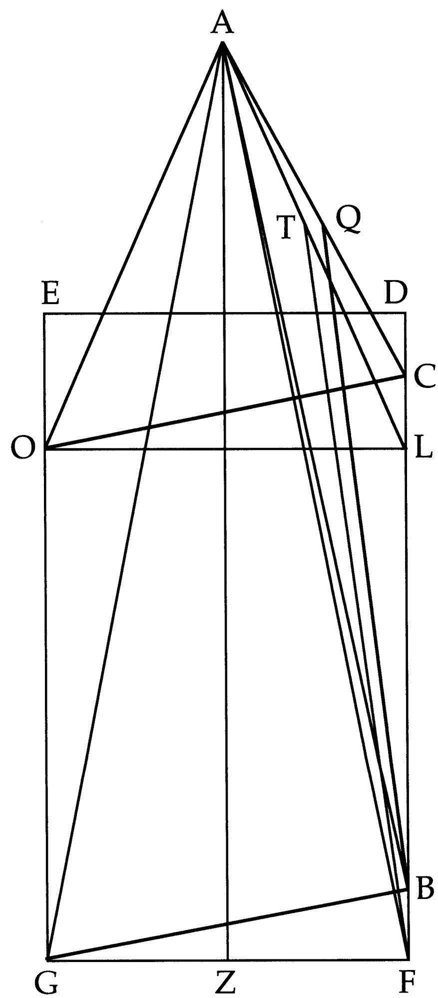


FIGURE 7.7.11a

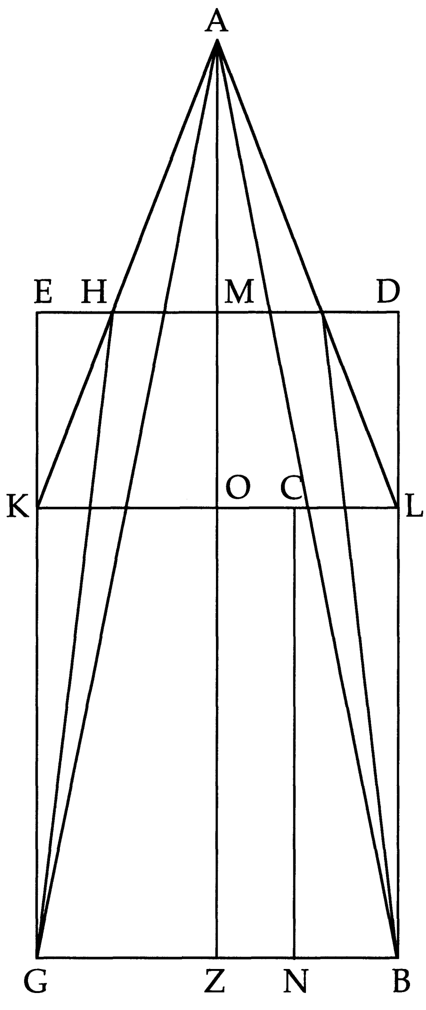


FIGURE 7.7.12

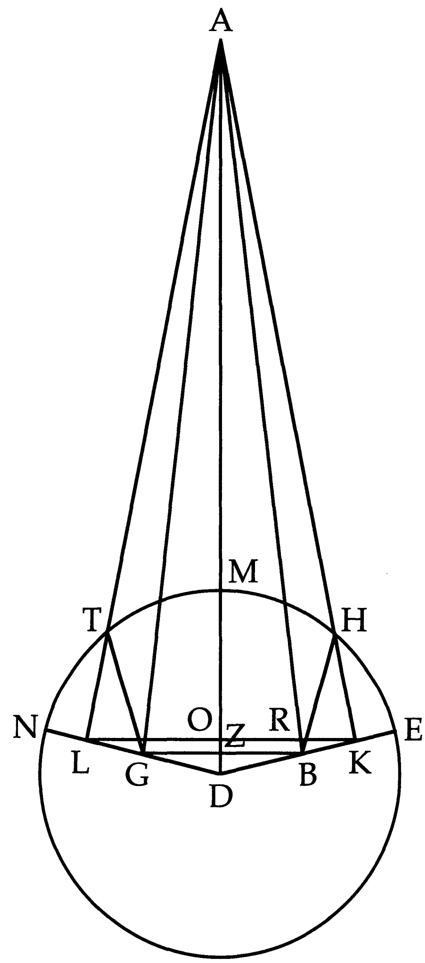


FIGURE 7.7.13

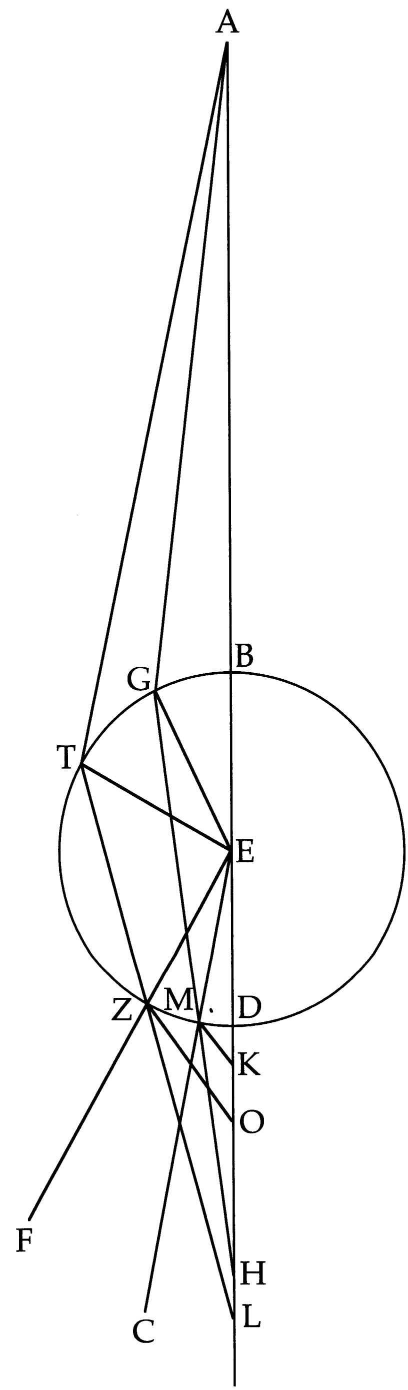


FIGURE 7.7.14

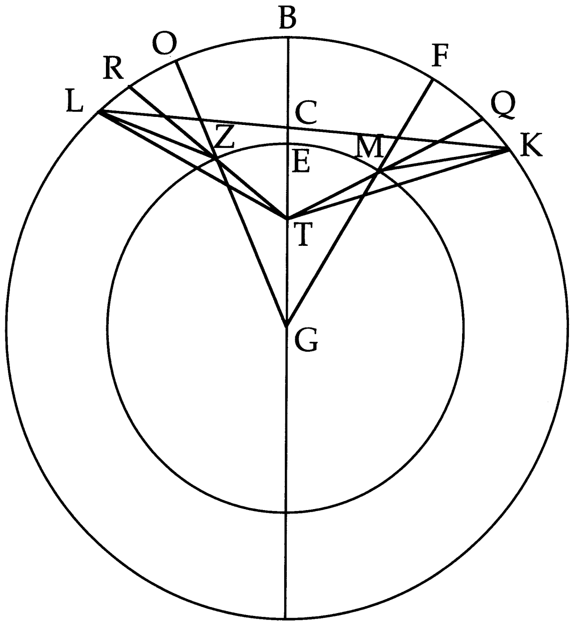


FIGURE 7.7.15

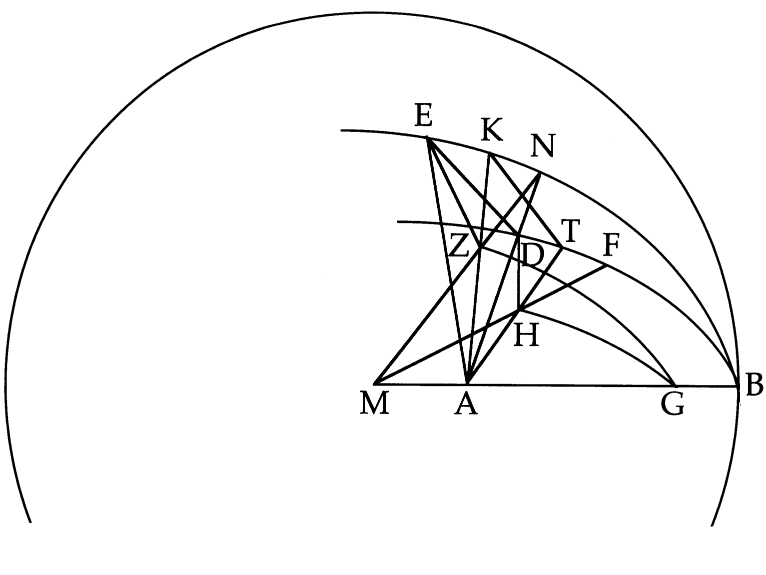


FIGURE 7.7.16

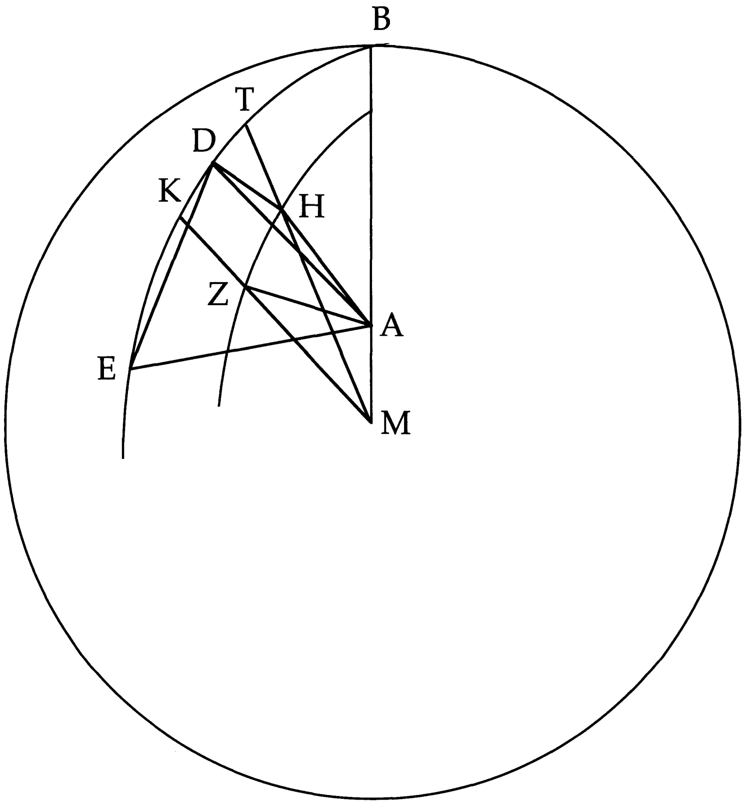


FIGURE 7.7.17

LATIN-ENGLISH INDEX

LATIN-ENGLISH INDEX

ENGLISH-LATIN GLOSSARY

ENGLISH-LATIN GLOSSARY

about/around circuitus
to abut sequi
accidental accidentalis, accidentaliter
to accommodate recipere
according to refraction reflexive
accordingly modus
across longitudo
to act efficere
actual vere, verus
actuality veritas
acute acutus
acute angle acuitas
to add addere
addition additio
adhesive inglutum
to adjust movere, mutare
adjustment motus
to affect mutare, pati
to affix applicare
aforementioned/aforesaid predictus
air aer
albugineous humor albuginea
alidade hora
to align (with) adequare, opponere, verificare
alike similis, similiter
all numerus
along (a line or path) verticatio
to alter mutare, transmutare
altitude altitudo, elevatio
amount quantitas
amount of refraction quantitas reflexionis
amplitude of a cone piramidalitas
analogous transumptivus
ancient antiquus
angle angulus
angle of inclination angulus declinationis
angle of intersection angulus sectionis

angle of refraction angulus reflexionis
aperture foramen
apparatus instrumentum
apparent manifestus
to appear apparere, habere, videre
appearance apparitio
to apply (to) applicare, ponere, sequi, superponere
to approach appropinquare
to appropriate appropriare
arc arcus
area locum, locus
area of intersection locus sectionis
area of refraction locus refractionis
to arise accidere, facere
armillary armilla
armillary apparatus instrumentum armillarum
armillary sphere instrumentum de armillis
to arrange complere, ordinare, ponere
arrangement ordinatio, positio
to arrive at invenire
to ascertain certificare
to assess perpendere
to assume opinari, ponere, preservare
astrolabe astrolabium
at an angle obliquus
to attach applicare, coniungere, paxillare, ponere, superponere
to attest to testari
to avoid evadere
away (from) contrarius
axis axis, linea, perpendicularis
axis of the eyes axis visuum

back/backside dorsus
ball pila, spera
base basis
to be existere, facere, habere
to be accidental accidere
to be apparent apparere
to be appropriate convenire
to be at center mediare
to be at rest quiescere
to be attached adherere
to be behind opponere
to be clear/evident/manifest/obvious patere
to be close to contingere
to be contiguous contingere

to be continuous continuare
to be dropped exire, existere
to be established constare
to be faint/poorly visible latere
to be feasible distinguere
to be hidden latere
to be in contact with contingere
to be in front of opponere
to be in place preservare
to be incident intendere
to be left remanere
to be like assimulare
to be nearly appropinquare
to be next to sequi
to be noticeable latere
to be obliged (need/ought/should) debere
to be oblique declinare
to be on the other side opponere
to be/remain parallel equidistare
to be tangent to contingere
to be the case that constare
to be up against contingere, sequi, tangere
to be up high eminere
to be visible apparere, videre
to become facere
to become larger crescere
to become smaller decrescere
to become still quiescere
to befall accidere
to begin incipere
beginning initium
to bend incurvare
bending incurvatio
to bisect dividere, secare, transire
bisection divisio
black/blackness niger, nigredo
block frustrum
to block delere
block of glass vitrum
bluish glaucus
bodily mass corporeitas
body corpus
book liber, tractatus
to bore perforare
bottom anterior, fundamentum, fundus, profundum
breadth latitudo

to break frangere
brim hora
to bring facere, mittere, ponere
to bring close/near appropinquare
broad latus, spatiosus
bronze es
brute nudus
by means of refraction reflexive
by the same token similis, similiter

calculation computatio
to calm quiescere
capacious amplus
case dispositio, positio, situs, status
cause causa
to cause facere
celestial mundus
celestial body celestis
celestial pole polus mundi
center centrum, medius
center of sight centrum, centrum visus, visus
center of the eye centrum visus
center of the universe centrum mundi
center of the world centrum mundi
centerpoint centrum, punctus centri
to change mutare
to change position movere
chapter capitulum, differentia, pars, tractatus
chickpea granum ciceris
chord corda
to choose eligere
circle circulus
circle of refraction circulus reflexionis
circle of the meridian circulus meridiei
circular circularis
circumference circumferentia
to circumscribe circumdare, continere
to clarify manifestare
clear clarus, evidens, manifestus
close propinquus
to close (off) claudere
cloud nubis
to cohere coniungere
to coincide contingere, superponere, tangere
color color
to color tingere

colored coloratus
to come to pervenire
common communis
common angle angulus communis
common axis axis communis
common body corpus communis
common nerve nervus communis
common section differentia communis
to compare comparare
to complete complere, perficere
to compose componere
to comprise continere
concave concavus
concave curvature concavitas
concave section concavus
concave surface concavitas, concavus
concavity concavitas, concavus
to concern solicitare
to conclude terminare
condition positio
to conduct facere
to conduct an experiment experiri
to confirm certificare, declarare
to confuse ambigere
cone piramis
cone of radiation piramis radialis
cone of refraction piramis reflexa
conical piramidalis
to conjoin superponere
to connect continuare, copulare
conspicuous manifestus
constant perpetualis, perpetuus
to constrict constringere, diminuere
to construct ponere
to contact continuare, tangere
to contain circumdare, continere
container vas
continuation continuatio
to continue extendere, extrahere, procedere, transire
continuous continuus
continuum successio
to converge congregare, coniungere
converse conversus
convex convexus
convex section convexus
convex surface convexus

convexity convexitas, convexus, gibbositas
to convey deferre
copper cupreus, cuprus
to copy iterare
cornea cornea
correct vere, verus
correctly oriented rectus
to correlate comparare
correlation comparatio
corresponding consimilis, similis
counterpart compar
cross-section diameter
crystal cristallinus, cristallus
crystalline cristallinus
cube cubicus
cube of glass vitrum
cubical cubicus
cubit cubitum
customary assuetus
cut sectio
to cut facere, secare, signare
cut edge locus sectionis
to cut into descendere
to cut off distinguere, dividere, secare
cylinder columpna, columpnalis
cylindrical/cylindrical object columpnalis

to darken with shadow obumbrare
day dies
daylight lux diei
to deal with declarare, negotiari, tractare
to deceive fallere
to define distinguere
to deflect deviare
deflection declinatio
to demarcate distinguere, signare
to demonstrate declarare, demonstrare, monstrare
demonstration demonstratio
denarius denarius
dense densus, grossus, spissus
density grossities
to depict depingere
depth altitudo, spissitudo
to describe signare
described/discussed earlier predictus

description doctrina
detail distinctio
determination certificatio
to determine (accurately or empirically) certificare, declarare, determinare, experimentare, experiri, mensurare, notare, sentire
to deviate exire
device instrumentum
diagram figura, forma
diameter diameter
to dictate exigere
to differ differre, diversare
difference differentia, diversitas, excessus
different types diversitas
to differentiate distinguere
differentiation differentia
digit digitus
dimension dimensio
to diminish demittere
diminished minor, minus
to dip intingere, tingere
direct directus, rectitudo
to direct apponere, opponere, precipere
direction pars, verticatio
directly facing rectus
directly overhead verticatio capitis
to discern distinguere, videre
to discover invenire
discrepancy distantia
to discuss declarare, loqui
disk circulus
disparity diversitas
to dispose preservare
disposition positio, situs
distance distantia, longitudo, remotio, spatium
distant remotus
to distinguish distinguere
to diverge diversare
divergence declinatio, diversitas
to divert referre, removere, transferre
to divide dividere
to do complere, facere, observare, perficere
to draw continuare, describere, extrahere, signare
to draw inward attrahere
to drill perforare
to drop cadere, egredi, extendere, extrahere
due causa

each other adinvicem
earlier predictus
earth terra
east oriens
eastern orientalis
edge acuitas, extremitas, finis, hora, latus, terminus
elevation elevatio
to eliminate destruere
to embrace continere
empirical demonstration/test experientia, inductio
empty vacuus
to enclose claudere, continere
to encounter invenire, occurrere
end/endpoint extremitas, finis, terminatio, terminus
end piece superfluitas
to ensure preservare
to entail dependere
to enter intrare, pervenire
entire universus
to envelop continere
equal equalis
equality equalitas
equator equinoctialis
equinoctial equinoctialis
equivalent consimilis, equalis, similis
to erase delere
erect rectus
to erect erigere, extrahere, statuere
to err errare
error error
to establish certificare, declarare
even equalis
to even out radere, verificare
evident manifestus
to examine aspicere, declarare, inspicere, intueri
to exceed addere, excedere, superfluere
excess additio
excess piece differentia, superfluitas
to excise scindere, secare
to exit egredi
experience experientia
experiment/experimentation experientia, experimentatio
experimental determination experimentatio
experimenter experimentator
to explain declarare, ostendere

to extend ducere, exire, existere, extendere, extrahere, pervenire, protrahere, transire
to extend beyond superfluere
extension extensio
extent magnitudo, quantitas
extreme magnus
eye oculus, visus
eyelid palpebra

face basis, facies, medius, superficies
to face opponere, sequi, stare
face of the Earth planities terre
facing position opponere
faint debilis
to fall cadere
far away magnitudo, remotio, remotus
fast velox
to fasten applicare
figure figura
to fill implere, infundere
to fill with effundere
to find invenire, sumere
fine minutus, subtilis, tenuis
finger index
to finish complere, perficere
fire ignis
first initium
to fix figere
flat equalis, planus
to follow accidere, incipere, movere, patere, sequi
force fortitudo, violentia, vis
forceful fortis
forefinger index
foregoing predictus
form figura, forma
to form constituere, continere, efficere, extrahere, facere, habere, tenere
to form a square circumdare
foundation fundamentum

general(-ly) universalis, universaliter
gentle modicus
given predictus
glacial humor glacialis
to glance off labi
glass vitreus, vitrum
to go outward removere

to graduate ordinare
grain of barley granum ordeacii, granum ordei
to grasp habere
great magnus
great circle circulus, circulus magnus
to grind abradere, adequare
grinding abrasio
grinding machine confrictorio
ground terra

hand manus
half medietas
half-segment medietas
to hang from superponere
to happen accidere, facere
to happen previously precedere
to have habere
having been made smaller diminutio
head capud
heavenly body stella
heavens celum, orbis
height altitudo, spissitudo
hemisphere medietas spere, semispera
to hold tenere
to hold fixed/stationary figere
to hold up erigere
hole foramen
hollow concavitas, concavus, foramen, vas
home domus
horizon orizon
hour hora
how (something occurs) modus, qualitas, via
how light refracts qualitas reflexionis
humor humor
to hurl eicere

illuminated lucidus
image fantasma, forma, ymago
image location locus ymaginis
image point punctus ymaginis
imaginary ymaginabilis
imagination ymaginatio
to imagine intelligere, ymaginare, ymaginari
immovability consolidatio
immutable immutabilis
to impede impedire

implement instrumentum
impossible/impossibility impossibilis
in contact with continuus
inability to see occultatio
to incise signare
incising tool scalpsio
inclination/incline declinatio, obliquatio
to incline declinare
inclined declinis, obliquus
to include continere
to increase addere, crescere
indefinite indeterminatus
to indicate signare
in fact vere, verus
in front of oppositio
initial point principium
ink incaustum
in line with equidistans, rectitudo
inner/inside anterior, interior
inner wall interior
to inscribe signare
inseparable inseparabilis
to insert imponere, mittere, ponere
to inspect intueri
in succession/in turn successivus
intense fortis
interface differentia communis
to interface distinguere
to interfere with impedire, occultare
to intermingle admiscere
to intersect concurrere, coniungere, intersecare, occurrere, pervenire, secare
intersection concursus, locus sectionis, sectio
interval distantia, remotio
introduction proemium
invariant perpetuus
to investigate experiri
iron ferreus, ferrum
to issue exire

to join continuare, copulare
to judge putare

to keep habere
to keep from prohibere
kind modus
to know scire

large amplus, magnus
large(ness) amplitudo
lathe retornativus, tornatorius
to lay (as a foundation) proponere
to leave exire, relinquere
to leave be existere, remanere
left (-hand side) sinister
length longitudo, quantitas
lengthwise longus
less than minor, minus
to lie existere, requiescere
to lie above elevare
to lie at a distance/far distare, removere
to lie between interesse, opponere
light lumen, lux
lighted lucidus, luminosus
like consimilis
likewise similis, similiter
limit finis
line linea
line of sight linea radialis, visus
to line up ponere
lip (of a vessel) circumferentia
to locate ponere
location locum, locus, situs
location (of living) locus habitationis
log lignum
long longitudo, longus
longitude longitudo
to look (at/into/through) aspicere, inspicere, intueri, respicere
loose levis
lower inferior
to lower demittere
lucent lucidus
luminous lucidus
lying at existentia
lying to the side declinis, obliquatio, obliquus

magnification additio, excessus, magnitudo
to maintain observare, preservare
to make continere, facere
to make a preliminary point premittere
to make certain/sure preservare
to make clear/evident/manifest/obvious/visible declarare, manifestare
to make disappear abscindere

to make firm/permanent/solid figere
to make flush with superponere
to mandate exigere
manifest manifestus
manifold multimodus
many times multiplex
to mark (off) distinguere, dividere, facere, separare, signare
mark signum
marked differentia
to match assimilare, assimulare, comparare
to mean intelligere
means modus
to measure (off) distinguere, dividere, mensurare, separare
to meet occurrere
to meet a condition preservare
to melt dissolvere
meridian meridies
meridian circle circulus meridiei
middle/midpoint medietas, medius, punctus medius
milk lac
to mingle miscere
minute minutum
mirror speculum
misperception deceptio, fallacia
to mix miscere
mode modus
moon luna
moonrise ortus lune
motion motus
to move incurrere, movere, mutare, removere
to move around revolvere
to move back elongare
moving body motus
much magnus
to multiply by two duplicare

narrow modicus
to narrow adunare
narrowness gracilitas, subtilitas
natural naturalis
natural characteristic natura
near/next to propinquus, proximus
nearly equal propinquus
nearness propinquitas
to need indigere
needle acus

neighboring propinquus
to nest against applicare, superponere
night nox
no/none/not any nullus
normal linea perpendicularis, perpendicularis
to notice videre
noticeable sensibilis
number numerus

object corpus, res
object point punctum, punctus
oblique declinis, obliquus
obolus obulus
observation inspectio
to observe aspicere, experiri, invenire, videre
obstacle impedimentum
to obtain adquirere, habere, sequi
obtuse obtusus
obvious manifestus
to occur accidere, facere
one numerus
one another adinvicem
opacity densitas, densitudo
opaque densus
opening foramen
to oppose opponere
opposite contrarius
opposition oppositio
order ordo
organ membrum
orientation ordinatio, pars, positio
origin principium
orthogonal(-ly) perpendicularis, perpendiculariter
other reliquus
outer/outside exterior
outer edge circumferentia
outer end extremitas
overarching generalis, generaliter
to overlap superfluere

to paint depingere
panel hora, lamina
paper bombax
parallel equidistans
parallelism equidistantia
to parse dividere

part pars
to pass egredi
to pass along/from/through/to exire, extendere, pertransire, transire
passage extensio, transitus
path linea
pen calamus, penna
people/person homo
to perceive certificare, comprehendere, patere
perceptible comprehensibilis, sensibilis
perception comprehensio, sensus
perfect vere, verus
to perforate perforare
perimeter circumferentia
periphery circumferentia
permeable passibilis
perpendicular perpendicularis
perpendicular line/path linea perpendicularis
physical naturalis
picture pictura
piece frustum, regula
pin stilus
place locum, locus, positio, situs
to place applicare, mittere, ponere
to place upon superponere
plane planus, superficies
plane of refraction superficies reflexionis
to plane down adequare
plank tabula
plate lamina
point capud, locum, locus, punctum, punctus, signum
point of bisection locus divisionis
point of intersection locus sectionis, punctus sectionis
point of reflection locus reflexionis
point of refraction locus reflexionis, punctus reflexionis
pointed edge acumen
pointed out earlier predictus
pole polus
to polish polire
polished tersus
portion portio
to parse dividere
to pose/position applicare, ponere
position positio, situs
pot cadus
pottery jar olla
to pour effundere, fundere, infundere

to precede precedere
preceding predictus
precise vere, verus
to presume ponere
previous predictus
to proceed procedere
to produce extrahere, facere
to project eminere
proof declaratio, demonstratio, probatio
to propagate oriri
proper(-ly) vere, verus
property res
proportion proportio
proposition figura
to protrude eminere
to prove declarare, demonstrare
proximity propinquitas
to pull off evellere
to purify purificare
to push mittere
to push against repellere
to put at/to applicare, mittere, ponere, superponere
to put back revertere
to put into place considerare, preservare

quadrant quarta
quick velox

radial radialis
radial line linea radialis
radiant light splendor
radiation radialis
radius medietas diametri, semibasis, semidiameter
to raise elevare
rare rarus, subtilis
rarity subtilitas
ratio proportio
ray radius
to reach cadere, exire, extendere, facere, pervenire, prevenire, venire
to read legere
reality veritas
reason causa, ratio
reasoning causa
to recapitulate iterare
to receive recipere
rectangle quadratus

rectilinear rectitudo
to redraw iterare
to reduce diminuere
reduction diminutio
reed calamus
to reflect convertere, reflectere, revertere
reflection conversio, reflexio
to refract reflectere
refracted vision reflexio
refraction reflexio
refractive interface decisum
to regard aspicere
regard to refraction locus reflexionis
register/register plate lamina
reinforcement additio
to reinsert reducere
to relate respicere
relation quantitas
relationship proportio
to remain existere, remanere
to remain the same permanere
remainder residuus
remaining/rest reliquus, residuus
remote remotus
remoteness remotio
removal ablatio
to remove auferre, diminuere, elevare, evellere, reducere, separare
to render facere
to replace ponere
to represent signare
repulsive force repulsio
to require exigere
to resist resistere
resistence resistentia
to respect respicere
respect to pars
to result accidere
to return redire
to reveal patere
revolution revolutio
to revolve revolvere
right rectus
right (-hand side) dexter
right angle angulus rectus
right at/on vere, verus
rim hora

ring anulus, armilla
to rise oriri
rising ascensio, ortus
rod lignum
room domus
to rotate circumvolvere, revolvere
round rotundus
to round off retornare
rounded rotunditas
roundness rotunditas
rubbing confricatio
ruler regula

same consimilis, equalis, similis
to satisfy continere
to scatter separare
to score signare
section differentia, linea, locus sectionis, sectio
to see videre
segment pars, portio
segment on a circle circulatio
semicircle dimidium, medius circulus, semicirculus
semicylinder semicolumpna
to send mittere
sensation sensus
to sense sentire
sense of sight sensus
sense perception sensus
sensitive sensibilis
sensitive power virtus sensibilis
series of tests experimentatio
setup positio
to set up componere, erigere, facere, ponere, preservare
several multitudo
to shade/shadow obumbrare
shadow umbra
shallow paucus
shape figura, forma, quantitas
to shift extrahere, movere, mutare
to shine exire, oriri
short parvus
to show declarare, ostendere, patere
to show beforehand/previously predeclarare, preostendere
side latitudo, latus, pars
sight radialis, visus
sighting hole hora

significant magnus
silver object argentum
similar consimilis, equalis, similis
similarly similiter
simple simplex
to sit sedere
situation positio, situs, status
size magnitudo, quantitas, remotio
size in relation proportio
size of [angle of] refraction quantitas reflexionis
sky celum
slant obliquatio
to slant declinare
slanted/slantwise declinis, obliquus
to slide off exire
slight minor, minus, modicus, parvus
small minor, minus, parvus, strictus, subtilis
to smooth out adequare
snow white nivius
solid solidus
somewhat modicus
sort modus
soul anima
space spatium
to span respicere
specific characteristic proprietas
specific to particularis
speed velocitas
sphere instrumentum, orbis, spera, spericus
sphere of glass vitrum
spherical spericus
spot locum, locus, particula
to spread out amplificare
square quadratus
to stand stare
to stand above preminere
to stand straight up/upright erigere
star stella
to situate ponere
to start incipere
stationary immobilis
to stay existere
to stay the same remanere
steel ferrum
stone lapideus, lapis
stood on edge erectus

straight directus, rectus
straight ahead oppositio
straight line linea recta, rectitudo, verticatio
straightness rectitudo
to strike existere, occurrere, percutere
strip regula
strong fortis
stylus acus, festuca, regula, stilus
to subdivide dividere
substance substantia
to substantiate certificare
to subtend cordare, facere, respicere
subtraction diminutio
successive successivus
to suffice sufficere
sun lux solis, sol
sunlight lumen solis, lux solis, sol
sunrise ortus solis
to suppose ponere
surface superficies
surface at which refraction occurs superficies reflexionis
surround circuitus
to surround circumdare, continere
swift velox
sword ensis

to take accepere, accipere, ponere, sumere
to take account of considerare, notare, preservare
to take together coniungere
tear duct lacrimal
tendency proprietas
terrestrial terrestria
to test empirically experimentare, experiri
test experimentatio
testing experientia
theorem figura
thick grossus
thickness grossities, spissitudo
thin gracilis, parvus, subtilis
thing res
those who live somewhere habitantes
throughout universus
time hora, tempus
tiny parvus
tooth dens
top ultimitas

to touch applicare
toward oppositio
to track preservare
transit transitus
translucency diaffonitas
translucent diaffonus
to transmit deferre, reddere
transparency diaffonitas
transparent diaffonus
transverse transversus
travel motus
treatise liber
triangle triangulus
true vere, verus
to truncate abscindere
turn revolutio
to turn movere, revolvere
to turn out to be invenire
turner figulus
turning mechanism tornatorius
twice duplus

to uncover detegere
to understand cognoscere, intelligere
unequal inequalis
uninterrupted continuus
to unite adunare
universal communis
universe mundus
upright erectus, rectus
usual mediocris
uvea uvea

valley vallis
vantage positio
vapor vapor
variation diversitas
various ways diversitas
to vary differre, diversare
vast magnus, spatiosus
to verify verificare
vertex capud, cornus
vertical verticalis
vessel vas
viewer aspiciens, visus
visible manifestus, visibilis

visible object res visa, res visibilis, visibilia, visibilis, visum
visible point punctus visus
vision visio, visus
visual axis axis visuum
visual faculty visus

to wait expectare
wall paries
to want volere
washbasin pelvis
water aqua
wax cera
way compositio, modus, ratio, situs, via
weak debilis
to weaken debilitare
weakening debilitas
weakness debilitas
where locum, locus
white albus
white(ness) albedo
white lead cerusa
why causa
wide amplus, latus
width latitudo, spissitudo
widthwise edge latitudo
window foramen
to wish volere
wooden ligneus
wooden stylus lignum
world mundus
to write scribere
writing scriptura

to yield cedere

zenith capud, cenit, cenit capitis, vertex capitis, verticatio, verticatio capitis

BIBLIOGRAPHY

Ackrill, J. L. *See* Aristotle, *Categories*.

Akdogan, Cemil. *Albert's Refutation of the Extramission Theory of Vision and His Defence of the Intromission Theory.* Kuala Lumpur: International Institute of Islamic Thought and Civilization, 1998.

Alberti, Leon Battista. *See* Spencer.

al-Kindī. *See* Björnbo and Vogel.

Alverny, M-Th. d', and F. Hudry, "Al-Kindī, *De Radiis*," *Archives d'histoire doctrinale et littéraire du moyen âge* 41 (1974): 139-260.

Anawiti, G. C., and A. Z. Iskandar, "Ḥunayn ibn Isḥāq," *Dictionary of Scientific Biography*, vol. 15, supplement 1, Charles Coulston Gillispie, ed., pp. 230-249. New York: Scribner's, 1978.

Apollonius of Perga. *Conics. See* Heiberg, Taliaferro.

Aristotle. *Categories*. Trans. J. L. Ackrill, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *De anima*. Trans. J. A. Smith, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *De memoria et reminiscentia*. Trans. J. I. Beare, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *De sensu et sensato*. Trans. J. I. Beare, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *Meteorology*. Trans. E. W. Webster, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *Physics*. Trans. R. P. Hardie and R. K. Gaye, in *The Complete Works* of *Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Aristotle. *Posterior Analytics*. Trans. Jonathan Barnes, in *The Complete Works of Aristotle: The Revised Oxford Translation*, Jonathan Barnes, ed. Princeton: Princeton University Press, 1984.

Ausonio, Ettore. *Theorica speculi concavi spaerici*. N.p., n.d.

Avicenna. *See* Van Riet.

Baarmann, J., "Abhandlung über das Licht von Ibn al-Haitam," *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 36 (1882): 195-237.

Bacon, Roger. *See* Lindberg.

Badawi, 'Abdurrahman. *Histoire de la Philosophie en Islam*. 2 vols. Paris: Vrin, 1972.

Barker, Peter, Hanne Andersen and Xiang Chen. *The Cognitive Structure of Scientific Revolutions*. Cambridge: Cambridge University Press, 2006.

Barnes, Jonathan, ed. *The Complete Works of Aristotle: The Revised Oxford Translation*. Princeton: Princeton University Press, 1984. *See also* Aristotle, *Posterior Analytics*.

Barrow, Isaac. *Isaac Barrow's Optical Lectures, 1667*. A. G. Bennett, D. F. Edgar, and H. C. Fay, eds. and trans. London: The Worshipful Company of Spectacle Makers, 1987.

Bartholomaeus Anglicus. *See* Long.

Beare, J. I. *See* Aristotle, *De memoria et reminiscentia, De sensu et sensato*.

Bellosta, Hélène, "Burning Instruments from Diocles to Ibn Sahl," *Arabic Sciences and Philosophy* 12 (2002): 285-303.

Bennett, A. G. *See* Barrow.

Biagioli, Mario. *Galileo Courtier*. Chicago: University of Chicago Press, 1993.

Bierens de Haan, D. *See* Huygens.

Björnbo, Axel, and Sebastian Vogel, ed. and trans. *Alkindī, Tideus und Pseudo-Euklid: Drei optische Werke*. Leipzig: Teubner, 1912.

Bloom, Jonathan M. *Paper Before Print: The History and Impact of Paper in the Islamic World*. New Haven: Yale University Press, 2001.

Blum, André. *On the Origin of Paper*. New York: R. R. Bowker, 1934.

Bosscha, J. *See* Huygens.

Boyer, Carl B. *The Rainbow: From Myth to Mathematics*. New York: Yoseloff, 1959.

Brownson, C. D., "Euclid's Optics and its Compatibility with Linear Perspective," *Archive for the History of Exact Sciences* 24: 165-194.

Burton, Dan, ed. and trans. *Nicole Oresme's* De visione stellarum (On Seeing the Stars). Leiden/Boston: Brill, 2007.

Cennini, Cennino. *Libro dell'arte. See* Thompson.

Clagett, Marshall, "A Medieval Latin Translation of a Short Arabic Tract on the Hyperbola," *Osiris* 11 (1954): 359-366.

Clagett, Marshall. *The Science of Mechanics in the Middle Ages*. Madison: University of Wisconsin Press, 1959.

Clagett, Marshall. *Archimedes in the Middle Ages*. 5 vols. Madison: University of Wisconsin Press, 1964, vol. 1. Philadelphia: American Philosophical Society, 1965-1984, vols. 2-5.

Copernicus. *De revolutionibus orbium caelestium*. *See* Duncan.

Crew, Henry. *The* Photismi de lumine *of Maurolycus: A Chapter in Late Medieval Optics*. New York: Macmillan, 1940.

Dales, Richard C., "Robert Grosseteste's Scientific Works," *Isis* 52 (1961): 394-402.

Dales, Richard C. *The Problem of the Rational Soul in the Thirteenth Century*. Leiden: E. J. Brill, 1995.

De Lacy, Phillip, ed. and trans. *Galen on the Doctrines of Hippocrates and Plato*. Corpus Medicorum Graecorum 4.4.1.2. Berlin: Akademie Verlag, 1980-1984.

Denery, Dallas. *Seeing and Being Seen in the Later Medieval World: Optics, Theology and Religious Life*. Cambridge: Cambridge University Press, 2005.

Donahue, William H., trans. *Johannes Kepler, Optics: Paralipomena to Witelo and Optical Part of Astronomy*. Santa Fe, NM: Green Lion Press, 2000.

Duncan, A. M., trans. *Copernicus, On the Revolution of the Heavenly Spheres*. New York: Barnes and Noble, 1976.

Dupré, Sven, "Ausonio's Mirrors and Galileo's Lenses: The Telescope and Sixteenth-Century Practical Optical Knowledge," *Galileana* 2 (2005): 145-180.

Dupré, Sven, "Optics, Pictures and Evidence: Leonardo's Drawings of Mirrors and Machinery," *Early Science and Medicine* 10 (2005): 211-236.

Eastwood, Bruce S., "Grosseteste's 'Quantitative' Law of Refraction: A Chapter in the History of Non-Experimental Science," *Journal of the History of Ideas* 28 (1967): 403-414.

Eastwood, Bruce S. *The Elements of Vision: The Micro-Cosmology of Galenic Visual Theory According to Ḥunayn Ibn Isḥāq*. Transactions of the American Philosophical Society 72.5. Philadelphia: American Philosophical Society 1982.

Eastwood, Bruce S., "Alhazen, Leonardo, and Late-Medieval Speculation on the Inversion of Images in the Eye," *Annals of Science* 43 (1986): 413-446.

Edgar, D. F. *See* Barrow.

Euclid. *Elements*. *See* Heath.

Euclid. *Optics*. *See* Heiberg, Kheirandish.

Falco, Charles, "The Art of the Science of Painting," *Proceedings of the Symposium on Effective Presentation and Interpretation in Museums*. Dublin: National Gallery of Ireland, 2003.

al-Fārisī, Kamāl al-Dīn. *Tanqīḥ al-Manāẓir*. *See* Wiedemann.

Favaro, Antonio, ed. *Le opere di Galileo Galilei*, vol. 3.2. Florence: Barbèra, 1907.

Fay, H. C. *See* Barrow.

Fried, Michael, and Unguru, Sabetai. *Apollonius of Perga: Text, Context, Subtext*. Leiden: Brill, 2001.

Frisius, Gemma. *De Radio Astronomico et Geometrico*. Antwerp/Louvain, 1545.

Galen. *De placitis Hippocratis et Platonis*. *See* De Lacy.

Galen. *De usu partium*. *See* May.

Galileo. *Siderius Nuncius*. *See* Van Helden.

Galileo. *Theorica speculi concavi sphaerici*. *See* Favaro.

Gaukroger, Stephen. *Descartes: An Intellectual Biography*. Oxford: Clarendon Press, 1995.

Gilson, Étienne, "Les sources Gréco-Arabes de l'Augustinisme Avicennisant," *Archives d'histoire doctrinale et littéraire du moyen âge* 4 (1929): 5-149.

Gilson, Étienne. *Reason and Revelation in the Middle Ages*. New York: Scribner's, 1938.

Gilson, Simon. *Medieval Optics and Theories of Light in the Works of Dante*. Lewiston, Queenston, Lampeter: Edwin Mellen, 2000.

Girke, Dorothea, ed. *Eilhard Wiedemann: Gesammelte Schriften zur arabisch-islamischen Wissenschaftsgeschichte*, vol. 1. Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften an der Johann Wolfgang Goethe-Universität, 1984.

Goldstein, Bernard R., trans. *The Arabic Version of Ptolemy's Planetary Hypotheses*. Transactions of the American Philosophical Society 57.4. Philadelphia. American Philosophical Society, 1967.

Gorini, Rosanna, "Al-Haytham the Man of Experience, First Steps in the Science of Vision," *Journal of the International Society for the History of Islamic Medicine* (2003): 53-55.

Granger, Frank, trans. *Vitruvius:* De architectura, *Books I-IV.* Cambridge, MA: Harvard University Press, 1931.

Grant, Edward, "Jordanus Nemorarius," *Dictionary of Scientific Biography*, vol. 7, Charles Coulston Gillispie, ed., pp. 189-210. New York: Scribner's, 1973.

Grant, Edward. *Much Ado About Nothing: Theories of Space and Vacuum from the Middle Ages to the Scientific Revolution*. Cambridge: Cambridge University Press, 1981.

Grant, Edward. *In Defense of the Earth's Centrality and Immobility: Scholastic Reaction to Copernicanism in the Seventeenth Century*. Transactions of the American Philosophical Society 74.4. Philadelphia: American Philosophical Society, 1984.

Grant, Edward. *The Foundations of Modern Science in the Middle Ages*. Cambridge: Cambridge University Press, 1996.

Hall, A. R., and M. B. *See* Oldenbourg.

Hanson, N. R., "The Copernican Disturbance and the Keplerian Revolution," *Journal of the History of Ideas* 22 (1961): 169-184.

Hardie, R. P., and R. K. Gaye. *See* Aristotle, *Physics*.

Harvey, E. Ruth. *The Inward Wits: Psychological Theory in the Middle Ages and the Renaissance*. Warburg Institute Surveys, 6. London: University of London Press, 1975.

Hawthorne, John G., and Cyril Stanley Smith, trans. On Divers Arts: *The Treatise of Theophilus*. Chicago: University of Chicago Press, 1963.

Heath, T. L. *Diophantus of Alexandria*, 2nd ed., 1910; reprint, New York: Dover, 1964.

Heath, T. L. *Aristarchus of Samos: The Ancient Copernicus.* Clarendon: Oxford University Press, 1913.

Heath, T. L., trans. *The Thirteen Books of Euclid's* Elements, 3 vols, 2nd ed., 1925; reprint ed., New York: Dover, 1956.

Heiberg, I. L., ed. *Euclidis opera omnia*, 8 vols. & supplement. Leipzig: Teubner, 1883-1916.

Heiberg, I. L., ed. *Apollonii Pergaei Quae Graece Existant Cum Commentariis Antiquis*. 2 Volumes. Leipzig: Teubner, 1891-1893.

Heiberg, I. L., ed. *Sereni Antinoensis opuscula*. Leipzig: Teubner, 1896.

Heiberg, I. L., and Eilhard Weidemann, "Ibn al-Haitams Schrift über parabolische Hohlspiegel," *Bibliotheca Mathematica*, ser. 3, 10 (1909-1910): 201-237.

Hero of Alexandria. *Catoptrica*. *See* Schmidt.

Hockney, David. *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters.* New York: Viking, 2001.

Hogendijk, Jan P. *Ibn al-Haytham's Completion of the Conics*. New York: Springer, 1984.

Huxley, G. L. *Anthemius of Tralles: A Study in Later Greek Geometry.* Cambridge, MA: Harvard University Press, 1959.

Huygens, Christiaan. *Oeuvres complètes*. D. Bierens de Haan, J. Bosscha, D. J. Korteweg and J. A. Vollgraff, eds. and trans., 22 vols. LaHaye: Nijhoff, 1988-1950.

Ibn al-Haytham. *De configuratione mundi*. *See* Langermann.

Ibn al-Haytham. *Kitāb al-Manāẓir*. *See* Sabra.

Ibn Isḥāq, Ḥunayn. *The Book of the Ten Treatises on the Eye*. *See* Meyerhof.

Ilardi, Vincent, "Renaissance Florence: The Optical Capital of the World," *Journal of European Economic History* 22 (1993): 507-541.

Ilardi, Vincent. *Renaissance Vision from Spectacles to Telescopes*. Memoirs of the American Philosophical Society 259. Philadelphia: American Philosophical Society, 2007.

Jolivet, Jean, "L'Intellect selon al-Fārābi: quelques remarques." *Philosophie médiévale arabe et latine*, pp. 211-219. Paris: Vrin, 1995.

Kelso, Carl J., Jr., ed. and trans., "Witelonis Perspectivae liber quartus," Ph.D. diss., University of Missouri, 2003.

Kemp, Martin. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*. New Haven: Yale University Press, 1990.

Kepler, Johannes. *Kepler's Conversation with Galileo's Sidereal Messenger*. Trans. Edward Rosen. New York: Johnson Reprint, 1965.

Kepler, Johannes. *Ad Vitellionem paralipomena*. *See* Donahue.

Kheirandish, Elaheh. *The Arabic Version of Euclid's Optics*. New York: Springer Verlag, 1999.

Kohl, Karl, "Über den Aufbau der Welt nach Ibn al-Haitam," *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, 54-55 (1922-1923): 140-179.

Kohl, Karl, "Über das Licht des Mondes, eine Untersuchung von Ibn al-Haitham," *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, 56-57 (1924-1925): 305-398.

Korteweg, D. J. *See* Huygens.

Krause, Max, ed. and trans. *Die Sphârik von Menelaos aus Alexandrien in der Verbesserung von Abū Naṣr Manṣūr.* Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen, philol.-hist. Kl., 3 Folge, vol. 17. Berlin: Weidmann, 1936.

Kretzmann, Norman, Anthony Kenney, and Jan Pinborg, eds. *The Cambridge History of Later Medieval Philosophy*. Cambridge: Cambridge University Press, 1982.

Kuhn, Thomas. *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press, 1962.

Langermann, Y. Tzvi. *Ibn al-Haytham's* On the Configuration of the World. New York: Garland, 1990.

Lee, Raymond, and Alistair Fraser. *The Rainbow Bridge: Rainbows in Art, Myth, and Science*. University Park: Pennsylvania State University Press, 2001.

Leff, Gordon. *The Dissolution of the Medieval Outlook: An Essay on Intellectual and Spiritual Change in the Fourteenth Century*. New York: New York University Press, 1976.

Lejeune, Albert. *L'Optique de Claude Ptolémée*. Leiden: Brill, 1989.

Leonardo da Vinci. *See* Richter.

Lindberg, David C., "Roger Bacon's Theory of the Rainbow: Progress or Regress?" *Isis*, 57 (1966): 235-248.

Lindberg, David C., "The Cause of Refraction in Medieval Optics," *British Journal for the History of Science* 4 (1968): 23-38.

Lindberg, David C., ed. and trans. *John Pecham and the Science of Optics*. Madison: University of Wisconsin Press, 1970.

Lindberg, David C., "Introduction" to the 1972 reprint of *Opticae thesaurus*. *See* Risner.

Lindberg, David C. *A Catalog of Medieval and Renaissance Optical Manuscripts*. Toronto: Pontifical Institute of Mediaeval Studies, 1975.

Lindberg, David C. *Theories of Vision from Al-Kindī to Kepler*. Chicago: University of Chicago Press, 1976.

Lindberg, David C., "The Transmission of Greek and Arabic Learning," *Science in the Middle Ages*, David C. Lindberg, ed., pp. 52-90. Chicago: Chicago University Press, 1978.

Lindberg, David C., "Medieval Latin Theories of the Speed of Light," *Roemer et la Vitesse de la Lumière*, René Taton, ed. Paris: Vrin, 1978.

Lindberg, David C., ed. and trans. *Roger Bacon's Philosophy of Nature: A Critical Edition, with English Translation, Introduction, and Notes, of* De multiplicatione specierum *and* De speculis comburentibus. Oxford: Clarendon Press, 1983.

Lindberg, David C., "Optics in Sixteenth-Century Italy," *Novità celesti e crisi del sapere*, supplement to *Annali dell'Istituto e Museo di Storia della Scienza* 2 (1983): 131-148.

Lindberg, David C., "The Genesis of Kepler's Theory of Light: Light Metaphysics from Plotinus to Kepler," *Osiris* 2 (1986): 5-42.

Lindberg, David C., "Lenses and Eyeglasses." *Dictionary of the Middle Ages*, vol. 7, Joseph Strayer, ed., pp. 538-541. New York: Scribner's, 1986.

Lindberg, David C. *The Beginnings of Western Science: The European Scientific Tradition in Philosophical, Religious, and Institutional Context, 600 B.C to A.D. 1450.* Chicago: University of Chicago Press, 1992.

Lindberg, David C., ed. and trans. *Roger Bacon and the Origins of* Perspectiva *in the Middle Ages*. Oxford: Clarendon Press, 1996.

Lohne, J. A., "Alhazens Spiegelproblem," *Nordisk Matematisk Tidskrift* 18 (1970): 5-35.

Long, R. James, ed. *Bartholomaeus Anglicus, De proprietatibus rerum, Books 3-4: On the Properties of Soul and Body*. Toronto: Pontifical Institute of Mediaeval Studies, 1979.

Maier, Anneliese. *Die Vorläufer Galileis im 14. Jahrhundert: Studien zur Naturphilosophie der Spätscholastik,* 2nd ed. Rome: Edizioni di storia e letteratura, 1966. *See also* Sargent.

Manetti, Vincent. *Vita di Filippo di Ser Brunelleschi*. *See* Saalmann and Enggass.

Maurer, Armand. *Medieval Philosophy*. New York: Random House, 1962.

Maurolyco, Francesco. *Photismi de lumine et umbra ad perspectivam et radiorum incidentiam facientes*. Naples, 1611.

May, Margaret Talmadge, trans. *Galen on the Usefulness of the Body*. Ithaca, NY: Cornell University Press, 1968.

McMurrich, J. Playfair. *Leonardo da Vinci, the Anatomist (1452-1519)*. Baltimore: Williams & Wilkins, 1930.

Menelaus of Alexandria. *Sphaerica*. *See* Krause.

Meyerhof, Max, ed. and trans. *The Book of the Ten Treatises on the Eye Ascribed to Ḥunain ibn Isḥāq*. Cairo, 1928.

Milhaud, Gaston. *Descartes savant*. Paris: Librairie Félix Alcan, 1921.

Millás Vallicrosa, José Maria. *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo*. Madrid, 1942.

Mogenet, Joseph, ed. *Autolycus de Pitane. Histoire du texte suivie de l'édition critique de Traitées de la sphère en mouvement and de levers et couches*. Receuil de travaux d'histoire et de philologie de l'Université de Louvain 3, 37. Louvain, 1950.

Moody, E. A., "Empiricism and Metaphysics in Medieval Philosophy," *Philosophical Review* 67 (1958): 145-163.

Morrow, Glenn R., trans. *Proclus: A Commentary on the First Book of Euclid's Elements*. Princeton: Princeton University Press, 1970.

Naẓīf, Muṣṭafā. *al-Ḥasan ibn al-Haytham, buḥūthuhu wa kushūfuhu al-baṣariyya*. Cairo: Fouad I University, 1942-1943.

Nebbia, Giorgio, "Ibn al-Haytham nel millesimo anniversario della nascita," *Physis* 9 (1967): 165-214.

Newhauser, Richard, "Der 'Tractatus moralis de oculo' des Petrus von Limoges und seine *Exempla,*" *Exempel und Exempel-sammlungen*, Walter Haug and Burghard Wachinger, eds., pp. 95-136. Tübingen: Max Niemeyer Verlag, 1991.

Nuñes, Pedro. *De crepusculis liber unus . . . Item Allacen . . . De causis crepusculorum liber unus, a Gerardo Cremonensi iam olim latinitate donatus*. Lisbon, 1542.

Oldenbourg, Henry. *The Correspondence of Henry Oldenbourg*. 9 vols. A. R. Hall and M. B. Hall, eds. Madison: University of Wisconsin Press, 1965-1973.

Omar, Saleh. *Ibn al-Haytham's Optics: A Study of the Origins of Experimental Science*. Minneapolis: Bibliotheca Islamica, 1977.

Panofsky, Erwin. *Meaning in the Visual Arts*. Garden City, NY: Doubleday Anchor, 1955.

Pecham, John. *Perspectiva communis*. See Lindberg.

Phillips, Heather, "John Wyclif and the Optics of the Eucharist," *From Ockham to Wyclif*, Anne Hudson and Michael Wilks, eds., pp. 245-258. Oxford: Basil Blackwell, 1987.

Porta, Giambattista della. *Magiae Naturalis libri XX.* Naples, 1589.

Porta, Giambattista della. *De refractione opticae partes: libri novem.* Naples, 1593.

Proclus. *See* Morrow.

Pseudo-Euclid. *See* Björnbo and Vogel.

Ptolemy. *Almagest. See* Toomer.

Ptolemy. *Optics. See* Smith.

Ptolemy. *Planetary Hypotheses. See* Goldstein.

Rashed, Roshdi, "Le 'Discours de la Lumière' d'Ibn al-Haytham (Alhazen)," *Revue d'histoire des sciences et de leurs applications* 21 (1968): 197-224.

Rashed, Roshdi, "La modèle de la sphère transparente et l'explication de I'arc-en-ciel," *Revue d'histoire des sciences et de leurs applications* 23 (1970): 109-140.

Rashed, Roshdi, "Futhitos et al-Kindī sur 'l'illusion lunaire'," in ΣΟΦΙΗ ΜΑΙΗΤΟΡΕΣ *"Chercheurs de sagesse": Hommage à Jean Pépin.* Paris: Institut d'Études Augustiniennes, 1972.

Rashed, Roshdi, "Lumière et vision: L'Application des mathématiques dans l'optique d'Ibn al Haytham," *Roemer et la vitesse de la lumière,* René Taton, ed. Paris: Vrin, 1978.

Rashed, Roshdi, "A Pioneer in Anaclastics: Ibn Sahl on Burning Mirrors and Lenses," *Isis* 81 (1990): 464-491.

Rashed, Roshdi. *Géometrie et dioptrique au Xe siècle: Ibn Sahl, al-Quhi, and Ibn al-Haytham.* Paris: Les Belles Lettres, 1992.

Rashed, Roshdi. *Les mathématiques infinitésimales du IXe au XIe siécle,* vol. 2. London: Al-Furqan, 1993.

Rashed, Roshdi, and Jean Jolivet, eds. *Oeuvres philosophiques et scientifiques d'al-Kindī,* vol. 2. Leiden: Brill, 1998.

Reeves, Eileen Adair. *Painting the Heavens: Art and Science in the Age of Galileo.* Princeton: Princeton University Press, 1997.

Reeves, Eileen Adair. *Galileo's Glassworks: The Telescope and the Mirror.* Cambridge, MA: Harvard University Press, 2008.

Rehren, Thilo, and Papakhristu, O., "Cutting-Edge Technology–The Ferghana Process of medieval crucible steel smelting," *Metalla* 7 (2000): 55-69.

Richter, Jean-Paul, ed. *The Literary Works of Leonardo da Vinci.* 2 vols. London/New York: Oxford University Press, 1939.

Riedl, Clare C. *Robert Grosseteste on Light.* Marquette University Press: Milwaukee, 1942.

Risner, Friedrich, ed. *Opticae thesaurus. Alhazeni arabis libri septem, nunc primum editi. Eiusdem liber De crepusculis et Nubium ascensionibus. Item Vitellonis thuringopoloni libri X.* Basel, 1572; reprint, New York: Johnson Reprint, 1972.

Risner, Friedrich. *Opticae libri quattuor ex voto Petri Rami novissimo . . .* Kassel, 1606.

Ronchi, Vasco. *Optics: The Science of Vision.* Trans. Edward Rosen. New York: New York University, 1957.

Rosen, Edward. *See* Kepler, Ronchi.

Ross, Helen E., and Cornelius Plug. *The Mystery of the Moon Illusion.* Oxford: Oxford University Press, 2002.

Rouse, Mary A., and Richard H. Rouse. *Authentic Witnesses: Approaches to Medieval Texts and Manuscripts.* Notre Dame, IN: University of Notre Dame Press, 1991.

Saalmann, Howard, and Catherine Enggass, ed. and trans. *Vita di Filippo di Ser Brunelleschi by Vincent Manetti.* University Park: Pennsylvania State University Press, 1970.

Sabra, A. I., "Ibn al-Haytham's Criticisms of Ptolemy's Optics," *Journal of the History of Philosophy* 4 (1966): 145-149.

Sabra, A. I., "The Authorship of the *Liber de crepusculis*, an 11th-Century Work on Atmospheric Refraction," *Isis* 58 (1967): 77-85.

Sabra, A. I., "Ibn al-Haytham," *Dictionary of Scientific Biography*, vol. 6, ed. Charles Coulston Gillispie, pp. 189-210. New York: Scribner's, 1972.

Sabra, A. I., ed., "Treatise on the Marks Seen on the Surface of the Moon," *Journal for the History of Arabic Science* 1 (1977): 5-19.

Sabra, A. I., "Form in Ibn al-Haytham's Theory of Vision," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 5 (1980): 115-140.

Sabra, A. I. *Theories of Light from Descartes to Newton*. London: Oldbourne, 1967. Reprint, Cambridge: Cambridge University Press, 1981.

Sabra, A. I., "Ibn al-Haytham's Lemmas for Solving 'Alhazen's Problem,'" *Archive for History of Exact Sciences* 26 (1982): 299-324.

Sabra, A. I., ed. *Ibn al-Haytham, Al-Manāẓir I-II-III = Kitāb-al-Manāẓir. Books I-II-III.—On Direct Vision*. Kuwait: National Council for Culture, Arts and Letters, 1983.

Sabra, A. I., "Psychology versus mathematics: Ptolemy and Alhazen on the moon illusion." *Mathematics and its applications to science and natural philosophy in the Middle Ages*, Edward Grant and John Murdoch, eds., pp. 217-247. Cambridge: Cambridge University Press, 1987.

Sabra, A. I., trans. *The Optics of Ibn al-Haytham: Books I-III on Direct Vision*. 2 vols. London: Warburg Institute, 1989.

Sabra, A. I., "On Seeing the Stars II: Ibn al-Haytham's 'Answers' to the 'Doubts' Raised by Ibn Ma'dan," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 10 (1995/96): 1-59.

Sabra, A. I., "One Ibn al-Haytham or Two? An Exercise in Reading the Bio-Bibliographical Sources," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 12 (1998): 1-40.

Sabra, A. I., "Ibn al-Haytham's Revolutionary Project in Optics: The Achievement and the Obstacle," *The Enterprise of Science in Islam: New Perspectives*, Jan P. Hogendijk and A. I. Sabra, eds., pp. 85-118. Cambridge, MA: MIT Press, 2003.

Saliba, George. *Islamic Science and the Making of the European Renaissance*. Cambridge, MA: MIT Press, 2007.

Sargent, Steven D., ed. and trans. *On the Threshold of Exact Science: Selected Writings of Anneliese Maier on Late Medieval Natural Philosophy*. Philadelphia: University of Pennsylvania Press, 1982.

Sarton, George. *An Introduction to the History of Science*. 2 vols. Baltimore: Williams & Wilkins, 1927.

Sarton, George. *A History of Science*. 2 vols. New York: Norton, 1970.

Schechner, Sara, "Between Knowing and Doing: Mirrors and Their Imperfections in the Renaissance," *Early Science and Medicine* 10 (2005): 137-162.

Schmidt, W., ed. and trans. *Heronis Alexandrini Opera Quae Supersunt Omnia*, vol. 2. Leipzig: Teubner, 1900.

Shastid, Thomas, "History of Ophthalmology." *The American Encyclopedia and Dictionary of Ophthalmology*, vol. 11, Casey A. Wood, ed., pp. 8524-8904. Chicago: Cleveland Press, 1917.

Simon, Gérard, "Derrière le miroir," *Le Temps de la Réflexion* 2 (1981): 298-332.

Simon, Gérard, "L'Optique d'Ibn al-Haytham et la tradition Ptoléméenne," *Arabic Sciences and Philosophy* 2 (1982): 203-235.

Simon, Gérard. *Le regard, l'être et l'apparence dans l'Optique de l'Antiquité.* Paris: Seuil, 1988.

Simon, Gérard. *Sciences et savoirs aux XVIe et XVIIe siècles.* Paris: Presses Universitaires du Septentrion, 1996.

Simon, Gérard. *L'Archéologie de la vision: L'optique, le corps, le peinture.* Paris: Seuil, 2003.

Simon, Gérard, "L'Expérimentation sur la réflexion et la réfraction chez Ptolémée at Ibn al-Haytham," *De Zenon d'Élée à Poincaré,* Régis Morelon and Ahmad Haznawi, eds., pp. 355-375. Louvain/Paris: Peeters, 2004.

Simon, Gérard, "The Gaze of Ibn al-Haytham," *Medieval History Journal* 9 (2006): 89-98.

Smith, A. Mark, "Getting the Big Picture in Perspectivist Optics," *Isis* 72 (1981): 568-589.

Smith, A. Mark, "Ptolemy's Search for a Law of Refraction: A Case-Study in the Classical Methodology of 'Saving the Appearances' and its Limitations," *Archive for History of Exact Sciences* 26 (1982): 221-240.

Smith, A. Mark, ed. and trans. *Witelonis Perspectivae liber quintus.* Studia Copernicana 23. Wrocław: Ossolineum Press, 1983.

Smith, A. Mark. *Descartes's Theory of Light and Refraction: A Discourse on Method.* Transactions of the American Philosophical Society 77.3. Philadelphia: American Philosophical Society, 1987.

Smith, A. Mark, "Alhazen's Debt to Ptolemy's Optics," *Nature, Experiment, and the Sciences,* T. H. Levere and W. R. Shea, eds., pp. 147-164. Dordrecht: Kluwer, 1990.

Smith, A. Mark, "Knowing Things Inside Out: The Scientific Revolution from a Medieval Perspective," *American Historical Review* 95 (1990): 726-744.

Smith, A. Mark, "Picturing the Mind: The Representation of Thought in the Middle Ages and Renaissance," *Philosophical Topics* 20 (1992): 149-170.

Smith, A. Mark, "The Latin Version of Ibn Mu'adh's Treatise 'On Twilight and the Rising of Clouds,'" *Arabic Sciences and Philosophy* 2 (1992): 83-132.

Smith, A. Mark, "Extremal Principles in Ancient and Medieval Optics," *Physis* 31 (1994): 113-140.

Smith, A. Mark. *Ptolemy's Theory of Visual Perception.* Transactions of the American Philosophical Society 86.2. Philadelphia: American Philosophical Society, 1996.

Smith, A. Mark, "Ptolemy, Alhazen, and Kepler and the Problem of Optical Images," *Arabic Sciences and Philosophy* 8 (1998): 8-44.

Smith, A. Mark, "The Physiological and Psychological Grounds of Ptolemy's Visual Theory: Some Methodological Consideration," *Journal of the History of the Behavioral Sciences* 34 (1998): 231-246.

Smith, A. Mark. *Ptolemy and the Foundations of Ancient Mathematical Optics.* Transactions of the American Philosophical Society 89.3. Philadelphia: American Philosophical Society, 1999.

Smith, A. Mark, "The Latin Source of the Fourteenth Century Italian Translation of Alhacen's *De aspectibus* (Vat. Lat. 4595)," *Arabic Sciences and Philosophy* 11 (2001): 27-43.

Smith, A. Mark, ed. and trans. *Alhacen's Theory of Visual Perception*. Transactions of the American Philosophical Society 91.5 and 6. Philadelphia: American Philosophical Society, 2001.

Smith, A. Mark, "Ptolemy, Alhacen, and Ibn Mu'adh and the Problem of Atmospheric Refraction," *Centaurus* 45 (2003): 100-115.

Smith, A. Mark, "What is the History of Medieval Optics Really About?" *Proceedings of the American Philosophical Society* 148 (2004): 180-194.

Smith, A. Mark, "Reflections on the Hockney-Falco Thesis: Optical Theory and Artistic Practice in the Fifteenth and Sixteenth Centuries," *Early Science and Medicine* 10 (2005): 163-185.

Smith, A. Mark, ed. and trans. *Alhacen on the Principles of Reflection*. Transactions of the American Philosophical Society 96.2 and 3. Philadelphia: American Philosophical Society, 2006.

Smith, A. Mark, "Le *De aspectibus* d'Alhazen: Révolutionnaire ou réformiste?" *Revue d'histoire des sciences* 60.1 (2007): 65-81.

Smith, A. Mark, ed. and trans. *Alhacen on Image-Formation and Distortion in Mirrors*. Transactions of the American Philosophical Society 98.1. Philadelphia: American Philosophical Society, 2008.

Smith, A. Mark, "Petrus Hispanus' 'Treatise on the Eyes'," *O tratado dos olhos de Pedro Hispano*, Smith, A. Mark and Arnaldo Pinto Cardoso, pp. 9-56. Lisbon: Alétheia Editores, 2008.

Smith, A. Mark, "The Measure of Vision in the Middle Ages," *Micrologus*. Forthcoming.

Smith, A. Mark, and Bernard R. Goldstein, "The Medieval Hebrew and Italian Versions of Ibn Mu'adh's 'On Twilight and the Rising of Clouds,'" *Nuncius* 8 (1993): 613-643.

Smith, J. A. *See* Aristotle, *De anima*.

Spencer, John R., trans. *Leon Battista Alberti on Painting*. New Haven: Yale University Press, 1966.

Steffens, Bradley. *Ibn al-Haytham: First Scientist*. Greensboro, NC: Morgan Reynolds, 2007.

Stock, Brian. *The Implications of Literacy: Written Language and Models of Interpretation in the Eleventh and Twelfth Centuries*. Princeton: Princeton University Press, 1983.

Stratton, George M., trans. *Theophrastus and the Greek Physiological Psychology Before Aristotle*. London: Allen and Unwin, 1917.

Tachau, Katherine. *Vision and Certitude in the Age of Ockham: Optics, Epistemology and the Foundations of Semantics, 1250-1345*. Leiden: E. J. Brill, 1988.

Takahashi, Ken'ichi. *The Medieval Latin Traditions of Euclid's* Catoptrica. Fukuoku: Kyushu University Press, 1992.

Taliaferro, R. Catesby. *On* Conic Sections Books I-III *by Apollonius of Perga*. Great Books of the Western World Reprints, 1952.

Taton, René, ed. *Roemer et la Vitesse de la Lumière*. Paris: Vrin, 1978. *See* Lindberg, Rashed.

Theoderic of Freiberg. *De iride et radialibus impressionibus*. *See* Boyer, Lee and Fraser, and Wallace.

Theophilus. *De diversis artibus. See* Hawthorne and Smith.

Theophrastus. *On the Senses. See* Stratton.

Thiesen, Wilfred. "The Mediaeval Tradition of Euclid's Optics," PhD dissertation: University of Wisconsin, 1974.

Thompson, Daniel V., Jr., trans. *Cennino d'Andrea Cennini,* The Craftsman's Handbook. New Haven: Yale University Press, 1933.

Tideus. *See* Björnbo and Vogel.

Toomer, G. J. *Diocles on Burning Mirrors.* Berlin, Heidelberg, New York: Springer, 1976.

Toomer, G. J., trans. *Ptolemy's Almagest.* Princeton: Princeton University Press, 1998.

Travaglia, Pinella. *Magic, Causality and Intentionality: The Doctrine of Rays in Al-Kindī.* Micrologus Library, vol. 3. Florence: Edizioni del Galuzzo, 1999.

Unguru, Sabetai, "A Very Early Acquaintance with Apollonius of Perga's Treatise on Conic Sections in the Latin West," *Centaurus* 20 (1976): 112-128.

Unguru, Sabetai, ed. and trans. *Witelonis* Perspectivae *liber Primus.* Studia Copernicana 28. Wrocław: Ossolineum Press, 1977.

Unguru, Sabetai, ed. and trans. *Witelonis* Perspectivae *liber secundus et liber tertius.* Studia Copernicana 15. Wrocław: Ossolineum Press, 1991.

Van Helden, Albert. *Measuring the Universe.* Chicago: University of Chicago Press, 1985.

Van Helden, Albert, trans. *The Sidereal Messenger.* Chicago: University of Chicago Press, 1989.

Van Riet, Simone, ed. *Avicenna Latinus: Liber de anima seu sextus de naturalibus I-II-III.* Louvain: E. Peeters, 1972.

Vitruvius. *De architectura. See* Granger.

Vollgraff, J. A. *See* Huygens.

Voss, Don L., "Ibn al-Haytham's *Doubts Concerning Ptolemy*: a Translation and Commentary," PhD dissertation: University of Chicago, 1985.

Wallace, William A. *The Scientific Methodology of Theodoric of Freiberg*, Studia Friburgensia, n.s. 26. Fribourg: Fribourg University Press, 1959.

Wallace, William A. *Causality and Scientific Explanation*, vol. 1. Ann Arbor: University of Michigan Press, 1972.

Webster, E. W. *See* Aristotle, *Meteorology*.

Wedin, Michael V. *Mind and Imagination in Aristotle*. New Haven: Yale University Press, 1988.

Wiedemann, Eilhard, "Über das Licht der Sterne nach Ibn Al Haitham," *Wochenschrift für Astronomie, Meteorologie und Geographie*, ns, 33 (1890): 129-133.

Wiedemann, Eilhard, "Ibn al-Haitam, ein arabischer Gelehrter," *Festschrift J. Rosenthal*. Leipzig, 1906, pp. 147-178.

Wiedemann, Eilhard, "Über ein Schrift von Ibn al Haitam 'Über die Beschaffenheit der Schatten,'" *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen* 39 (1907): 226-248.

Wiedemann, Eilhard, "Ibn al-Haitams Schrift über die sphärischen Hohlspiegel," *Bibliotheca Mathematica* ser. 3, 10 (1909-1910): 293-307.

Wiedemann, Eilhard, "Theorie des Regensbogens von Ibn al Haitam," *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen* 46 (1914): 39-56.

Wiedemann, Eilhard, "Über die Camera obscura bei Ibn al Haitam," *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen* 46 (1914): 155-169.

Wiedemann, Eilhard. *Aufsätze zur arabischen Wissenschafts-Geschichte*, vol. 2. Hildesheim: Georg Olms, 1970.

Willach, Rolf. *The Long Route to the Invention of the Telescope*. Philadelphia: American Philosophical Society, 2008.

Winter, H. J. J., and W. 'Arafat, "Ibn al-Haitham on the Paraboloidal Focussing Mirror," *Journal of the Royal Asiatic Society of Bengal* ser. 3, 15 (1949): 25-40.

Winter, H. J. J., and W. 'Arafat, "A Discourse on the Concave Spherical Mirror of Ibn al-Haitham," *Journal of the Royal Asiatic Society of Bengal* ser. 3, 16 (1950): 1-16.

Wippel, John F., "The Condemnations of 1270 and 1277 at Paris," *Journal of Medieval and Renaissance Studies* 7 (1977): 169-201.

Witelo. *See* Donahue, Kelso, Smith, Unguru.

Wolf-Devine, Celia. *Descartes on Seeing: Epistemology and Visual Perception.* Carbondale and Edwardsville, IL: Southern Illinois University, 1993.

Wolfson, H. A., "The Internal Senses in Latin, Arabic, and Hebrew Philosophical Texts," *Harvard Theological Review* 25 (1935): 69-133.

GENERAL INDEX

Alhacen on Refraction: A Critical Edition, with English Translation and Commentary, of Book 7 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*

A. Mark Smith

The completion of the Latin version of Alhacen's *De Aspectibus* represents a monumental achievement in the history of optical theory. The influence of this work on the Perspectivist tradition in the West cannot be underestimated. Professor Smith's edition and translation will take its place alongside other essential texts in the history of optics by Roger Bacon and John Pecham, both of whom were influenced by Alhacen.

Richard Newhauser
Professor of English
Arizona State University

Mark Smith's publication of the Latin version of Ibn al-Haytham's *Optics* is one of the great contributions to the history of science of our time. The edition is based upon an exhaustive examination of the manuscripts, the translation of the difficult, and at times obscure, text a model of clarity, and the introduction and commentary are exemplary in placing the work within the history of optics and explaining its technicalities and difficulties. This is a work of scholarship that will endure and be consulted for ages.

Noel Swerdlow
Professor Emeritus of Astronomy and Astrophysics and of History
University of Chicago
Visiting Professor, Division of Humanities and Social Sciences
California Institute of Technology

Earlier books on Alhacen's optics by A. Mark Smith

Alhacen's Theory of Visual Perception: A Critical Edition, with English Translation and Commentary, of the First Three Books of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*—Transactions of the American Philosophical Society, Volume 91, Parts 4 (ISBN: 0-87169-914-1) and 5 (ISBN: 0-87169-915-X)

AWARDED THE 2001 JOHN FREDERICK LEWIS AWARD

Alhacen on the Principles of Reflection: A Critical Edition, with English Translation and Commentary, of Books 4 and 5 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*—Transactions of the American Philosophical Society, Volume 96, Parts 2 (ISBN: 0-87169-962-1) and 3 (ISBN: 0-87169-963-X)

Alhacen on Image-Formation and Distortion in Mirrors: A Critical Edition, with English Translation and Commentary, of Book 6 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*—Transactions of the American Philosophical Society, Volume 98, Part 1, sections 1 (ISBN: 978-1-60618-981-8) and 2 (ISBN: 978-1-60618-986-3)

WINNER OF THE JOHN FREDERICK LEWIS AWARD FOR 2010

ISBN 978-160618006-8
53500
9 781606 180068

Presented by the American Philosophical Society